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State of the regarding stiffness modulus for reinforced concrete elements

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Summary

The paper aims a comparative synthesis of the values currently considered to affect the stiffness modulus of the reinforced concrete elements. Both national and international norms were considered, but reference research works is also presented. The article concludes with a case study that shows that there are behaviour differences for a reinforced concrete frame in the national and international codes.

Keyword: stiffness modulus, elasticity modulus, moment of inertia.

1. INTRODUCTION

Most of the reinforced concrete structures are statically indeterminate structures, so the structural elements stiffness influence not only the displacements but also the distribution of the forces in the elements. The structural walls, columns and beams respond in the second working stage, the cracked state. For this reason, it is necessary to consider the stiffness corresponding to stage II, cracked, when designing such a structure.

If approximate values of stiffnesses, obtained by affecting the stiffness modules of the section with subunit factors, are used, the structure's overall displacements assessment have acceptable results from the point of view of reflecting the actual behaviour. (Postelnicu, 2012)

The subunit factors given in the national and international codes practically reduce the values of the elasticity modulus or geometric characteristics of the section.

2. RESEARCH METHODOLOGY

2.1. In national codes

In Romania, according to the seismic design code, Part I, Indicative P100-1 / 2013 [7], the reduction subunit factor of the stiffness modulus is given by the structure type structure (frames or structural walls). For reinforced concrete frames structures, the nature of the connections between the non-structural components and the reinforced concrete structure is taken into account. Table 2.1. present the possible values of this factor.

Table 2.1. Stiffness modulus design values according to P100-1/2013

Structure type	The number of connections between the non-structural components and the reinforced concrete structure	
	The non-structural components account for the assembly stiffness	The non-structural components do not influence the structure
Reinforce concrete structures		
Frame structures	$E_c I_g$	$0{,}50 \cdot E_c I_g$
Walls structures	$E_c I_g$	

Where E_c - concrete modulus of elasticity and I_g - the moment of inertia for the uncracked concrete section.

2.2. In international codes

The values from the Canadian design code for reinforced concrete buildings, indicative CAN/CSA-A23.3-04 [3], revised July 2007, are based on constituent elements for reinforced concrete structures, Table 2.2.

Table 2.2. Stiffness modulus design values according to CAN/CSA23.3-04/2007

Beams	$0{,}35 \cdot I_g$
Columns	$0{,}70 \cdot I_g$
Walls (uncracked)	$0{,}70 \cdot I_g$
Walls (cracked)	$0{,}35 \cdot I_g$
Slabs	$0{,}25 \cdot I_g$

Where I_g is the moment of inertia of the uncracked concrete section.

The *American Concrete Structure Design Code*, ACI 318-08 [1], provides either the values from Table 2.3, which depend on the element type, or are calculated based on element load (compression or bending).

Table 2.3. Stiffness modulus design values according to ACI 318-08

Beams	$0{,}35 \cdot I_g$
Columns	$0{,}70 \cdot I_g$
Walls (uncracked)	$0{,}70 \cdot I_g$
Walls (cracked)	$0{,}35 \cdot I_g$
Slabs	$0{,}25 \cdot I_g$

For elements subjected to compression, the effective moment of inertia can be computed based on (2.1) relation, and those subjected to bending by (2.2) relation.

$$0{,}35 \cdot I_g \geq I = \left(0{,}8 + 0{,}25 \cdot \frac{A_{st}}{A_g}\right) \cdot \left(1 - \frac{M_u}{P_u \cdot h} - 0{,}5 \cdot \frac{P_u}{P_o}\right) \cdot I_g \leq 0{,}875 \cdot I_g \quad (2.1)$$

$$0{,}25 \cdot I_g \geq I = \left(0{,}1 + 0{,}25 \cdot \rho\right) \cdot \left(1{,}2 - 0{,}5 \cdot \frac{b_w}{d}\right) \cdot I_g \leq 0{,}5 \cdot I_g \quad (2.1)$$

Where: I_g - the moment of inertia of the uncracked concrete section, A_g - the area of the uncracked concrete section, A_{st} - the uncompressed reinforcement area, M_u - the bending moment resulted from loads combination, P_u - the axial force resulted loads combination, P_o - the nominal axial strength at zero eccentricity, h - section height, b_w - section width, and d - distance from compressed area to the center of tensioned reinforcement.

The American Society of Civil Engineers, using the existing seismic rehabilitation standard for existing buildings, ASCE / SEI 41-06 [2], recommends values depending the type of structural elements, similar to ACI 318-08 [1], but with larger subunit factors, Table 2.4.

Table 2.4. Stiffness modulus design values according to ASCE/SEI 41-06

Beams	$0{,}50 \cdot I_g$
Columns	$0{,}70 \cdot I_g$
Walls (uncracked)	$0{,}80 \cdot I_g$
Walls (cracked)	$0{,}50 \cdot I_g$

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The *New Zealand standard*, NZS 3101-1/2016 [6], provides (2.3) equation, which takes into account the elasticity modulus and the moment of inertia of the uncracked concrete section in relation to the normalized axial force. For the calculation accuracy, the characteristics of the reinforcement are considered in equation (2.4).

$$EI = \frac{\left(E_c I_g / 2{,}5 \right)}{1 + \beta_d} \tag{2.3}$$

$$EI = \frac{\left(E_c I_g / 5 + E_e I_{se} \right)}{1 + \beta_d} \tag{2.4}$$

Where: I_g is the moment of inertia of the uncracked concrete section, I_{se} - the reinforcement moment of inertia, E_c - concrete modulus of elasticity, E_s - steel modulus of elasticity and β_d - axial force:

$$\beta_d = \frac{N_{Ed}}{A_c \cdot f_{cd}} \tag{2.5}$$

f_{cd} - design value for concrete compression strength;

A_c - area section for the concrete section;

N_{Ed} - design value for the axial force;

2.3. In the literature

Professor Tudor Postelnicu, in his book "Design of reinforced concrete structures in seismic areas", Volume III, 2012, presents an example of designing a high rise reinforced concrete structure, with a mixed structural system with a central core from structural walls, and an outlined from frame, on all four sides. For this structure, the design values of the stiffness modules for all categories of elements were chosen according to CR 2-1 provisions, but also based on the engineering judgment and are presented in table 2.6.

Table 2.6. Stiffness modulus design values according to Postelnicu/2012

Beams	$0{,}60 \cdot E_c I_g$
Columns	$0{,}80 \cdot E_c I_g$
Walls	$0{,}60 \cdot E_c I_g$
Slabs	$0{,}50 \cdot E_c I_g$
Coupling beams (reinforcement with diagonal cases)	$0{,}30 \cdot E_c I_g$

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3. QUANTITATIVE AND QUALITATIVE RESULTS. DISSCUSSIONS

A modal and a pushover analysis were performed where the stiffness modulus of the frame elements was affected according to the synthesis presented in chapter 2 of the paper. For this, it was considered a single opening reinforced concrete frame structure. The bay dimensions is of 6 m and the structure has four levels of 3 m height. The column section is 50x50cm, reinforced with 12 longitudinal bars of Ø16 and Ø8 stirrups. The beams cross-section is 30x60 cm, reinforced both, at the top and bottom, with 2 Ø16 and 1 bar Ø10, and Ø8 stirrups. The live loads are represented by a uniformly distributed load of 50 kN / m applied on the beams, meanwhile the dead loads are of 50 kN / m.

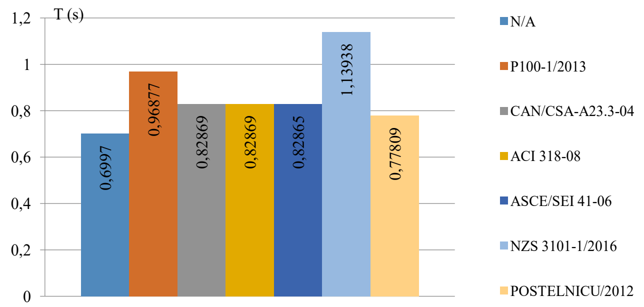

Fig. 3.1. Results synthesis – fundamental period comparison

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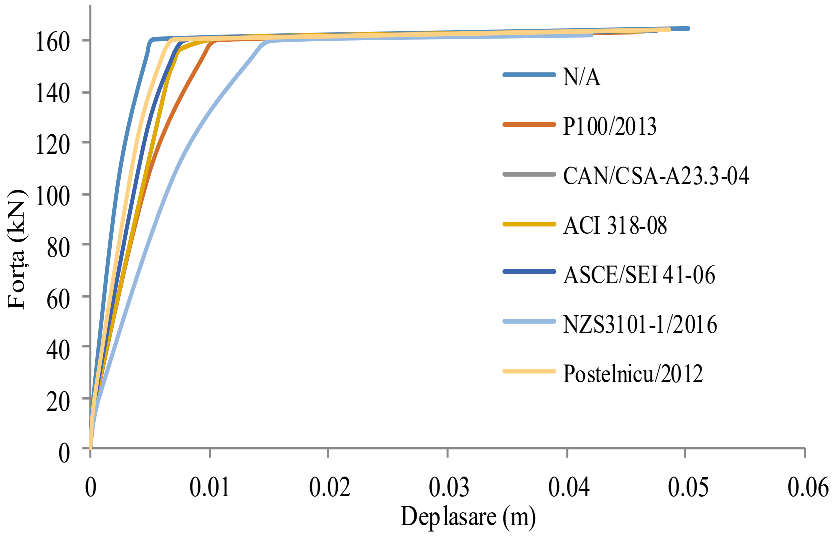

Fig. 3.2. Results synthesis, capacity curves comparison (pushover analysis)

Table 3.1. Results synthesis, pushover capacity curve comparison

	Displacement [m]	Force [kN]
N/A	0.050218	164.632
P100-1/2013	0.045761	163.27
CAN/CSA-A23.3-04	0.046566	163.65
ACI 318-08	0.046566	163.65
ASCE/SEI 41-06	0.04755	163.864
NZS 3101-1/2016	0.042101	162.098
POSTELNICU/2012	0.048618	164.167

4. CONCLUSIONS AND FURTHER RESEARCH DIRECTIONS

Reinforced concrete frames structures are flexible ones. It is specific for this type of structure that the larger the vibration period is, higher dynamic gains occur. A modal and a static non-linear analysis (pushover), were considered, in which the no damage structure, N/A, and the damaged structure were considered according to different international codes, respectively specialized literature. Based on the obtained results on the analyzed structure it is noticed that the values for the fundamental period are significantly different from the American codes, and they are higher by 15.5% compared to the no damage structure, (N/A). According to the Romanian code, the fundamental period of vibration is higher by 27.8% compared to the no damaged structure and by 14.4% respectively against the American codes.

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There is a significant increase in the period according to the New Zealand code, which is 38.6% higher than the N/A case.

Based on the obtained results from the nonlinear static analysis, the maximum lateral force recorded is sensitively differentiated between the simple frame and the damaged one according to the example given by T. Postelnicu, but the maximum displacement is lower by about 2.8%, in the latter case. A higher difference is observed when applying the provisions of the New Zealand Code, lateral force and displacement being by 1.5%, respectively by 19.2% lower than N/A.

As future research directions, it is desirable to determine a efficient value for the subunit reduction factor of the stiffness modulus

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