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Analytical Evaluation of the Flexural Capacity of Steel Beams Strengthened with Bonded CFRP Composite Strips

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Summary

Traditional strengthening solutions of steel beams usually consist in supplementing the base material in specific locations, for which, certain parameters are not fulfilled any more. For the most common cases, these strengthening solutions imply large consumptions of construction materials and high costs. Strengthening solutions based on carbon fibre reinforced polymer (CFRP) composites have gained much interest in the last years, because of the superior mechanical properties of the latter and due to their simple and rapid technique of application. This paper presents the analytical evaluations of the flexural capacity of a steel beam which has been strengthened using a (CFRP) strip.

KEYWORDS: steel beams, flexural capacity, adhesively bonded CFRP composite strips.

1. INTRODUCTION

Traditional strengthening schemes of steel beams consist in welding or mechanically fastening steel plates or profiles in those sections for which the resisting capacity is smaller than the effective applied forces. Other conventional solutions consist in encasing the steel element, either partially or totally, in concrete, or changing the static scheme of the element by introducing supplementary components, like bracings or props and even intermediate supports [1, 2, 3]. Also, for the cases in which the strength or the stiffness of the beam are affected due to local degradations, the strengthening solution my consist in replacing the damaged sections with new ones.

For most of the traditional solutions, the strengthening process implies large consumptions of materials, labour and costs. Moreover, the strengtheing process usually comes with the incapacity of using the building, which may amplify the financial impact of the project. Using composite materials in strengtheing processes of steel beams may turn into a feasable solution, first because of the great mechanical properties of these materials (high ratio between strength and specific weight) and second, because of the execution phase which can be done in a very short time with minimum impact upon the activities that are run inside the building.

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Strengthening solutions of steel beams based on CFRP composite elements are among the newest methods, the first steel structure which has been strengthened with this technique is the Tickford Bridge from UK, in 1999 [4].

Steel beams flexurally strengthened with CFRP composite elements have been analysed by numerous research teams, by either analytical and numerical methods or experimental tests [5, 6, 7, 8]. The results of the experimental programs have confirmed the efficiency of this strengthening solution, the flexural capacities of the beams being improved from 15% up to 85%, depending on the amount and quality of the CFRP material that was used and being also influenced by the bonding agent. Also, in many cases, a good correlation between the analytical models and the experimental results has been confirmed [9].

2. DESIGN EXAMPLE

For a better illustration of the strengthening effect for a steel beam, a design example is presented in this section. Thus, a simply supported steal beam is considered, with geometrical characteristics being presented in Figure 1. The bottom flange has been strengthened with a CFRP composite strip bonded with an adhesive characterized by a non-linear bond-slip model presented in Figure 2. The mechanical properties of the materials are briefly inserted in Table 1. The beam is subjected to a concentrated force, in the midspan and in order to emphasise the effects of the strengthening scheme, the lateral buckling of the beam and the premature debonding failure of the adhesive are not taken into account.

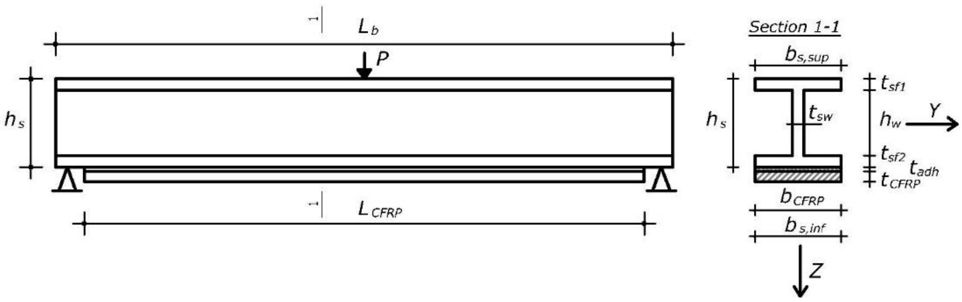

Figure 1. Layout of the FRP plated steel beam

Table 1. Geometrical and mechanical properties of the components

Property	Notation	Value	Unit
Steel beam			
Span	L_b	4100	mm
Height	h_s	270	mm
Top flange width	$b_{s,sup}$	150	mm
Bottom flange width	$b_{s,inf}$	150	mm

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Thickness of the top flange	t_{sf1}	12	mm
Thickness of the bottom flange	t_{sf2}	12	mm
Thickness of the web	t_{sw}	10	mm
Height of the web	h_w	246	mm
Cross-sectional area	A_s	6060	mm^2
Yield strength	f_y	235	MPa
Young's modulus	E_s	200000	MPa
Yielding strain	ε_{sy}	0.001175	-
Safety factor	γ_s	1	-
Elastic modulus in Y direction	$W_{y,el}$	535974.66	mm^3
Plastic modulus in Y direction	$W_{y,pl}$	615689.94	mm^3
Capable elastic bending moment	$M_{y,el,cap}$	125.954	KNm
Capable plastic bending moment	$M_{y,pl,cap}$	144.687	KNm
Epoxy adhesive			
Thickness	t_{adh}	1	mm
Modulus of elasticity	E_{adh}	1750	MPa
Tensile strain energy	R	0.139	MPa mm/mm
Slip at the end of the elastic stage	δ_1	0.0163	mm
Slip at the end of the constant stage	δ_2	0.809	mm
Ultimate slip	δ_f	1.36	mm
Safety factor	γ_{adh}	1.25	-
CFRP composite strip			
Thickness	t_{cfrp}	1.4	mm
Length	L_{cfrp}	4000	mm
Width	b_{cfrp}	150	mm
Tensile strength	f_{cfrp}	2900	MPa
Modulus of elasticity	E_{cfrp}	170000	MPa
Safety factor	γ_{cfrp}	1.25	-

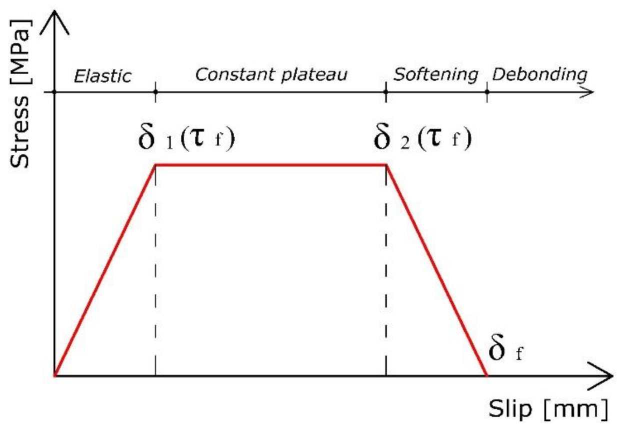

Figure 2. Bond-slip model for non-linear adhesive

2.1. Calculating the ultimate strain of the CFRP composite strip

$$\varepsilon_{cfrpl,d} = \min(\varepsilon_{cfrp,rup,d}; \varepsilon_{cfrp,l,d}) \tag{1}$$

where:

- - $\varepsilon_{cfrpl,d}$ is the design ultimate strain of the CFRP composite strip;
- - $\varepsilon_{cfrp,rup,d}$ is the design strain corresponding to the rupture of the CFRP composite strip;
- - $\varepsilon_{cfrp,l,d}$ is the design strain corresponding to the intermediate debonding of the CFRP composite strip.

The design strain corresponding to the rupture of the CFRP composite strip is calculated with Equation 2.

$$\varepsilon_{cfrp,rup,d} = \frac{f_{CFRP}}{\gamma_{CFRP} E_{CFRP}} = 0.013647 \tag{2}$$

The design strain corresponding to the intermediate debonding of the CFRP composite strip is calculated with Equations 3 and 4.

$$\varepsilon_{cfrp,l,d} = \frac{1}{\gamma_{adh}} \sqrt{\frac{2G_f}{E_{CFRP} t_{CFRP}}} = 0.00807 \tag{3}$$

G_f is the interfacial fracture energy and is calculated with Equation 4.

$$G_f = 628 t_{adh}^{0.5} R^2 = 12.1335 \text{ Nmm/mm}^2 \tag{4}$$

Equation 3 is valid only if the anchorage length of the CFRP strip (L_a) is greater than the effective one (L_e).

Because the critical section of the beam is at the mid-span, the anchorage length is calculated with Equation 5.

$$L_a = \frac{L_{CFRP}}{2} = 2000 \text{ mm} \tag{5}$$

The effective anchorage length (L_e) is calculated with Equation 6.

$$L_e = a_d + b_e + \frac{1}{\lambda_1} \ln \frac{1+C}{1-C} = 187.318 \text{ mm} \tag{6}$$

Factors λ, λ_1, λ_3, a_d, b_e and C are obtained with Equations 8, 9, 10, 11 and 12.

$$\lambda = \sqrt{\frac{\tau_{\max}^2}{2G_f}\left(\frac{1}{E_{CFRP} t_{CFRP}} + \frac{b_{CFRP}}{E_S A_s}\right)} = 0.00569 \text{ mm}^{-1} \tag{7}$$

where:

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- $\tau_{\max} = 0.9 f_{adh}$.

$$\lambda_1^2 = \frac{2G_f}{\tau_{\max}\delta_1}\lambda^2 = 0.05985\ \text{mm}^{-1} \tag{8}$$

$$\lambda_3 = \sqrt{\lambda^2 \frac{2G_f}{\tau_{\max}(\delta_f - \delta_2)}} = 0.01029\ \text{mm}^{-1} \tag{9}$$

$$a_d = \frac{1}{\lambda_1}\sqrt{((2\frac{\delta_2}{\delta_1} - 1) - 1)} = 148.910\ \text{mm} \tag{10}$$

$$b_e = \frac{1}{\lambda_3}\arcsin\left[\frac{\lambda_3\lambda}{0.97\delta_1\lambda_1^2}(\delta_f - \delta_2)\right] = 59.006\ \text{mm} \tag{11}$$

$$C = \frac{\lambda_3}{\lambda_1\delta_1}(\delta_f - \delta_2)\cot(\lambda_3 b_e) - \lambda_1 a_d = -0.5486\ \text{mm} \tag{12}$$

Since L_e is smaller than L_a, Equation 3 can be used and the design ultimate strain of the CFRP strip $\varepsilon_{cfrpl,d} = \varepsilon_{cfrp,l,d} = 0.00807$.

2.2. Determining the capable bending moment of the composite section

The capable bending moment of the composite sections can be obtained using the classical mechanical theories by imposing the assumption of total plasticization of the steel section. Still, for steel sections reinforced with CFRP strips, it has been observed that when the capable moment is reached, the steel section is not completely plasticized, the mid-part of the web being in elastic stage [1]. Figure 3 presents the variation of the stresses and strains on the composite cross-section.

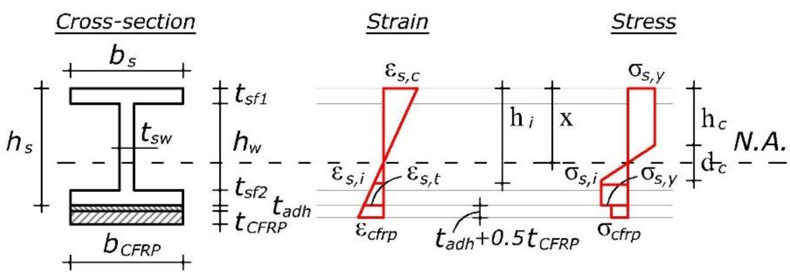

Figure 3. Variation of strains and stresses on the composite cross-section

where:

- - $\varepsilon_{s,c}$ is the strain at the limit of the compressed steel section;
- - $\varepsilon_{s,t}$ is the strain at the limit of the tensioned steel section;

- - ε_{cfrp} is the strain in the mid-section of the CFRP strip;
- - σ_{cfrp} is the average axial stress in the CFRP strip;
- - $\sigma_{s,y}$ is the yielding stress of the steel;
- - $\varepsilon_{s,i}$ is the strain in the steel at h_i height;
- - $\sigma_{s,i}$ is the stress in the steel at h_i height;
- - x is the depth of the neutral axis;
- - h_c is the height of the plasticized steel section;
- - d_e is the height of the elastic steel section;

The depth of the neutral axis is unknown and, for that reason, in the first stage, its value is imposed as x = 150mm. This value will be verified when the equilibrium conditions are check on the composite cross-section. Based on the design ultimate strain of the CFRP strip and on the depth of the neutral axis, the strains at the boundaries of the steel sections, $\varepsilon_{s,c}$ and $\varepsilon_{s,t}$, can be calculated, using Equations 13 and 14, respectively.

$$\varepsilon_{s,c} = \varepsilon_{cfrpl,d} \frac{-x}{h_s + t_{adh} + \frac{t_{cfrp}}{2} - x} = 0.0099 \text{ mm} \tag{13}$$

$$\varepsilon_{s,t} = \varepsilon_{cfrpl,d} \frac{h_s - x}{h_s + t_{adh} + \frac{t_{cfrp}}{2} - x} = -0.0079 \text{ mm} \tag{14}$$

The strain of the steel at h_i depth can be calculated with Equation 15.

$$\varepsilon_{s,i} = \varepsilon_{cfrpl,d} \frac{h_i - x}{h_s + t_{adh} + \frac{t_{cfrp}}{2} - x} \tag{15}$$

The height of the compressed zone, is obtained by assuming in Equation 15 that $\varepsilon_{s,i} = \varepsilon_{s,y}$.

$$h_c = \frac{\varepsilon_{s,y}(h_s + t_{adh} + t_{cfrp}/2 - x)}{\varepsilon_{cfrpl,d}} + x = 132.2982 \text{ mm} \tag{16}$$

The height of the elastic zone, is obtained with Equation 17.

$$d_e = 2(x - h_c) = 35.4036 \text{ mm} \tag{17}$$

The position of the neutral axis is checked by balancing Equation 18.

$$\int_0^h \sigma_{s,i} b_i dh_i + \sigma_{cfrp} b_{cfrp} t_{cfrp} = 0 \tag{18}$$

which, in an explicit form, can be written as:

$$\frac{f_y}{\gamma_s} t_{sf1} b_{s,\sup} + \frac{f_y}{\gamma_s}(h_c - t_{sf1}) t_{sw} + \frac{f_y}{2\gamma_s}(x - h_c) t_{sw} - \frac{f_y}{2\gamma_s}(x - h_c) t_{sw} -$$
$$-\frac{f_y}{\gamma_s}(h_s - t_{sf2} - d_e - h_c) t_{sw} - \frac{f_y}{\gamma_s} t_{sf2} b_{s,\inf} - \varepsilon_{cfrp} E_{cfrp} b_{cfrp} t_{cfrp} = -217889.4339 \neq 0 \tag{18*}$$

Since the equilibrium of Equation 18 has not been achieved, the correct value of the depth is obtained using an iterative process, by applying successive values for x.

It has been found that, for x = 196.36, Equation 18 becomes:

$$\frac{f_y}{\gamma_s} t_{sf1} b_{s,\sup} + \frac{f_y}{\gamma_s}(h_c - t_{sf1}) t_{sw} + \frac{f_y}{2\gamma_s}(x - h_c) t_{sw} - \frac{f_y}{2\gamma_s}(x - h_c) t_{sw} -$$
$$-\frac{f_y}{\gamma_s}(h_s - t_{sf2} - d_e - h_c) t_{sw} - \frac{f_y}{\gamma_s} t_{sf2} b_{s,\inf} - \varepsilon_{cfrp} E_{cfrp} b_{cfrp} t_{cfrp} = 2.6131 \cong 0 \tag{18*}$$

where:

- - $\varepsilon_{s,c}$ = 0.02105;
- - $\varepsilon_{s,t}$ = -0.00789;
- - h_c = 185.4014 mm;
- - d_e = 21.9170 mm

The capable moment of the composite section is evaluated with Equation 19.

$$M_{ib,d} = \int_0^h \sigma_{s,i} b_{s,i} \left(\frac{h_s}{2} - h_i\right) dh_i + E_{cfrp} \varepsilon_{cfrp} b_{cfrp} t_{cfrp} \left(\frac{h_s}{2} + t_{adh} + \frac{t_{cfrp}}{2}\right) \tag{19}$$

Which, in explicit form, can be written as:

$$
\begin{aligned}
M_{ib,d} = {} & t_{sf1} b_{s,\sup} \frac{f_y}{\gamma_s}\left(\frac{h_s}{2} - \frac{t_{sf1}}{2}\right) + (h_c - t_{sf1}) t_{sw} \frac{f_y}{\gamma_s}\left(\frac{h_s}{2} - \left(t_{sf1} + \frac{h_c - t_{sf1}}{2}\right)\right) + \\
& + \frac{d_e}{2} t_{sw} \frac{f_y}{2\gamma_s}\left(\frac{h_s}{2} - \left(t_{sf1} + (h_c - t_{sf1}) + \frac{2}{3}\frac{d_e}{2}\right)\right) + \\
& + \frac{d_e}{2} t_{sw} \left(-\frac{f_y}{2\gamma_s}\right)\left(\frac{h_s}{2} - \left(x + \frac{d_e}{2}\frac{1}{3}\right)\right) + \\
& + (h_s - h_c - d_e - t_{sf2}) t_{sw} \left(-\frac{f_y}{2\gamma_s}\right)\left(\frac{h_s}{2} - \left(x + \frac{d_e}{2} + \left(\frac{h_s - h_c - d_e - t_{sf2}}{2}\right)\right)\right) + \\
& + t_{sf2} b_{s,\inf} \left(-\frac{f_y}{2\gamma_s}\right)\left(\frac{h_s}{2} - \left(h_s - \frac{t_{sf2}}{2}\right)\right) + \\
& + E_{cfrp}(-\varepsilon_{cfrp}) b_{cfrp} t_{cfrp}\left(\frac{h_s}{2} - \left(h_s + t_{adh} + \frac{t_{cfrp}}{2}\right)\right) = 175.256\,\text{KNm}
\end{aligned}
\tag{19*}
$$

3. CONCLUSIONS

The application of CFRP composite strips on the bottom flange of I-shaped steel beams subjected to bending, is an efficient strengthening solution. For the case st that has been presented in this paper, the 1.4 mm thick and 150 mm width CFRP composite strip that has been adhesively bonded to the bottom side of the steel beam, produced an increase of the capable bending moment of 21.13%.

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