

Uncertainties in seismic hazard probabilistic analysis

Dan Cretu¹, Andrei Pricopie¹, Liviu Crainic²

¹Department of Strength of Materials, Bridges and Tunnels, Technical University of Civil Engineering, Bucharest, 020396, Romania

²Department of Reinforced Concrete Structures, Technical University of Civil Engineering, Bucharest, 020396, Romania

Structural codes for seismic analysis and design imply an accurate (as far as possible) assessment of the parameters which quantify the severity of seismic action as felt on each location of the national territory. Probabilistic seismic hazard analysis (PSHA) is nowadays preferred by most codes. Cornell-McGuire methodology is usually accepted.

Although PSHA seems to be an advanced and logical approach, its intensive use along many years evidenced intrinsic sensitivities leading to unexpected distortions with respect to a real phenomenon.

The goal of the paper is to examine the sensitivity of PSHA, according to the model used within the Romanian Seismic Design Code P100, for different input data. A MATLAB program has been written for this purpose, which allowed determining the mean recurrence interval (MRI) for different earthquake magnitudes.

KEYWORDS: structural code, seismic analysis, seismic hazard, probabilistic seismic hazard analysis (PSHA), mean recurrence interval (MRI), earthquake magnitude..

1. INTRODUCTION

Parameters which describe earthquake features and their effects within a seismic zone show a pronounced random character. Consequently, the use of statistic/probabilistic concepts and methods was a logical trend in quantifying the seismic hazard for structural analysis and design. For the last decades, probabilistic seismic hazard analysis (PSHA) according to Cornell-McGuire methodology ([2],[15]) is preferred by most structural codes.

Although PSHA seems to be an advanced and logical approach, its intensive use along many years evidenced intrinsic sensitivities leading to unexpected distortions with respect to a real phenomenon. Two families of uncertainties define the random character of the PSHA: (a) aleatory uncertainties related to the variability of natural phenomena and (b) epistemic uncertainties - due to the insufficient accuracy of the modelling. Epistemic uncertainties can be improved through



Dan Cretu, Andrei Pricopie, Liviu Crainic

alternative assumptions, better theoretical models or use of more reliable parameters within an accepted model.

Romanian seismic design codes [5] and [6] are based on “modern” PSHA procedure for seismic hazard assessment. The PSHA model implemented in these codes is the “classical” Cornell-McGuire one. The recurrence relationships and the predictive (attenuation) relations are those proposed by Lungu [9], [12].

In an attempt to suggest a further way to enhance the actual codes version, the present paper aims to identify some sources of sensitivity of the PSHA model implemented within them. A MATLAB program was compiled and thoroughly tested for this purpose. The influence of the following parameters was examined: the value of the maximum credible earthquake moment magnitude, the site distance to the hypocenter, the focal depth, the site distance to the epicentre and the standard deviation of the ground motion attenuation relationship.

Only the seismic hazard due to Vrancea sub-crustal earthquakes has been investigated.

2. THEORETICAL BACKGROUNDS

According to Cornell-McGuire methodology ([2], [11], [15]), the probability that a ground shaking parameter Y exceeds a certain threshold y^* can be determined using the total probability theorem:

$$P[Y > y^*] = P[Y > y^* | X] P[X] = \int P[Y > y^* | X] f_x(X) dx \quad (1)$$

where X is a vector of the random variable Y .

The following parameters have been considered within the vector X : the moment magnitude M_w , the epicentre distance R and the focal depth H . These are accepted to be independent variables and, as consequence, the probability density function $f_x(X)$ can be determined as a product of three independent functions, i.e. the product of three independent integrals:

$$P[Y > y^*] = \int \int \int P[Y > y^* | m, r, h] f_M(m) f_R(r) f_H(h) dm dr dh \quad (2)$$

$P[Y > y^* | m, r, h]$ is obtained from the predictive (attenuation) function itself depending on the three random independent functions $M(m)$, $R(r)$ and $H(h)$. The Lungu's relationship has been accepted ([12]):

$$\ln PGA = b_0 + b_1 M_w + b_2 \ln R + b_3 R + b_4 h + \varepsilon \quad (3)$$



Uncertainties in seismic hazard probabilistic analysis

where:

$PGA = y^*$ is the peak ground acceleration at the site,

M_w - the earthquake moment magnitude,

$R = \sqrt{h^2 + d^2}$ - the site distance to the focus,

h, d - the focal depth and the site distance to the epicentre, respectively,

$b_0 \div b_4$ - regression coefficients,

ε - random variable with zero mean value and the standard deviation $\sigma_{\ln,PGA}$.

Using the records of the earthquakes of 1977, 1986 and 1990, Lungu determined through multiple regression the follow coefficients for Bucharest [9]:

$$b_0 = 1.685 \quad b_1 = 1.181 \quad b_2 = -1.0 \quad b_3 = 0.002 \quad b_4 = -0.005 \quad \varepsilon = 0.461 \quad (4)$$

It should be noted that d represents the site distance to any point of the seismogenic zone. It is assumed that any point within the seismogenic zone has the same occurrence probability of an earthquake having the moment magnitude between $M_{w,\min} = 6.3$ and $M_{w,\max} = 8.1$.

$f_M(m), f_R(r), f_H(h|m)$ are normal probability density functions for moment magnitude, epicentre distance and focal depth.

Recurrence function $f_H(h|m)$ shows the relationship between moment magnitude and focal depth. Using the linear regression of data for M_w within domain 6.3-8.1, one gets:

$$\ln h = -0.866 + 2.846 \ln M_w \quad (5)$$

with a standard deviation $\sigma_{\ln,h} = 0.18$.

For a site with N_S zones with potential occurrence of earthquakes, each of them having a magnitude exceeding mean rate of

$$v_i = e^{\alpha_i - \beta_i M_{w,\min}} \quad (6)$$

$$\alpha_i = 8.657 \quad \beta_i = 1.687$$

$$M_{w,\min} = 6.3$$

the number of earthquakes in one year is

$$n = 10^{3.76 - 0.73m} \quad (7)$$

The mean rate of exceeding for all considered seismogenic zones will be



Dan Cretu, Andrei Pricopie, Liviu Crainic

$$\lambda_y = \sum_{i=1}^{N_S} v_i \int \int \int P[Y > y^* | m, r, h] f_{M_i}(m) f_{R_i}(r) f_{H_i}(h|m) dm dr dh \quad (8)$$

In order to solve the integrals, each density functions is accepted to be a normal distribution. Functions h and Y are supposed to be lognormal, but applying the logarithm they become a normal distribution.

The integrals are solved through numerical procedure accepting that the functions are constant over the integral steps $\Delta m, \Delta r, \Delta h$. Accordingly, the mean exceeding rate is:

$$\lambda_y = \sum_{i=1}^{N_S} \sum_{j=1}^{N_M} \sum_{k=1}^{N_R} \sum_{l=1}^{N_H} v_i P[Y > y^* | m_j, r_k, h_l] f_{M_i}(m_j) f_{R_i}(r_k) f_{H_i}(h_l | m_j) \Delta m \Delta r \Delta h \quad (9)$$

relationship equivalent with:

$$\lambda_y = \sum_{i=1}^{N_S} \sum_{j=1}^{N_M} \sum_{k=1}^{N_R} \sum_{l=1}^{N_H} v_i P[Y > y^* | m_j, r_k, h_l] P(M = m_j) P(R = r_k) P(H = h_l | m_j) \quad (10)$$

The procedure accepts that each source generates only N_M earthquakes of moment magnitude m_j , over only N_R distances source-site r_k and only N_H focal distances h_l .

For a single seismogenic source results:

$$\lambda_{y,i} = \sum_{j=1}^{N_M} \sum_{k=1}^{N_R} \sum_{l=1}^{N_H} v_i P[Y > y^* | m_j, r_k, h_l] P(M = m_j) P(R = r_k) P(H = h_l | m_j) \quad (11)$$

The Gutenberg-Richter truncated probability density function has been implemented:

$$f_M(m) = \frac{\beta e^{-\beta(m - M_{w,\min})}}{1 - e^{-\beta(M_{w,\max} - M_{w,\min})}} \quad (12)$$

where $\beta = 1.687$, $M_{w,\min} = 6.3$ and $M_{w,\max} = 8.1$.

3. COMPUTER MATLAB PROGRAM

The probabilistic seismic hazard analysis (PSHA) performed within the present research uses a MATLAB program specifically developed for this purpose. The program involves two modules: POZITIE and PSHA01.



Uncertainties in seismic hazard probabilistic analysis

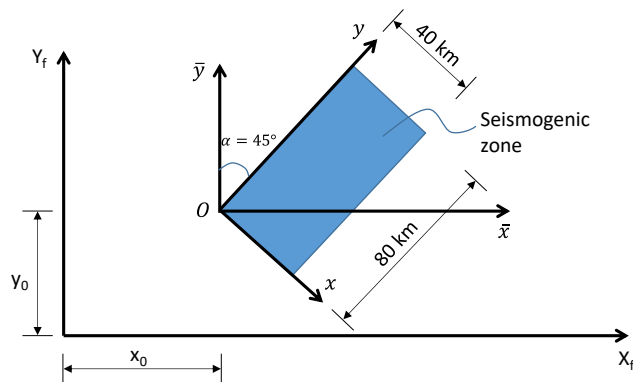


Figure 1. Seismogenic zone Vrancea

The first module, POZITIE, provides the coordinates of potential epicentres which could be developed within the seismogenic zone VRANCEA, supposed to be a rectangle of 40 x 80 km x km according to [9] (Figures 1 and 2).

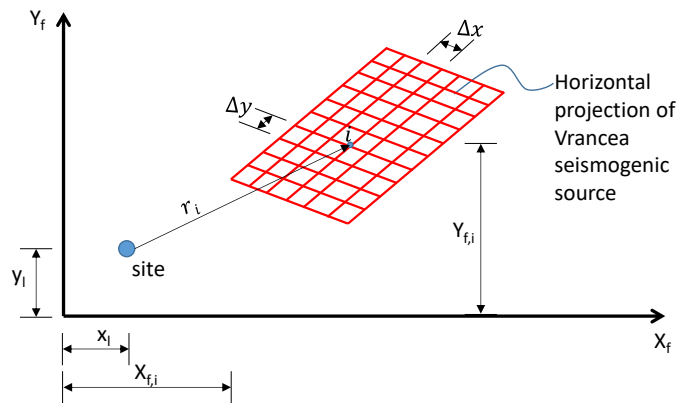


Figure 2. Horizontal projection of fault rupture surface

The second module, PSHA01, determines the probability that the peak ground acceleration (PGA) in a location exceeds a certain threshold, according to the Cornell-McGuire methodology above described. In a first step, the PGA probability density function is determined as a product of three random independent functions, i.e. the site distance to the epicentre d , the site distance to the hypocenter r and the focal depth h (Figure 3):



Dan Cretu, Andrei Pricopie, Liviu Crainic

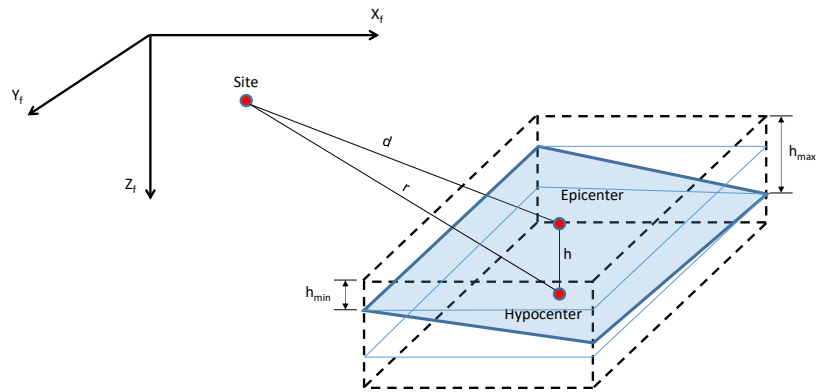


Figure 3. Three dimensional seismogenic zone

The next program step determines the temporal distribution of earthquakes using the Gutenberg-Richter recurrence law with the parameters α and β according to [9]. Supposing that the moment magnitude is a random function of normal distribution within the interval $M_{w,\min} - M_{w,\max}$, the density moment function is determined through Gutenberg-Richter truncated relationship.

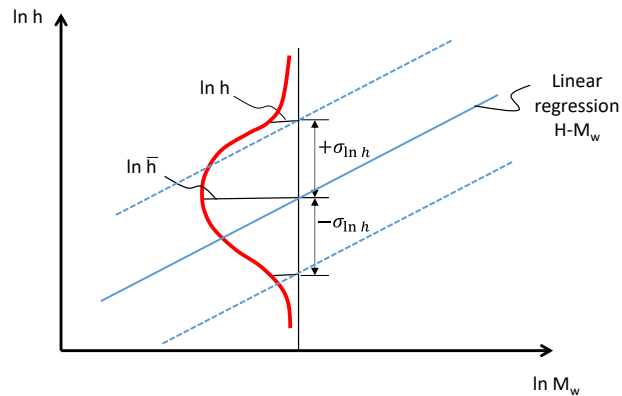


Figure 4. Linear regression between focal depth and moment magnitude

The random variable H of the focal depth is accepted to be a lognormal function (Figure 4). The relative frequency of the ratio h/m is obtained after applying the logarithm of the function H with the relationship:

$$f_H(h|m)\Delta h = \frac{\Delta h}{\sqrt{2\pi} \cdot h \cdot \sigma_{\ln h}} e^{-\frac{1}{2} \left(\frac{\ln h - \ln \bar{h}}{\sigma_{\ln h}} \right)^2} \quad (13)$$



Uncertainties in seismic hazard probabilistic analysis

The peak ground acceleration (PGA) is a random variable too, having a spatial distribution quantified through attenuation (predictive) relationships. Within present study, the attenuation relationship deduced by Lungu [9] has been used.

The mean rate of exceeding for all considered seismogenic zones is obtained through probability's summation of all random variables involved, according to the theorem of total probability. The result is the value of mean exceeding the rate for each discrete magnitude of PGA. The inverse of this value is the mean recurrence interval (MRI).

Thus, the program calculates the seismic hazard curve $PGA = f(MRI)$ and then, through linear interpolation, the discrete PGA magnitudes for MRI = 50, 100, 225, 475, and 975 years, respectively.

4. PERFORMED INVESTIGATIONS AND COMMENTS

The uncertainties in determining the parameters magnitude which defines the ground movement are generally associated with the location on which earthquake of significant magnitude occur, to the moment magnitude of potential earthquakes, to their recurrence rate, and to the accepted attenuation (predictive) relationship.

Other uncertainties are generated by the adopted calculation algorithm i.e. are of mathematical nature. Accordingly, the first step in the present research was to elaborate a computer program using the Cornell-McGuire Methodology [11] and to test it thoroughly. The attenuation relationships and the coefficients of nonlinear regression are those described in [9].

Table 1 contains the values of PGA for different MRI given by the computer program and, between brackets, the reference values taken from [20]. A difference of 5-15% between these values can be noticed.

In modern approach, PSHA uses, in attenuation relationship, the constant ε which expresses the uncertainty in prediction of PGA through independent random variables

$$\ln(PGA) = f(M, R, h) + \varepsilon = f(M, R, h) + n\sigma_{\ln PGA} \tag{14}$$

The magnitude of the standard deviation has a significant influence on the prediction of PGA, being a source of underestimation or overestimation of the parameters which define the seismic movement of the ground.

The step size of the numerical integration, namely Δm , Δr , Δh and ΔPGA have a small influence on the value of PGA, around 1%.



Dan Cretu, Andrei Pricopie, Liviu Crainic

Table 1. Calculated and reference PGA values for different MRI

Site-city	MRI =50	MRI=100	MRI=225	MRI=475	MRI=975	Data
Bucharest	158.9754	208.0345	271.6799	335.7867	402.2741	Bucharest
	158.3220	203.3545	261.0905	319.5041	380.7808	All data
	130.0899	157.6611	191.2836	222.8324	253.7478	0.50e
	142.1885	176.7198	219.8386	261.8401	304.6103	0.75e
	201.1098	280.1626	388.8639	506.6457	637.9049	1.50e
	259.4	395.2	598.8	836.9	1119.600	2.00e
		(213.97)	(289.54)			
Focșani	31.5139	41.8058	55.1549	69.6843	84.2528	Moldova
	55.6168	325.7026	414.1323	502.8896	595.6003	All data
	216.0891	277.5522	356.1851	435.5087	518.2142	Bucharest
		(292.487)	(392.6)			
Craiova	122.7789	162.4327	213.9023	265.6092	318.9603	Bucharest
	91.5325	118.4423	152.9939	188.2252	225.0098	All data
			(130.54)	(175.69)		
Caracal	126.2931	166.9326	219.8266	272.7046	327.3241	Bucharest
	98.6700	127.5023	164.6501	202.4717	242.1101	All data
			(140.36)	(189.43)		
Alexandria	134.2207	177.0261	232.5675	288.4301	345.9634	Bucharest
	113.6824	146.8124	189.4942	232.5494	277.8805	All data
			(163.91)	(214.95)		
Giurgiu	137.1806	180.8080	237.2512	294.0542	352.7319	Bucharest
	119.2197	153.6020	198.0142	242.9569	290.2834	All data
			(165.87)	(223.78)		
Ploiești	186.7080	242.1606	313.5630	385.6461	460.6751	Bucharest
	205.3517	262.9570	336.1176	410.0404	487.3537	All data
			(261.08)	(343.53)		
Iași	18.1899	24.3180	33.2640	42.6143	52.5674	Moldova
	129.4446	166.6598	214.5745	263.2086	314.2280	All data
			(169.8)	(229.67)		
Bacău	24.4484	33.1260	44.4248	56.3256	69.4168	Moldova
	191.5639	245.2579	313.7019	382.8400	455.3249	All data
			(247.34)	(321.93)		
Suceava	15.2868	21.6276	30.1162	37.8275	46.4803	Moldova
	108.1574	139.6631	180.0319	220.8840	263.6801	All data
			(145.262)	(196.3)		
Galați	172.1986	224.2652	291.6845	359.6702	430.1375	Bucharest
	180.4735	231.0209	295.4076	360.6649	429.0020	All data
			(229.671)	(291.506)		
Constanța	105.5233	136.1400	175.4553	215.3162	257.1190	All data
	107.4000	143.4325	189.8742	235.0499	280.8351	Dobrogea
			(147.225)	(194.337)		
Mangalia	94.5187	122.2251	157.7652	193.7246	231.5442	All data
	124.1572	164.2037	216.2364	268.5336	322.2857	Dobrogea
			(154.096)	(192.374)		



Uncertainties in seismic hazard probabilistic analysis

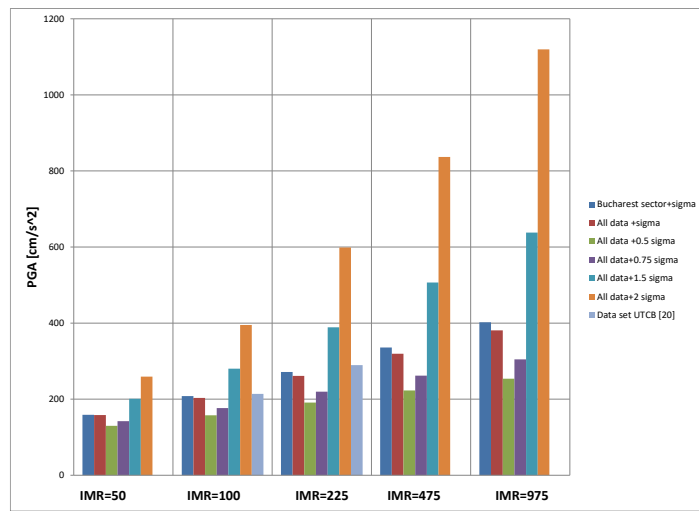


Figure 5. PGA values for different MRI and uncertainty levels ϵ in Bucharest

Figure 5 shows the PGA values for different MRI and uncertainty levels ϵ in Bucharest.

The inferior moment magnitude doesn't have a significant influence on the calculated PGA while the considered maximum moment magnitude is particularly important (see Figure 6). For this reason, it would be important to accept $M_{w,max} = 7.9$ as Cliff Frohlich suggests [10] instead of $M_{w,max} = 8.1$.

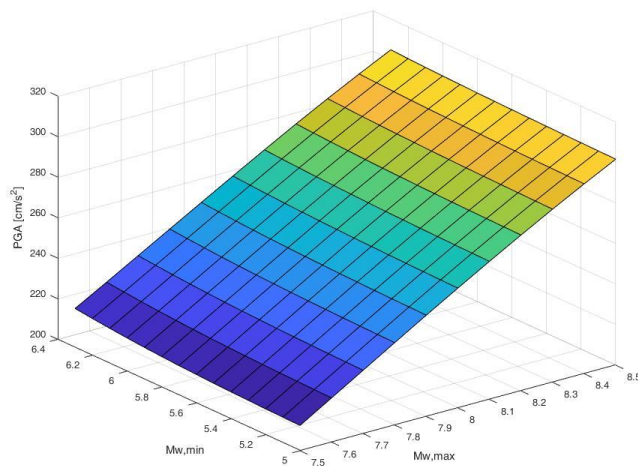


Figure 6. Variation of PGA with $M_{w,min}$ and $M_{w,max}$



Dan Cretu, Andrei Pricopie, Liviu Crainic

5. DIRECTIONS FOR FURTHER RESEARCHES

The research described in the paper, limited to the zones prone to the Vrancea earthquakes, should be considered as the first step in a more comprehensive effort aimed to improve the quantification of design seismic hazard to be implemented in next generations of codes. For this purpose, the next step is intended to be an extended research on the seismic hazard of the seismogenic zones of the Transylvanian Basin.

The Transylvanian Basin, spanning over about 40% of the Romanian territory, shows a moderate toward low seismic activity. However, it requires a specific attention. In contrast with the zones subjected to Vrancea earthquakes, the Transylvanian Basin seismicity is generated mainly by seismogenic crustal sources even though the effect of Vrancea subcrustal earthquakes could be felt (with moderate intensity) within its Eastern part. Consequently, for Transylvania, appropriate new predictive laws (attenuation relations) have to be developed in order to accurately apply PSHA. Moreover, for certain Transylvanian sites, two or more seismogenic zones should be considered according to appropriate procedures.

6. POTENTIAL ALTERNATIVE APPROACHES FOR DESIGN SEISMIC HAZARD ASSESSEMENT

The use of PSHA for seismic design codes is nowadays almost generalised throughout Europe countries as well as in the USA. However, its use along many years evidenced some important shortcomings of this approach. They have to be addressed.

A first issue which has been remarked was expressed in the following question [1]: *“Why Do Modern Probabilistic Seismic-Hazard Analyses often Lead to Increased Hazard Estimates?”* The first paper about that was followed by substantial comments [21]. It can be added that in the last version of the Romanian code the hazard estimation according to PSHA approach led to a substantial increase of design seismic force justified by an MRI transition from 100 to 225 years [4]. It is behind the scope of the present paper to extend more the discussion about this issue but it has to be mentioned.

In Italy, a country with strong seismic activity, the forecast of destructive earthquake occurrence according to seismic hazard maps compiled through PSHA procedures was dramatically contradicted by the reality of the last decades [17]. In zones considered with low seismicity according to seismic zonation maps, destructive earthquakes occurred with numerous casualties.



Uncertainties in seismic hazard probabilistic analysis

Similar situations have been encountered in Japan (Tohoku earthquake of 2011 with a Richter magnitude of 9 in a zone considered with low seismicity produced 15.000 victims and severe damage to Fukushima nuclear plant) and in Haiti - 2010.

The shortcomings of PSHA approach led to a trend of reconsidering the deterministic methods, obviously in a modern, improved manner. The NDSHA (Neo-Deterministic Seismic Hazard Analysis) developed at the University of Trieste / Italy [17] is a good example of such approach. It capitalises the updated knowledge about the Earth Geophysics and modern computational methods in Geodynamics for developing an original method to analyse the local seismic hazard which could be directly implemented in structural codes.

7. CONCLUSIONS

The accurate prediction - as far as possible - of the severity of earthquakes which potentially occur in a location within a given recurrence period is a basic key to improving the seismic structural design.

PSHA is the up-to-date approach to seismic hazard analysis implemented almost generalised in codes. However, its use along many years evidenced several shortcomings of this approach.

The present paper is focused on identify some sources of PSHA epistemic uncertainties, to quantify them, and to suggest implicitly ways to mitigate them.

The USA project Next Generation of Ground Motion Attenuation Models, a multidisciplinary research program, can be considered as a model approach to a key problem of the seismic hazard analysis for design purposes [19]. In order to mitigate the inherent subjectivity of approaches, five sets of ground motions models were developed by five teams working independently. However, interaction meetings were organised between the teams aimed to lead gradually to a consensual decision.

It is supposed that extending this research in several ways suggested within the paper could contribute to a better understanding and a more accurate assessment of the seismic hazard for design purposes.

References

1. Bommer, J., Abrahamson, N. A., *Why Do Modern Probabilistic Seismic-Hazard Analyses often Lead to increased Hazard Estimates?*, Bulletin of the Seismological Society of America, Vol. 96, No. 6, pp. 1967–1977, December 2006.
2. Cornell, C., *Engineering seismic risk analysis*, Bulletin of the Seismological Society of America, 58, 1968.



Dan Cretu, Andrei Pricopie, Liviu Crainic

3. Chen W.F., Lui E.M., *Earthquake engineering for structural design*, CRC Press, Taylor & Francis, 2006.
4. *Cod de proiectare seismică – Partea I – Prevederi de proiectare pentru clădiri, indicativ P100-1/2013*, Monitorul Oficial al ROMÂNIEI, 2013, nr. 338 bis/3.IX.2013. (in Romanian)
5. *Cod de proiectare seismică – Partea I – Prevederi de proiectare pentru clădiri, indicativ P100-1/2006*, R. A. Monitorul Oficial, 2006. (in Romanian)
6. *Cod de proiectare seismică – Partea a III-a – Prevederi pentru evaluarea seismică a clădirilor existente, indicativ P100-3/2008*, Monitorul Oficial al ROMÂNIEI, nr. 647 bis/1.X.2009. (in Romanian)
7. Douglas, J., *Ground-motion prediction equations 1964-2010*, PEER Report 2011/102, BRGM.
8. Douglas J., *Estimation of Strong Ground Motion: Aleatory Variability and Epistemic Uncertainty*, Proceedings of 5-th National Conference on Earthquake Engineering and 1-st National Conference on Earthquake Engineering and Seismology Bucharest, Romania, 19-20 June 2014, Conspress.
9. Dubină, D., Lungu, D. (coordinators), *Construcții amplasate în zone cu mișcări seismice puternice*, Ed. Orizonturi Universitare, Timișoara, 2003. (in Romanian)
10. Frohlich, C., *Deep Earthquake*, Cambridge University Press, 2009.
11. Kramer, S. L., *Geotechnical Earthquake Engineering*, Prentice Hall, 1996.
12. Lungu, D., Mazzolani, F. & Savidis, S., (coordinators), *Calculul structurilor în zone seismice - EUROCODE 8, Exemple de calcul*, BRIDGEMAN Ltd. Timișoara, 1997.
13. Mărmureanu, G., Cioflan, C.O., Mărmureanu, Al., Ionescu, C., *How long time will we go with linear seismology?*, Romanian Journal of Physics, vol. 60, pg. 613-625, București, 2015.
14. Măndrescu, N., Mărmureanu, G., Radulian, M., Ionescu C., *Studiul integrat al datelor geologice, geodezice și seismice pentru evaluarea răspunsului local în zona orașului București*, Ed. Academiei Române, 2008.
15. McGuire, R., *FORTRAN computer program for seismic risk analysis*. U.S. Geological Survey Openfile Report 76-67, 1976.
16. McGuire, R. L., *Seismic Hazard and Risk Analysis*, edited by Earthquake Engineering Research Institute, 2004.
17. Peresan, A., Panza, G. E., *Improving Earthquake Hazard Assessments in Italy: An Alternative to "Texas Sharpshooting"*, EOS, no. 51, American Geophysical Union, 18 December 2012.
18. Sokolov, V. Yu, Wenzel, F., Mohindra, R., *Probabilistic seismic hazard assessment for Romania and sensitivity analysis: A case of joint consideration of intermediate-depth (Vrancea) and shallow (crustal) seismicity*, Soil Dynamics and Earthquake Engineering 29, pp. 364-381, Elsevier, 2009.
19. Tapan, K. Sen, *Fundamentals of Seismic Loading on Structures*, John Wiley&Sons, 2009.
20. Văcăreanu, R., Pavel, F., Aldea, A., Arion, C., Neagu, C., *Noi perspective și rezultate ale analizei probabilistice de hazard seismic pentru România - partea I*, AICPS Review, nr. 3/2015.
21. Wang, Z., Mai, Z., *Comment on "Why Do Modern Probabilistic Seismic-Hazard Analyses often Lead to Increased Hazard Estimates?" by Julian J. Bommer and Norman A. Abrahamson*, *Bulletin of the Seismological Society of America*, Vol. 96, No. 6, Bulletin of the Seismological Society of America, Vol. 97, No. 6, pp. 2212-2214, December 2007.
22. Wang, Z., *Seismic Hazard and Risk Assessment and Mitigation Policy in USA*, ICTP Advanced Conference on Seismic Risk Mitigation and Sustainable Development, 10-14 May 2010.

