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# Analytical differences between seven prediction models and the description of the rail track deterioration process through these methods

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Summary

*The quality of the rail track is one of the most important indicators to manage a reliable maintenance system so that plan the costs and the technical interventions. In this study, the track state will be defined by the qualifying numbers of the track, which shows the geometrical condition of a given rail section. More than one million measured data with the car (FMK-004) were processed than analysed and defined by configuring and programming a new regression method. The aim was the perfection of an analytic examination, which describes the differences between models of the track deterioration process, through characterized the correspondences more precisely and better to use in practice. Seven models were built and tested for prediction of the track qualifying numbers. One conventional non-linear regression model based on Vaszary's model using VBA, four new model using basic predictable equations, one new model using linear regression in VBA and one new model using an artificial neural network in MATLAB. When compare the predictions of models, the result shows that exist some basic models with more accurate predictions than a complex model.*

KEYWORDS: rail geometric deterioration, track dimensioning factor, measuring and qualifying numbers, curve fitting, linear and non-linear regression, artificial neural network, visual basic, matlab

## 1. INTRODUCTION

The paper starts with a presentation of predictions models problems that shows a solution for this question. Furthermore, seven prediction model will be presented and these prediction values models will be compared with the real degradation values. Finally, it will be shown results and suggestions regarding some remark about the differences between the old and some new investigated methods.

There are two geometrical denominations in connection with rail tracks. The first one is the absolute geometry, which is planned by the rail designing engineer in case of newly-built rails or rail reconstructions with plot data, cross segment data



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and length segment data and realises based on geodetic alignment [1], [2]. The second denomination is the relative rail geometry, which describes the position of rails to each other and to the basic lines and plane, so it shows the deviation compared to the error-free position [3]-[5]. The deterioration process of relative geometry is affected by several factors, which mostly come from traffic on it and from environmental effects [6]-[11]. The deterioration of the track without any regulation works can be described with the following equation:

$$C = C_0 \cdot e^{\alpha\, m\, v^2}, \qquad (1)$$

where, $C$ is the general characteristic of the geometric condition of the rail; $C_0$ is the number describing the initial condition; $\alpha$ is the so-called track dimensioning factor depending on the structural formation; $m$ is the rolling load in tonnes and $v$ is the equivalent speed.

From a technical point of view, this quality deterioration is firstly due to the interaction between the running rail vehicle and the rail track, secondly to the effect of the weather. The condition of the rail track suffers structural deterioration from the implementation time. As a result of the rolling friction the wheels and rails wear away, the tight rail fastenings loosen because of the sinking - rising movements of the track, the sleepers are pushed increasingly into the ballast bed, the rail ballast granules are forced into the protection layer or if there is no one, into the subgrade. In the case of the superposing of all these effects there will be size differences and later on, location errors evolve in the rail. The relative geometry of the rail track is described with three quantities along with others:

- alignment;
- longitudinal level;
- track twist.

Using the measuring numbers created for these three quantities, the Summary qualifying number weighted with ADded track twist (SAD) qualifying number describes the general geometric condition of the track. It is calculated to a given qualifying length (earlier 500 m, currently 200 m).

## 2. CRITICAL OBSERVATIONS

Until now in the literature, the model was given by the equation (1) created by Dr. Vaszary Pál [12] (Fig. 1). This is a clear model, the process is undisturbed by operational interferences, but differs from reality more and more as it is moved forward along the axis. The condition of the rail track cannot deteriorate forever, sometime later the deterioration function has to be tended asymptotically towards a border condition characterised by Ch value.

In reality, the geometric deterioration of the track is not an undisturbed process, because from time to time regulation work has to be performed on the line as a function of the condition (Fig. 2).

In connection with Vaszary’s formula, equation (1), it must be stated that the product is difficult to describe numerically in a lack of values of α factor, and the condition of the substructure also plays a huge role in the rail track deterioration process.

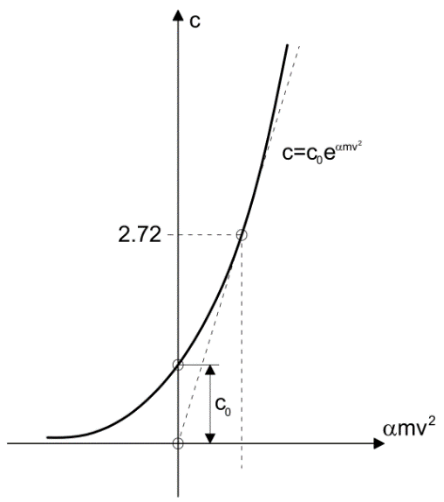


Figure 1. The equation of the track geometric deterioration according to [6]


Figure 2. The effect of the track regulation works on the deterioration process

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## 3. CREATING A NEW PROCEDURE

The FMK-004 measuring car of Hungarian State Railways Central Superstructure Examiner Ltd (MÁV KFV Ltd) provided measuring and qualifying numbers about the railway tracks at a stated frequency from the 2nd half of 1999 until the 2nd half of 2013. In the year 2013, the evaluation system has been changed, so the work with long data series is possible only with values, which were processed by FMK-004 measuring car until this year. Measuring and qualifying numbers are provided for each 500 meter length of the tracks. In Hungarian railway network there are 16.250 pieces qualifying sections with 500 meter length and altogether the data of 29 terms are available - counting with 2 measurements a year. That is 471.250 data to each measuring and qualifying number, and altogether (longitudinal level, alignment, twist and SAD) 1.885.000 data, which shows the extensiveness of the examination. After optimising the data chart and removing the incomplete lines there are 1.072.420 data remained for analysis.

It can easily be understood that it is an impossible task to analyse this amount of data with manual methods. So a method of progress was created, which correctly and valuably handles and processes this huge amount of data in a controlled way with given initial figures and boundary conditions.

### *3.1 The bases of the procedure*

As mentioned earlier, the product factors cannot be handled numerically correctly in the model, described by equation (1), so the model of the rail geometric deterioration is reformed according to the theory used at the Technical University of Graz [13], [14]:

$$C_{SAD_i} = C_{SAD_0} \cdot e^{bt} \tag{2}$$

where C value is the qualifying number C=SAD; C0 is the value of the qualifying number determined during the first measurement after maintenance.

Instead of the product $\alpha m v^2$ should be used $bt$, where b value is dimensioned factor, which depends on the super-structural parameters (e.g. rail profile, sleepers type and distance, ballast thickness) and the sub-structural parameters (e.g. E2 bearing capacity values on protection layer/sub-base) [15]-[17]; t value has elapsed time. (It should be mentioned that the worse the quality of track geometry, the deterioration process is the faster. Due to low financing of maintenance of railway tracks in Hungary, all the track faults cannot be eliminated. The extant track faults will be deteriorated forward due to the cancelled (or delayed) maintenance, and other new faults can be evolved. In case the size of the track faults exceeds the prescribed value contained the maintenance regulations related to railway tracks speed restrictions have to be used, i.e. a reduced speed is allowed in this section. At the end of this kind of sections, additional acceleration energy demand comes forward as compared to state if the train can be driven with constant speed [18]).

*3.2 The course of the procedure*

A large Excel table was created which contains all the measuring and qualifying numbers of each rail line in Hungary recorded with the FMK-004 measuring car from the second half of 1999 to the second half of 2013. The compiled large table was reduced to 1.072.420 data due to the changes mentioned above, which is the number of data taken into account.

*3.2.1 The process of the automatic procedure*

The automatic procedure (program henceforth) makes a catalogue of the lines found in the large table. It collects the initial and the final segments of the given line, the row-number of the initial and final segment in the large table and provides the number of the 500-meter sections in the given line. From this catalogue, the program knows, which fields to read when it analyses the given rail line. After reading the first line it analyses the data related to the 500-meter sections according to the description in the paragraph beginning with the analysis process of a 500-meter long qualifying section. If the analysis is completed the program goes on to the next 500-meter section.

When the last 500-meter section is analysed, all the regression equations are completed for the b dimensioned factor related to all qualifying sections.

The program is able to calculate the curve fitting with regression according to linear, exponential, power or natural based logarithm deterioration.

The program draws the distribution (n and m) of the equation parameters according to the type of curve fitting with regression one by one.

The result of linear regression calculation in parametrically:

$$y_i = n_i x + m_i \, . \qquad (3)$$

The result of exponential regression calculation parametrically:

$$y_i = m_i \, e^{n_i x} \, . \qquad (4)$$

The result of power regression calculation parametrically:

$$y_i = m_i \, x^{n_i} \, . \qquad (5)$$

The result of natural based logarithm calculation parametrically:

$$y_i = n_i \ln(x) + m_i \, . \qquad (6)$$

Currently the examinations are continuing according to the exponential fitting.

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*3.2.2 Distribution of the results of exponential regression calculation*

The program calculates the (2)-(5) equations for each 500-meter section of the indicated line and describes them in a frequency histogram, where there are data from at least 7 consecutive terms without track regulation.

In *Fig. 3* and *Fig. 4* the frequency diagrams of parameters *n* and *m* in the exponential deterioration equations of SAD values can be seen. It was measured along the railway line No. 1, where sections were built with track system 54.

In *Fig. 5* and *Fig. 6* there are the frequency diagrams of parameters *n* and *m* in the exponential deterioration equations of SAD values measured along the railway line No. 1, where sections were built with track system 60.

There are differences between the examined sections not only in the rail systems (54E1 and 60E1), but in the sleeper types (LM and LW) and the types of rail fastenings (Skl-3 and Skl-1).

In the functions, the value of parameter *n* is the *b* dimensioned factor itself, while parameter *m* shows where the function crosses *y*-axis so that it gives the value of $SAD_0$.

When the function describing the geometric deterioration process of a given railway track is unknown, it is mostly the speed of deterioration depending on the type of the curve fitting with regression is what we look for. The parameter *n* shows the deterioration speed in linear and non-linear cases as well.

The parameter *m* also provides interesting information, it shows the condition of the given rail track as a result of the regulation work.

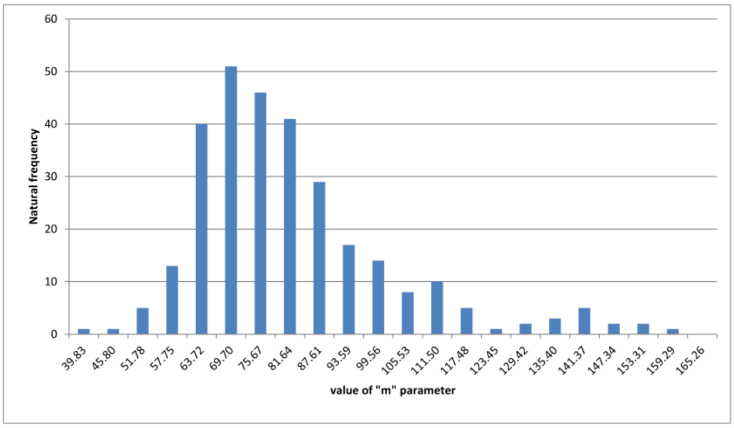


*Figure 3.* The natural frequency diagram of parameter *m* in the exponential deterioration equation for railway line No. 1, where sections were built with track system 54

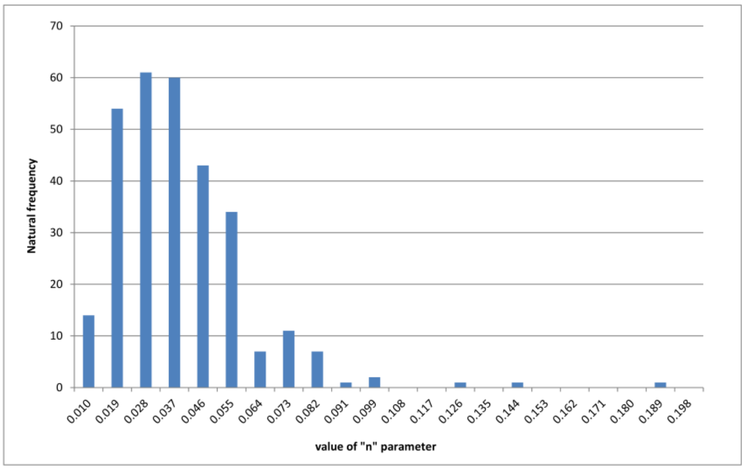


*Figure 4.* The natural frequency diagram of parameter *n* in the exponential deterioration equation for railway line No. 1, where sections were built with track system 54

To continue the evaluation, the values belonging to n parameter are selected, which are used by the program to calculate a more exact result.

After appointing the value set the program draws up the distribution of the given parameter. After this, the mean value of the given parameter weighted with R2 value can be calculated with the narrowed value set.

The program collects the mathematical and graphic results in a separate file, gives a name to the file, deletes the chart and calls in for the data of the following line with the help of a catalogue and the analysis starts again.

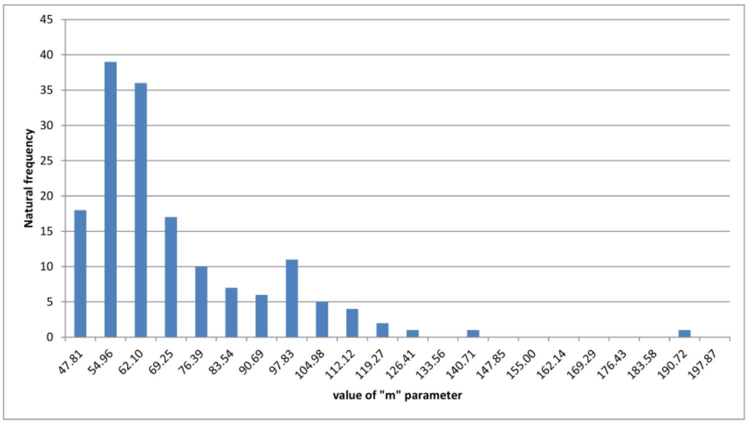


*Figure 5.* The natural frequency diagram of parameter *m* in the exponential deterioration equation for railway line No. 1, where sections were built with track system 60

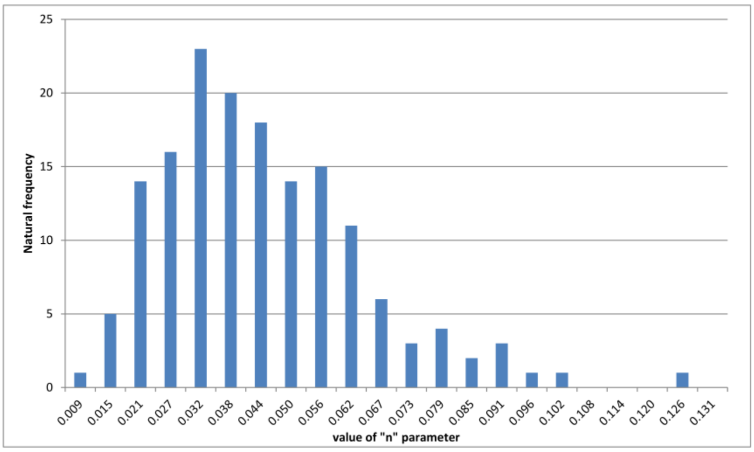


*Figure 6.* The natural frequency diagram of parameter in the exponential deterioration equation for railway line No. 1, where sections were built with track system 60

### *3.2.3 The analysis process of a 500-meter long qualifying section*

The process starts with the program looking for consecutive terms without regulation work in the given 500-meter section. It can be determined separately how many consecutive operation-free terms should be the minimum that the program takes into account during the analysis. The program indexes each inspected datum related to the term with 0 where there is a condition improvement (characterised by SAD value) higher than 5% compared to the previous term. This value indexed with 0 will be the SAD0 value during the analysis when the examination of the qualifying number is described.

Transforming the formula above (2) the b sub- and super-structural dimensioned factors are received as

$$b = \frac{\ln\left(C_{SAD_i} / C_{SAD_0}\right)}{t} \quad . \tag{7}$$

Based on the SAD numbers the program determines the equation of the given data series and the determination coefficient describing the rate of the fitting by the setting of linear (2) and non-linear (3), (4), (5) regression functions.

Hereinafter only the regression equations having a determination coefficient of

$$R^2 = 0.75, \tag{8}$$

or higher will be taken into consideration.

As the program completes the analysis of a 500-meter section, it continues with the next 500-meter rail section.

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When the last 500-meter track section has been already analysed and the analysis has been completed according to the scheme in Fig. 7, the program goes on to the next line.

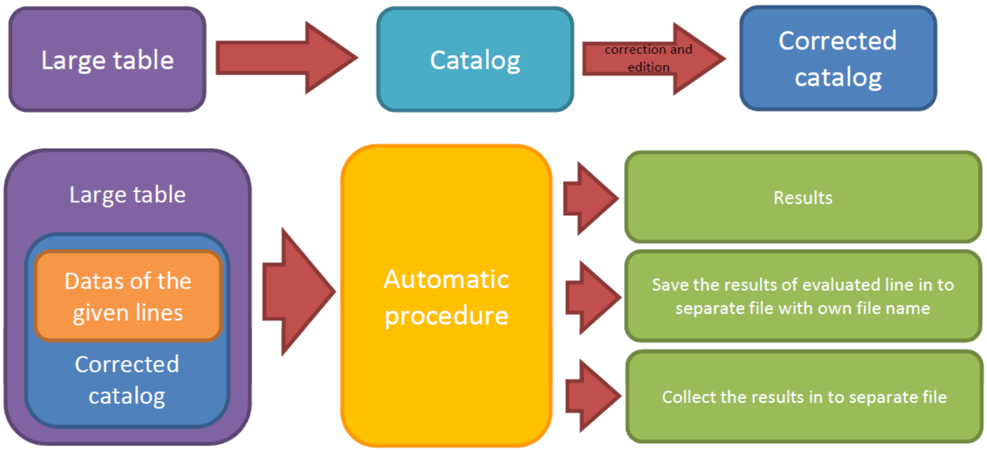


Figure. 7. The automatization figure of the program

*3.2.4 Assigning other parameters*

The procedure described above determines the deterioration equation of the given line on the basis of the unfiltered data.

It is a fact that a lot of parameters are available for the examined lines, including horizontal geometry, speed, the track system, the rail system, the type of sleeper, the sleeper spacing and the ballast thickness, etc. Despite this, there are factors in the analysis that the program cannot take into account.

The parameters mentioned above must be assigned to the 500-meter rail sections of the lines examined, which required separate programming and procedure creation over again. The following Table I provides the information for railway line No. 1 about the diversity of rail system, currently, it would be 70 rows. It can be simplified with creating a separate program.

Table 1. Sections of rail systems

| Zone | Initial section | Zone | Final section | Rail system |
|---|---|---|---|---|
| 1 | 0 | 1 | 1340 | MÁV48,5 normal |
| 1 | 1340 | 1 | 11800 | 54E1 normal |
| 1 | 11800 | 1 | 12300 | 60E1 normal |
| 1 | 12300 | 1 | 17299 | 54E1 normal |
| 1 | 17299 | 1 | 17600 | 60E1 normal |
| … | … | … | … | … |
| 3 | 188370 | 3 | 188495 | 60E1 normal |
| 3 | 188495 | 3 | 189115 | 54E1 normal |
| 3 | 189115 | 3 | 189476 | 60E1 normal |
| 3 | 189476 | 3 | 192720 | 54E1 normal |

If this assignment has been completed, it is possible to call the 500-meter long railway track sections that should be evaluated into the program according to the filtered parameters.

The results of the linear and the exponential regression from this program are going to be countered in the followings.

## 4. PREDICTION MODELS

Four simple models was created for control, two regression model and the artificial neural network was built to overcome the more simple models.

It can be seen that a higher orders model is endured if it predicts more precise data than more simple models.

Basic models show that some simple model can predict processes, although it is proved that a complex model has better prediction values.

*4.1 Basic models*

All of the basic models are linear and low ordered, meanwhile, it is necessary to compare with the higher ordered models.

Basic model 1

$$SAD_{t=n+1} = SAD_{t=n} + \frac{(SAD_{t=n} - SAD_{t=1})}{n} \tag{9}$$

Basic model 2

$$SAD_{t=n+1} = SAD_{t=n} + (SDA_{t=n} - SAD_{t=n-1}) \tag{10}$$

Basic model 3

$$SAD_{t=n+1} = SAD_{t=n} + \frac{(SAD_{t=n} - SAD_{t=n-1})}{2} + \frac{(SAD_{t=n-1} - SAD_{t=n-2})}{2} \qquad (11)$$

Basic model 4

$$SAD_{t=n+1} = SAD_{t=n} + \frac{(SAD_{t=n} - SAD_{t=n-1})}{4} + \frac{(SAD_{t=n-1} - SAD_{t=n-2})}{4} + \frac{(SAD_{t=n-2} - SAD_{t=n-3})}{4} + \frac{(SAD_{t=n-3} - SAD_{t=n-4})}{4} \qquad (12)$$

where $SAD_{t=n+1}$ is the value of the next half year's qualifying number.

*4.2 Regression models*

Hereinafter only the given railway line's regression equations having a determination coefficient of R2=0.75 or higher will be taken into consideration.

*4.2.1 Linear regression*

general equation: $y_i = \alpha + \beta x_i + u_i$ (13)

current equation: $SAD_t = SAD_0 + \mathrm{bt}$ (14)

where the

$SAD_t$ is the predicted SAD value in the tth half year,

$SAD_0$ is the predicted value of the qualifying number after maintenance

b is the degradation gradient (here it is the dimensioned factor),

t is the number of elapsed terms since the last maintenance.

*4.2.2 Exponential regression*

general equation: $y_i = \alpha + e^{\beta x_i} + u_i$ (15)

current equation: $SAD_t = SAD_0 + e^{bt}$ (16)

where the

$SAD_t$ is the predicted SAD value in the tth half year,

$SAD_0$ is the predicted value of the qualifying number after maintenance

b is the degradation gradient (here it is the dimensioned factor),

t is the number of elapsed terms since the last maintenance.

*4.3 Artificial Neural Networks*

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Artificial Neural Networks (ANNs) known as connectionist models, are systems that try to make use of some of the known or expected organising principles of the human brain. They consist of a number of independent, simple processors - the neurons. These neurons communicate with each other via weighted connections - the synaptic weights.

An ANN is such a mapping device, which assigns to each input exactly one output (both input and output may be vectors of values). The function is determined by the network's weights which are set while training the network. A neural network is trained, roughly, as follows. The network is shown a set of examples, each consisting of inputs and outputs. It learns the connections among them by assigning weights to connections. This is done by continuously changing weights to get closer to the desired outputs [19].

The X matrix was pre-ordained in order to the correct comparability, only the regression equations were used having a determination coefficient of R2=0.75 or higher, what caused the reduction of the input matrix to 1490 rows. The aim vector is always the value of the next missing half year; the hidden neurons number are three.

*4.3.1 The examined railway line*

Such a railway line had to be found in which the time between two maintenance is long enough to examine the model's predictions.

In order to find this railway line, the author program was used with some modifications. It countered the number of the half years between two maintenance line by line.

After the examination, the line No.60 (Pécs to Gyékényes) had been chosen because this line has the longest periods without maintenance, which provided long degradation processes to test the models.

This line has many 500 meter sections in which the time between two maintenance is more than 15 half years.

The learning sources and the validation process of the models are shown in Table 2. The first seven half years after the maintenance are the learning SAD values, while from the eighth half year the predicted and the real values were compared to count the difference.

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Table 2. SAD values of a given model

| | Learning SAD values | | | | | | | Predicted SAD values | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of the 500 meter long sections | half years | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ... | 16 |
| 1 | Real SAD values | | | | | | | Real SAD values | | | |
| 1 | | | | | | | | Predicted SAD values | | | |
| 1 | | | | | | | | The value of the error | | | |
| 2 | Real SAD values | | | | | | | Real SAD values | | | |
| 2 | | | | | | | | Predicted SAD values | | | |
| 2 | | | | | | | | | | | |
| ... | ... | | | | | | | ... | | | |
| 1490 | Real SAD values | | | | | | | Real SAD values | | | |
| 1490 | | | | | | | | Predicted SAD values | | | |
| 1490 | | | | | | | | The value of the error | | | |

The differences between the real and the predicted values are countered with the [1] equation, then these values are averaged to the given half years.

$$\sqrt{\frac{1}{n}\sum_{i=i}^{n}(SAD_i - \widehat{SAD_i})^2} \qquad (17)$$

This process was run for all seven models, the root mean squared error values are shown in the 3rd table.

Table 3. RMSE values

| | 8th half-year | 9th half-year | 10th half-year | 11th half-year | 12th half-year | 13th half-year | 14th half-year | 15th half-year | 16th half-year |
|---|---|---|---|---|---|---|---|---|---|
| Basic 1 | 10,67 | 15,94 | 17,28 | 24,30 | 33,51 | 17,95 | 28,41 | 34,24 | 43,22 |
| Basic 2 | 14,53 | 21,27 | 26,87 | 38,55 | 40,58 | 60,89 | 55,55 | 66,74 | 79,25 |
| Basic 3 | 12,64 | 15,71 | 18,82 | 24,56 | 22,10 | 30,81 | 33,88 | 30,30 | 52,11 |
| Basic 3 | 11,84 | 14,83 | 17,90 | 21,47 | 20,99 | 28,53 | 34,04 | 33,96 | 45,25 |
| Linear regression | 10,49 | 14,83 | 15,49 | 20,66 | 26,28 | 19,05 | 28,75 | 36,13 | 48,98 |
| Exponential regression | 17,11 | 29,48 | 39,71 | 52,42 | 85,28 | 44,21 | 70,72 | 85,10 | 47,88 |
| Artificial neural network | 10,86 | 11,54 | 12,71 | 17,17 | 17,29 | 19,95 | 22,47 | 18,95 | 26,46 |

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The next figures (8. to 11.) show the root mean squared error values of the prediction models as a function of the number of terms.

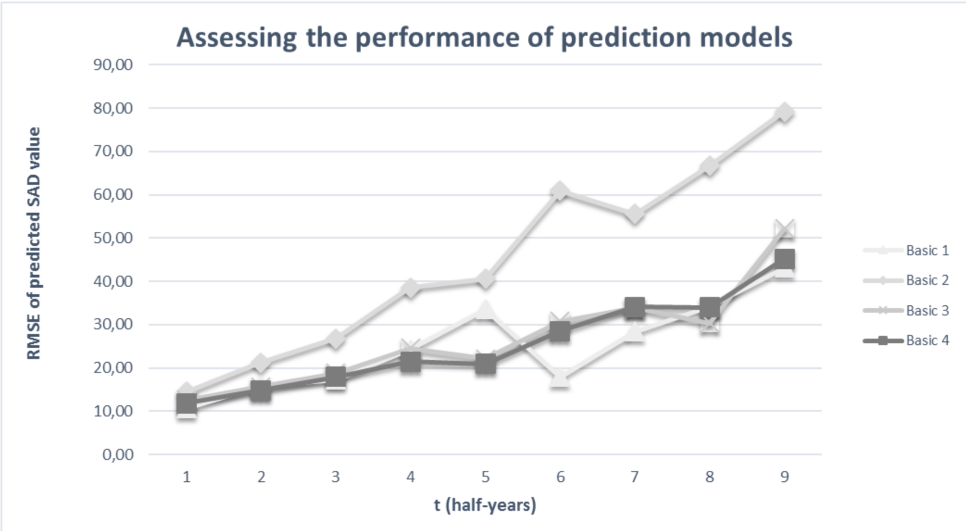


Figure 8. Basic models

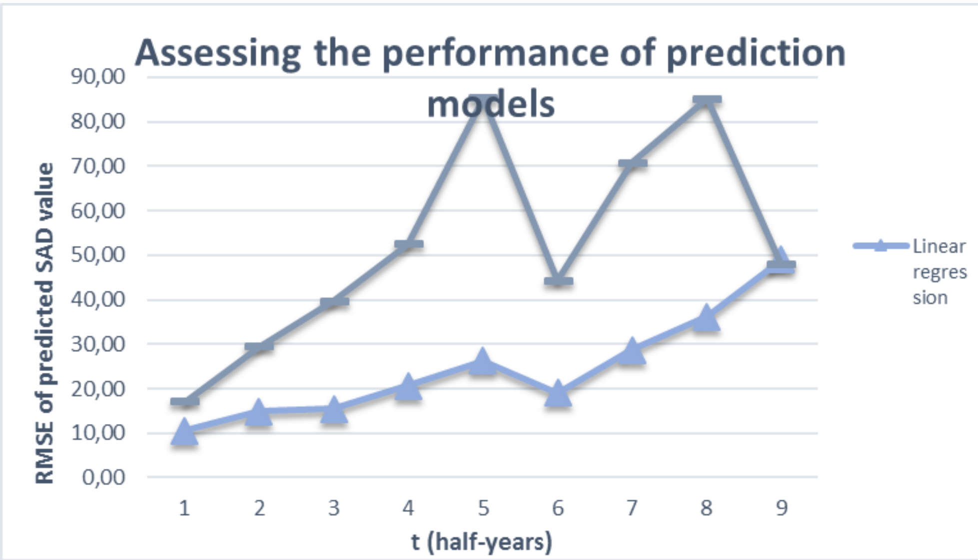


Figure 9. Regression models

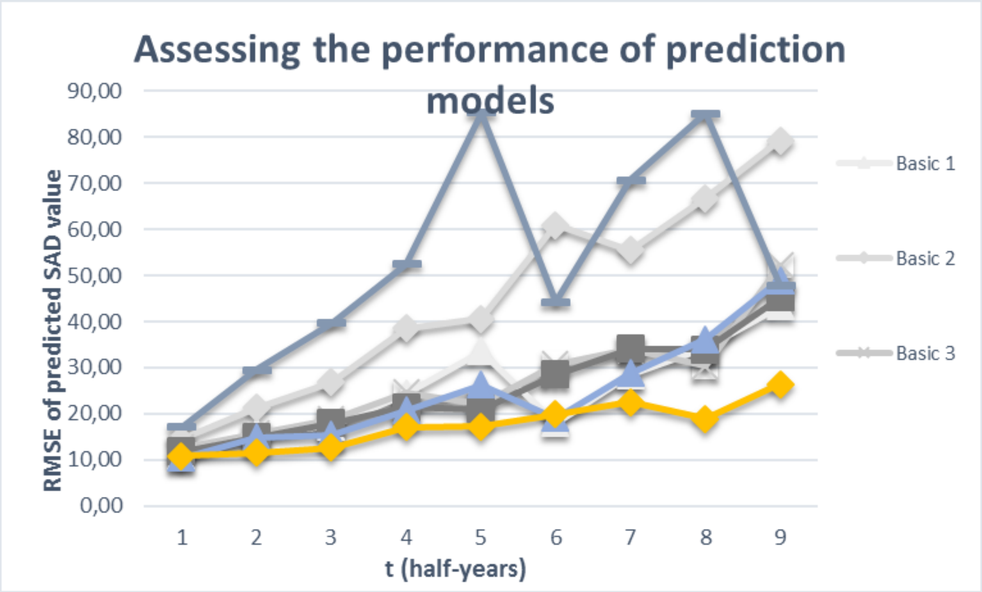


Figure 10. Prediction models

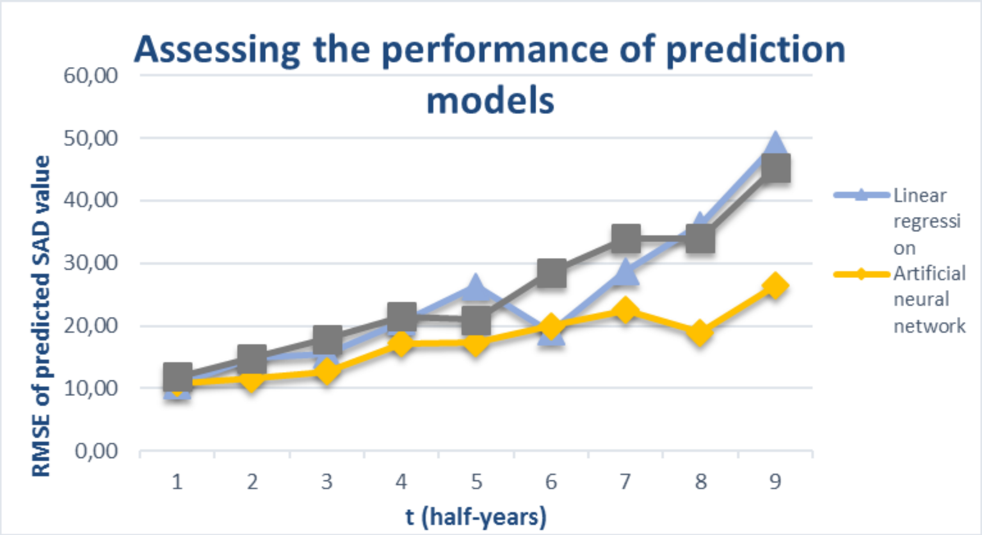


Figure 11. The chosen models

The root mean squared error values (RMSE) of the models relative to each other are shown with numbers in the 4th table.

Table 4. Excess error compare

| Excess error compared to the linear regression model | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Half years | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | cumulatively |
| Basic 4 | 1,34 | 0 | 2,41 | 0,81 | -5,29 | 9,48 | 5,29 | -2,17 | -3,73 | 8,14 |
| Neural Network | 0,37 | -3,28 | -2,78 | -3,49 | -8,99 | 0,89 | -6,27 | -17,17 | -22,53 | -63,25 |
| Cumulatively: | | | | | | | | | | -55,11 |
| | | | | | | | | | | |
| Excess error compared to the Basic 4 model | | | | | | | | | | |
| Half years | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | cumulatively |
| Basic 4 | -1,34 | 0 | -2,41 | -0,81 | 5,29 | -9,48 | -5,29 | 2,17 | 3,73 | -8,14 |
| Neural Network | -0,97 | -3,29 | -5,19 | -4,29 | -3,7 | -8,59 | -11,56 | -15 | -18,79 | -71,39 |
| Cumulatively: | | | | | | | | | | -79,53 |
| | | | | | | | | | | |
| Excess error compared to the neural network model | | | | | | | | | | |
| Half years | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | cumulatively |
| Basic 4 | 0,97 | 3,29 | 5,19 | 4,29 | 3,7 | 8,59 | 11,56 | 15 | 18,79 | 71,39 |
| Neural Network | -0,37 | 3,28 | 2,78 | 3,49 | 8,99 | -0,89 | 6,27 | 17,17 | 22,53 | 63,25 |
| Cumulatively: | | | | | | | | | | 134,64 |

The frequency of RMSE values of the predictions models is shown in the next figures (12th -14th figure).

These box plot diagrams show that the frequency of the predicted data has almost the same value. Thus the neural network model has a better prediction than the others.

It has to be mentioned that generally known box plot diagram was used to setup in the following figures.

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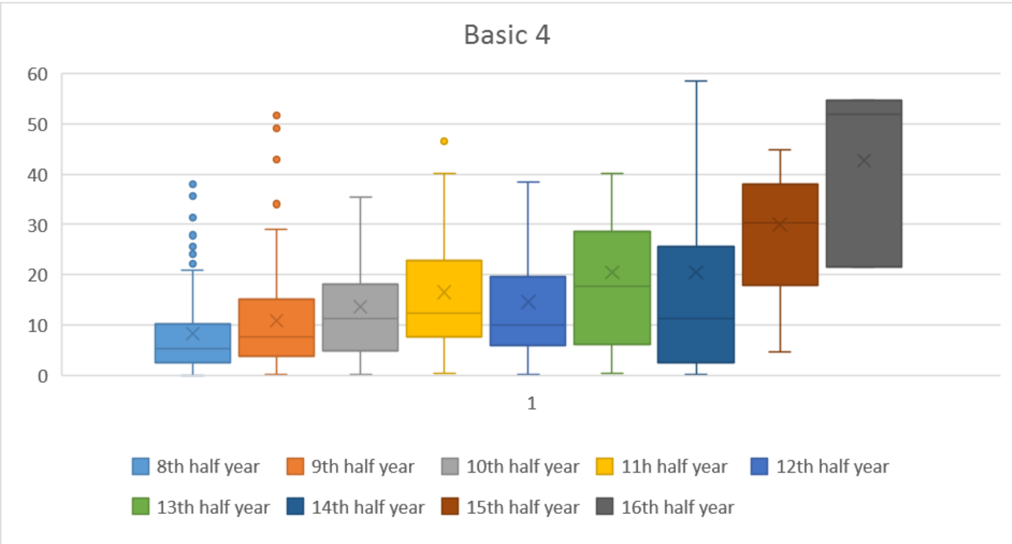
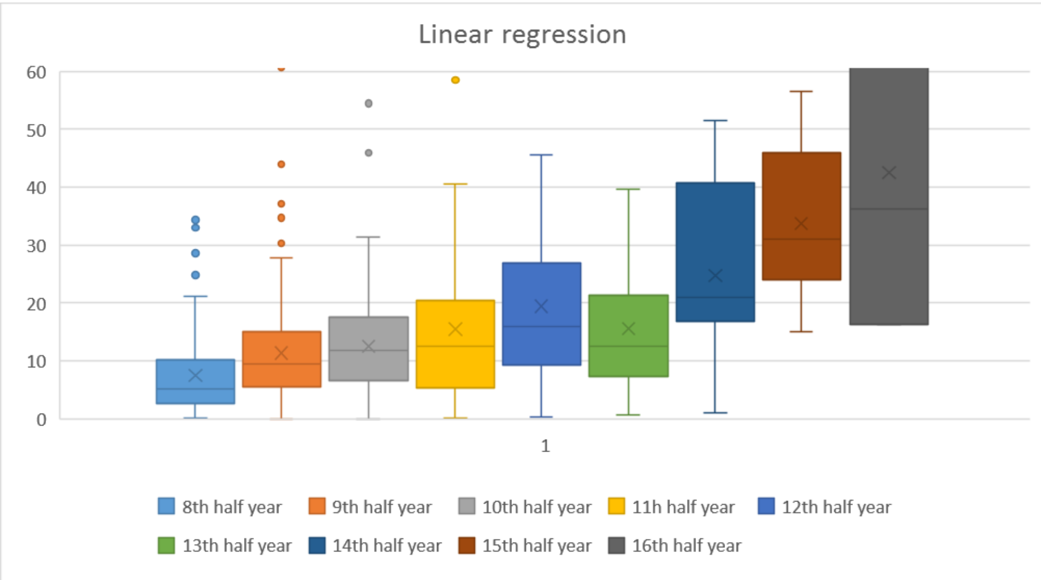
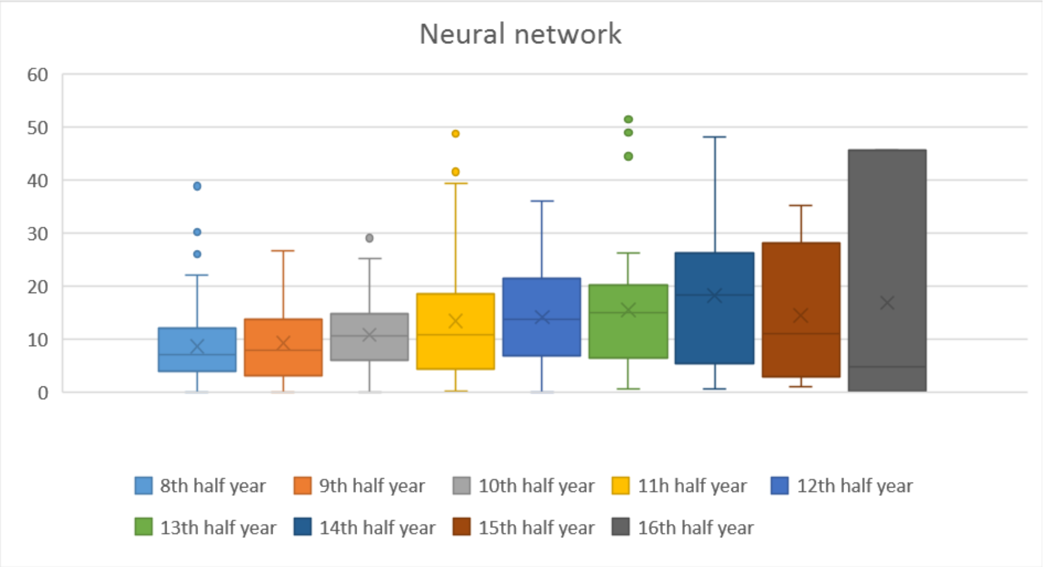



Figures 12. The frequency of the predicted values of the Basic 4 model




Figures 13. The frequency of the predicted values of the Linear regression model




Figures 14. The frequency of the predicted values of the Neural Network model

It can be seen that the artificial neural network is the model, among the others, which has the lowest RMSE values in long term prediction in the line No. 60.

## 5. CONCLUSION

On the one hand by analysing the deterioration processes the date of the required intervention (practices or restriction) became plan-able in an exact way. On the other hand, it supports the creating of economical calculations, which is significant when decisions are taken between investment and maintenance. The reviewed procedure calculates this progress automatically with the modern computing devices.

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