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Modelling the Geometrical Characteristics of Fabric Reinforced Composites

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Summary

Some micro-mechanical models are in use today to accurately describe the shape of textile reinforced composite materials. Physically, the textile reinforced composites modelling is more demanding than those of traditional composite (unidirectional) materials, given the fact that their geometric architecture of wire connections is quite sophisticated. Refined and accurate models are required to analyse these complicated structures. The numerical modelling techniques can be broken down into three main categories: classical laminate theory (CLT), stiffness averaging method (SAM) and finite element method (FEM). Of all the methods, finite element method (FEM) is the most promising because it allows analysis of nonlinear systems with general boundary conditions and can be adapted to complex geometries. Each model is based on micro-mechanical analysis, because all mechanical properties are affected by microscopic variables (geometric lengths, areas and volumes) and their properties. This paper is an attempt to find an efficient method of modelling the geometry, by analytical/numerical means, in order to save time and costs associated with the analysis of these composites. The method used here is a compromise between the continuous and pure discrete approaches and is associated with a mesoscopic analysis of the repetitive unit cell (UC).

KEYWORDS: composite material, woven composite, repetitive unit cell.

1. INTRODUCTION

In order to minimize the cost of analytical/numerical modelling of a woven composite material, only a small representative cell, the so-called unit cell (*UC*), that repeats itself along the entire material, is analysed. One of the first geometric patterns, extremely idealized, of the unit cell of a plain weave fabric was presented by Peirce [1]. Peirce's model (Fig. 1a) is based on a circular cross-section and incompressible wire. Kemp [2] modified Peirce's model, using elliptical wire cross sections (Fig. 1b), thereby achieving a more realistic representation of the geometry of the fabric.

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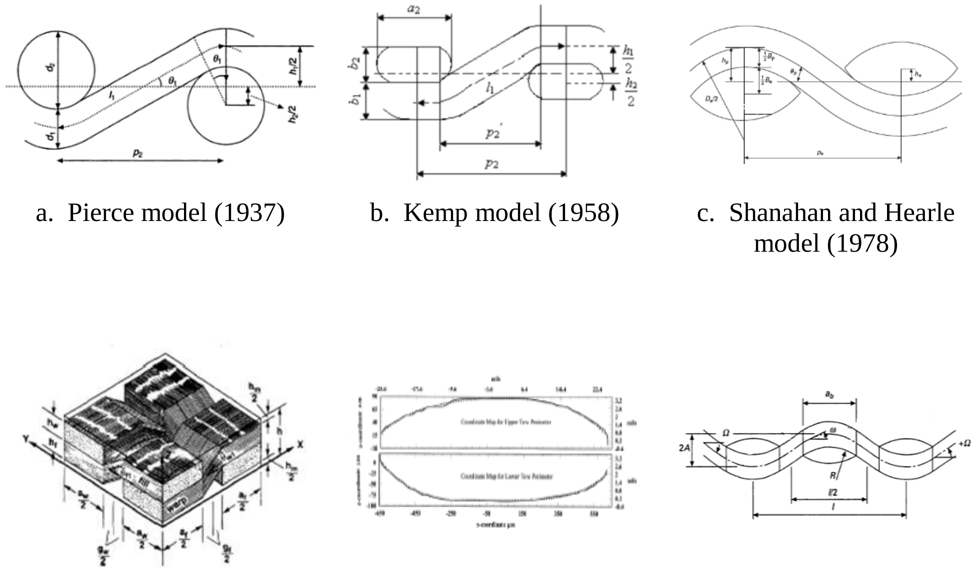

a. Pierce model (1937)

b. Kemp model (1958)

c. Shanahan and Hearle model (1978)

d. Ning and Chou model (1998)

e. Searles, Odegard and Kumosa model (2001)

f. Hofstee and van Keulen model (2001)

Fig. 1 The unit cell modelling evolution

Shanahan and Hearle [3] presented a geometric pattern using wire cross-section of lenticular shape (Fig. 1c) and introduced energy calculation methods. Ning and Chou [4] have developed a model with a more idealized unit cell (Fig. 1d) to estimate the effective plan thermal conductivity of the composite. Another representation of the wires was proposed by Searles [5], which introduced new procedures to characterize the upper and lower half of the cross section of the wires, relative to the centre of gravity (Fig. 1e). Hofstee and Van Keulen [6] proposed a geometric model of unit cells based on woven wire variable section (Fig. 1f).

An important feature of the available models is that they are all developed with certain textile architecture in mind. There is a need for a generalized model of fabric, in which one of the parameters should be the fabric pattern itself [7]. Also, the correlations between geometric parameters have not been fully investigated in published studies, in order to determine accurately how the change in a parameter affects the other design parameters [8].

2. MODELLING THE SATIN REINFORCED COMPOSITES

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The micromechanical study was performed on an epoxy lamina, reinforced with satin fabric. The method adopted is a hybrid method, which was proposed by Barbero [9] and is a good compromise between the accuracy of finite element methods and the simplicity of analytical methods. To find the elastic constants of the material, in-depth characterization of lamina geometry was necessary, pursuing the need to make an accurate estimation of both these constants and the factors influencing them. Due to the complexity of geometry, calculus become complicated and require the use of a computer backed-up by an appropriate programming language. Thus, all functions that describe the fabric geometry and equations for determining various parameters of the composite may be implemented in a MATLAB® like environment.

The *UC* is used for geometric and mechanical analysis and it is the smallest part of the fabric (2D) that contains all the features necessary to define the composite material. Therefore, the entire element can be rebuild by replicating the *UC* along the fill and warp pathways. Fig. 2 shows the *UC* of satin fabric 5/2/1. Fibres exposure on material faces is unbalanced, thus being dominated by fill or warp on opposite sides.

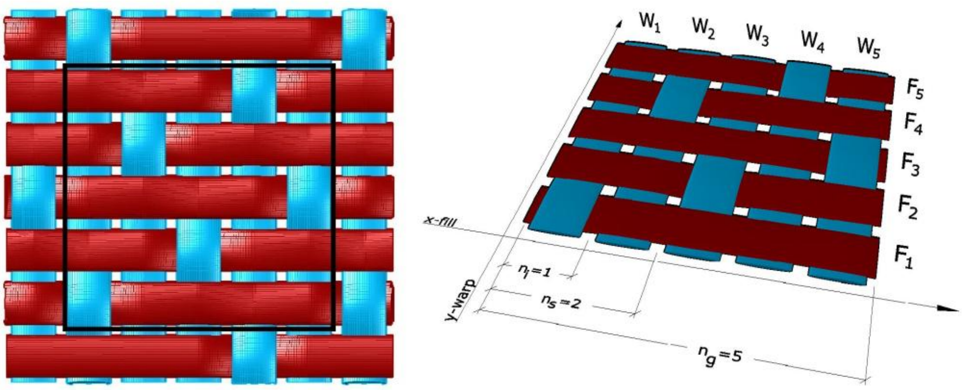

Fig. 2 The *UC* of satin fabric 5/2/1 and geometrical parameters

To identify the *UC*, three geometric parameters are introduced to facilitate the modelling and implementation of specific geometry in software. These parameters are n_g, n_s, n_i, defined as follows (Fig. 2):

- n_g is the number of subcells along one direction of the UC (harness). In case of 5/2/1 satin, n_g=5;
- n_s is the number of subcells between consecutive interlacing regions (shift). For 5/2/1 satin, n_s=2;
- n_i is the number of subcells in the interlacing region (interlacing). For satin 5/2/1, $n_i = 1$.

These parameters are proposed and used as a new coding system for 2D fabrics (biaxial orthogonal). Under this scheme, any style of fabric is described by combining the three parameters $n_g/n_s/n_i$, with no need to generate special geometric descriptions for individual cases.

The satin fabric is made by fill (x axis), and warp (y axis). They are described by the following geometric parameters, as seen on Fig. 3:

- tow width for fill a_f and warp a_w;
- tow height for fill h_f and warp h_w;
- gap between two consecutive tows for fill g_f or warp g_w;
- neat-matrix thickness h_m.

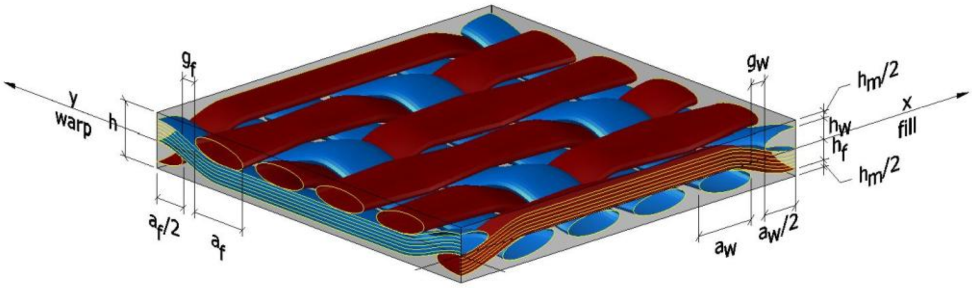

Fig. 3 Geometrical parameters of the *UC*

In the first step, the *UC* is split along the (x, y) plane in 2D smaller units subcells, in order to better consider the influence of curling wires. As the number of subcells is higher, the results are more accurate, but computing power needed increases with a quadratic rate (Fig. 4).

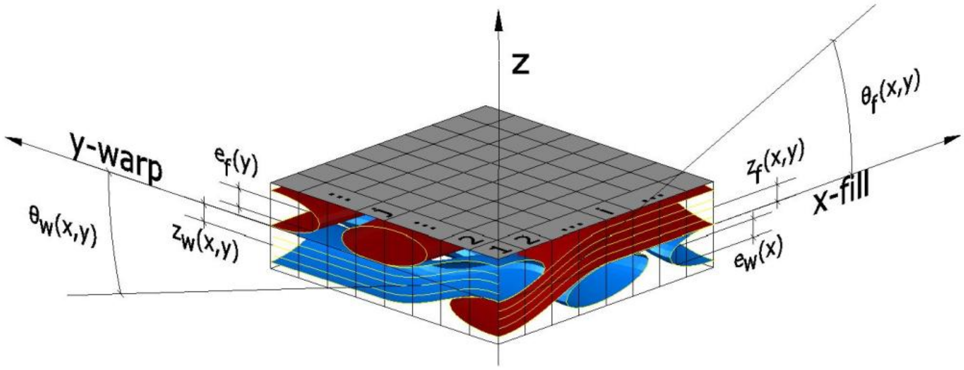

Fig. 4 Intermediate geometrical parameters of the woven textile

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For each subcell, thus for each location (x, y), the geometry of the two tows (fill and warp) may be depicted by the complex functions. It starts from undulation along the wires, taking into account the space between them g_f and g_w, which is expressed in terms of z coordinate of midpoint of the tow thickness. For example, in the case of the satin type fabric, the undulation of the fill $z_f(x,y,j)$ is given by Eq. 1 and for the warp by Eq. 2, where $i \; and \; j \in [1, n_g]$ are the interlacing positions for the warp and fill:

$$z_f(x,y,j) =$$

$$\begin{cases} \textbf{if} \; (j-1)w_f + \frac{g_f}{2} \le y \le jw_f - \frac{g_f}{2} \\ \begin{cases} \textbf{for } k \in 1 \ldots\ldots \textbf{trunc} \left[\frac{n_s(j-1)+1}{n_g}\right] + 1 \\ \begin{cases} (-1)^{(kn_g+1)} \frac{h_f}{2} sin\left[\frac{\pi x}{w_w} + n_s(j-1)\pi\right], & \textbf{if} \left[n_s(j-1) - \frac{1}{2} - kn_g\right] w_w \le x \le \left[n_s(j-1) + \frac{3}{2} - kn_g\right] w_w \\ \frac{h_f}{2}, & \textbf{if} \; (k-1) \ge 1 \; \textbf{and} \; \left[n_s(j-1) + \frac{3}{2} - kn_g\right] w_w < x < \left[n_s(j-1) - \frac{1}{2} - (k-1)n_g\right] w_w \end{cases} \\ \frac{-h_f}{2} sin\left[\frac{\pi x}{w_w} + n_s(j-1)\pi\right], \quad \textbf{if} \left[n_s(j-1) - \frac{1}{2}\right] w_w \le x \le \left[n_s(j-1) + \frac{3}{2}\right] w_w \\ (-1)^{(n_g+1)} \frac{h_f}{2} sin\left[\frac{\pi x}{w_w} + n_s(j-1)\pi\right], \quad \textbf{if} \left[n_s(j-1) - \frac{1}{2} + n_g\right] w_w \le x \le \left[n_s(j-1) + \frac{3}{2} + n_g\right] w_w \\ \frac{h_f}{2}, \quad \textbf{if} \left[n_s(j-1) + \frac{3}{2} - n_g\right] w_w < x < \left[n_s(j-1) - \frac{1}{2}\right] w_w \; \textbf{or} \\ \qquad \left[n_s(j-1) + \frac{3}{2}\right] w_w < x < \left[n_s(j-1) - \frac{1}{2} + n_g\right] w_w \end{cases} \\ 0, \quad \textbf{if} \left[(j-1)w_f \le y < (j-1)w_f + \frac{g_f}{2} \; \textbf{or} \; jw_f - \frac{g_f}{2} < y \le jw_f\right] \textbf{and} \; g_f \ne 0 \end{cases} \tag{1}$$

$$z_w(x,y,i) =$$

$$\begin{cases} \text{if } (i-1)w_w \le x \le iw_w \\ \begin{cases} \text{for } k \in 1 \ldots\ldots \text{fix}\left[\frac{(n_g-n_s)(i-1)+1}{n_g}\right]+1 \\ \begin{cases} (-1)^{(kn_g+1)}\frac{h_w}{2}\sin\left[\frac{\pi y}{w_f}+(n_g-n_s)(i-1)\pi\right], & \text{if } \left[(n_g-n_s)(i-1)-\frac{1}{2}-kn_g\right]w_f \le y \text{ and } \\ & y \le \left[(n_g-n_s)(i-1)+\frac{3}{2}-kn_g\right]w_f \\ \frac{-h_w}{2}, & \text{if } (k-1)\ge 1 \text{ and } \left[(n_g-n_s)(i-1)+\frac{3}{2}-kn_g\right]w_f < y \text{ and } \\ & y < \left[(n_g-n_s)(i-1)-\frac{1}{2}-(k-1)n_g\right]w_f \end{cases} \\ \frac{h_w}{2}\sin\left[\frac{\pi y}{w_f}+(n_g-n_s)(i-1)\pi\right], \quad \text{if } \left[(n_g-n_s)(i-1)-\frac{1}{2}\right]w_f \le y \text{ and } \\ \qquad y \le \left[(n_g-n_s)(i-1)+\frac{3}{2}\right]w_f \\ -(-1)^{(n_g+1)}\frac{h_w}{2}\sin\left[\frac{\pi y}{w_f}+(n_g-n_s)(i-1)\pi\right], \quad \text{if } \left[(n_g-n_s)(i-1)-\frac{1}{2}+n_g\right]w_f \le y \text{ and } \\ \qquad y \le \left[(n_g-n_s)(i-1)+\frac{3}{2}+n_g\right]w_f \\ \frac{-h_w}{2}, \quad \text{if } \left[(n_g-n_s)(i-1)+\frac{3}{2}-n_g\right]w_f < y < \left[(n_g-n_s)(i-1)-\frac{1}{2}\right]w_f \text{ or } \\ \qquad \left[(n_g-n_s)(i-1)+\frac{3}{2}\right]w_f < y < \left[(n_g-n_s)(i-1)-\frac{1}{2}+n_g\right]w_f \end{cases} \\ 0, \quad \text{if } \left[(i-1)w_w \le x < (i-1)w_w+\frac{g_w}{2} \text{ or } iw_w-\frac{g_w}{2} < x \le iw_w\right] \text{ and } g_w \ne 0 \end{cases} \tag{2}$$

where:

- $w_f = a_f + g_f$ - the distance between two consecutive fill tows;
- $w_w = a_w + g_w$ - the distance between two consecutive warp tows.

The thickness of the cross section for the fill and warp tows is specified in the Eq. 3 and Eq. 4).

$$e_f(y,j) = \begin{cases} \left| h_f \sin\frac{\pi\left(-(j-1)w_f+y-\frac{g_f}{2}\right)}{a_f}\right|, & \text{if } (j-1)w_f+\frac{g_f}{2} \le y \le jw_f-\frac{g_f}{2} \\ 0, & \text{if } (j-1)w_f \le y < (j-1)w_f+\frac{g_f}{2} \text{ or } jw_f-\frac{g_f}{2} < y \le jw_f \end{cases} \tag{3}$$

$$e_w(x,i) = \begin{cases} \left| h_w \sin\frac{\pi\left(-(i-1)w_w+x-\frac{g_w}{2}\right)}{a_w}\right|, & \text{if } (i-1)w_w+\frac{g_w}{2} \le x \le iw_w-\frac{g_w}{2} \\ 0, & \text{if } (i-1)w_w \le x < (i-1)w_w+\frac{g_w}{2} \text{ or } iw_w-\frac{g_w}{2} < x \le iw_w \end{cases} \tag{4}$$

Given the above parameters, undulation and cross-sectional thickness of the wires for the entire *UC* can be computed (Eq. 5, 6).

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$$\textit{if } (j-1)w_f \leq y \leq jw_f \rightarrow \begin{cases} z_f(x,y) = z_f(x,y,j) \\ e_f(y) = e_f(y,j) \end{cases} \quad (5)$$

$$\textit{if } (i-1)w_w \leq x \leq iw_w \rightarrow \begin{cases} z_w(x,y) = z_w(x,y,i) \\ e_w(x) = e_w(x,i) \end{cases} \quad (6)$$

The upper and lower surface boundaries of the fill and warp yarns can be evaluated using Eq. 7, 8.

$$\begin{cases} z_f^{top}(x,y) = z_f(x,y) + \frac{1}{2}e_f(y) \\ z_f^{bot}(x,y) = z_f(x,y) - \frac{1}{2}e_f(y) \end{cases} \quad (7)$$

$$\begin{cases} z_w^{top}(x,y) = z_w(x,y) + \frac{1}{2}e_w(x) \\ z_w^{bot}(x,y) = z_w(x,y) - \frac{1}{2}e_w(x) \end{cases} \quad (8)$$

Undulation angle of the tows θ (Fig. 5), is then calculated with Eq. 9.

$$\begin{aligned} \theta_f(x,y) &= \arctan\left(\frac{\partial}{dx} z_f(x,y)\right) \\ \theta_w(x,y) &= \arctan\left(\frac{\partial}{dy} z_w(x,y)\right) \end{aligned} \quad (9)$$

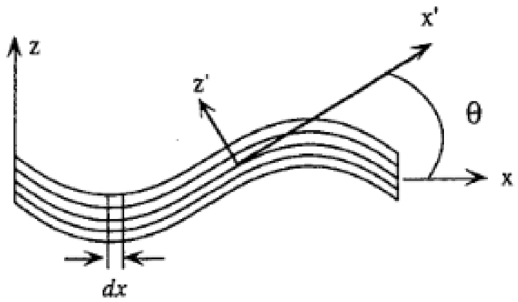

Fig. 5 Undulation angle θ of the fill and warp tows

Cross sectional area of the fill and warp tows of *UC* can be estimated by Eq. 10.

$$\begin{aligned} A_f &= \int_0^{w_f} e_f(y)dy \\ A_w &= \int_0^{w_w} e_w(x)dx \end{aligned} \quad (10)$$

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Wire length is determined in *UC* using the description of undulation (Eq. 11):

$$L_f = \int_0^{n_g \cdot w_w} \sqrt{1 + \left(\frac{\partial}{dx} z_f(x,y)\right)^2}\, dx$$
$$L_w = \int_0^{n_g \cdot w_f} \sqrt{1 + \left(\frac{\partial}{dy} z_w(x,y)\right)^2}\, dy \qquad (11)$$

Based on the above relationships, the volume of the fill tows is $v_f = n_g A_f L_f$ and that of the warp tows is $v_w = n_g A_w L_w$.

The mechanical characteristics of the composite are determined by the properties of the matrix and those of the fill and the warp tows. The mechanical properties of the yarns (i.e. E_1, F_{1t}, etc.) are calculated based on the apparent resistance of the constituent material (fibre and matrix), taking into account the volume fractions of the fibres in the yarns.

3. CONCLUSIONS

Mechanical performance of the textile composites can be determined either experimentally or through simulations (modelling); the latter is the less expensive approach, commonly used when calculating the mechanical properties of fabrics. In order to minimize the cost of analytical/numerical modelling of a woven composite material, only a small representative cell, the so-called unit cell (*UC*), that repeats itself along the entire material, is analysed.

The micromechanical study was performed on an epoxy lamina, reinforced with satin fabric. The method adopted is a hybrid method and is a good compromise between the accuracy of finite element methods and the simplicity of analytical methods. To find the elastic constants of the material, in-depth characterization of lamina geometry was necessary, pursuing the need to make an accurate estimation of both these constants and the factors influencing them. Due to the complexity of geometry, calculus become complicated and require the use of a computer backed-up by an appropriate programming language. Thus, all functions that describe the fabric geometry and equations for determining various parameters of the composite may be implemented in a MATLAB® like environment.

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