INTERSECTII
INTERSECTIONS
http://www.intersections.ro

Computational Methods in Predicting the Elastic Engineering Constants for Multi-layered Composites

Iuliana Dupir (Hudișteanu), Vlad Lupășteanu
Department of Civil and Industrial Engineering, Technical University "Gheorghe Asachi", Iasi, Zip code 700050, Romania

Summary

The paper presents the development of the computational equations which describe the elastic behaviour of multi-layered composites. The prediction of the elastic engineering constants is studied based on classical lamination theory, both for an orthotropic lamina and for a symmetric angle-ply laminate. The stiffening effect of stacking two or more laminas is demonstrated based on a case study, where two different types of composite material were analysed. Graphical distributions of the engineering constants with respect to different fibre volume fractions, for unidirectional off-axis laminas and symmetric angle-ply laminates, were represented. The influence of the fibre volume fractions on the elastic behaviour of composite laminates is also investigated.

KEYWORDS: classical lamination theory, orthotropic lamina, angle-ply laminate, composite laminate, fibre volume fraction, fibre orientation angle, engineering elastic constants.

1. INTRODUCTION

The classical lamination theory (CLT) derives from the classical plate theory proposed by Kirchhoff-Love [1] and it is the most commonly used theory for analysing composite laminates. The extension of this theory is valid for thin laminates, with small displacement in the transverse direction. The basic assumptions which govern the lamination theory are presented by Herakovich in [2], as it follows:

- Perfect bonding between the layers is assumed.
- Each ply is a homogeneous material with known effective properties.
- Each lamina is in a plate stress.
- The individual lamina can be isotropic, orthotropic or transversely isotropic.
- The laminate deforms according to the Kirchhoff-Love assumptions for bending and stretching.

Classical lamination theory presented in the paper is applicable to orthotropic continuous fibre laminated composites only.

ISSN 1582-3024

The laminate engineering elastic constants are important in predicting the stiffening behaviour of a composite structure. Therefore, it is necessary to know the influencing parameters as the fibres and matrix properties and the ply microstructure, the shape and size of the fibres' cross-sections, fibre volume fraction, fibre orientation angles and stacking sequence.

2. GOVERNING EQUATIONS OF ENGINEERING CONSTANTS

2.1. Off-axis unidirectional lamina

A ply or lamina is the simplest element of a composite material and it is considered an elementary layer of unidirectional fibres embedded in a matrix [3]. The off-axis unidirectional lamina or the general orthotropic lamina shown in Fig. 1 is defined as a composite ply where the principal material axes 1(L) and 2(T) make a different angle with the global system of reference (x, y) [4].

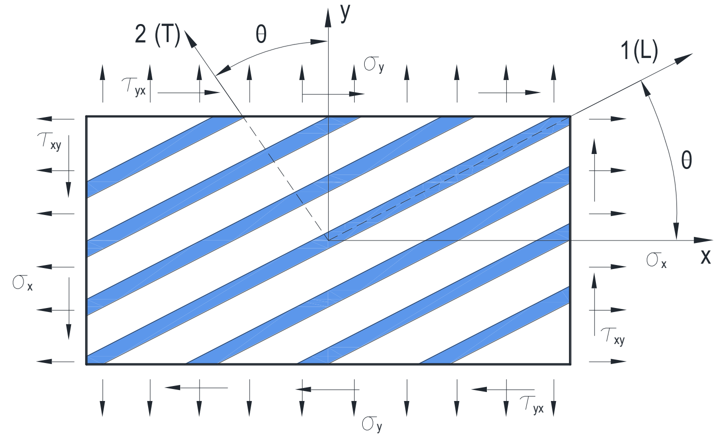


Figure 1. Off-axis unidirectional lamina

The effects of fibre orientation angles are studied with corresponding transformations between principal and global axes presented in Equation (1):

$$E_x = \frac{1}{\frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}s^4}$$

$$E_y = \frac{1}{\frac{1}{E_1}s^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}c^4}$$

ISSN 1582-3024

INTERSECTIONS
INTERSECTII
http://www.intersections.ro

$$G_{xy} = \frac{1}{2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^2c^2 + \frac{1}{G_{12}}\left(s^4 + c^4\right)} \tag{1}$$

$$\nu_{xy} = -\frac{\left[c^2s^2\left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) - \left(c^4 + s^4\right)\nu_{12}\right]}{\left[c^4 + c^2s^2\left(-2\nu_{12} + \frac{E_1}{G_{12}}\right) + s^4\frac{E_1}{E_2}\right]}$$

$$\nu_{yx} = -\frac{\left[c^2s^2\left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) - \left(c^4 + s^4\right)\nu_{12}\right]}{\left[s^4 + c^2s^2\left(-2\nu_{12} + \frac{E_1}{G_{12}}\right) + c^4\frac{E_1}{E_2}\right]}$$

where: E_1, E_2 and G_{12} are the axial and shear moduli with respect to the principal material axes, ν_{12} and ν_{21} are the Poisson's ratios and θ is the fibre orientation angle; $c = \cos\theta$, $c^2 = \cos^2\theta$, $c^4 = \cos^4\theta$; $s = \sin\theta$, $s^2 = \sin^2\theta$, $s^2 = \sin^2\theta$.

The in-plane engineering elastic constants with respect to the principal material directions are determined according to the micromechanics of fibre reinforced composites [4-6], as follows in Equation (2):

$$E_1 = E_f \cdot V_f + E_m \cdot V_m, \quad E_2 = \frac{E_f \cdot E_m}{V_f \cdot E_m + V_m \cdot E_f},$$

$$G_{12} = \frac{G_f \cdot G_m}{V_f \cdot G_m + V_m \cdot G_f} \tag{2}$$

$$\nu_{12} = \nu_f \cdot V_f + \nu_m \cdot V_m, \quad \nu_{21} = \nu_{12} \cdot \frac{E_2}{E_1},$$

where: E_f and E_m are the longitudinal Young's modulus of the fibre and matrix respectively; G_f and G_m are the shear modulus of the fibre and matrix; ν_f and ν_m are the Poisson's ratios of the fibre and matrix; V_f and V_m are the fibre and matrix volume fractions.

2.2. Angle-ply laminates

By definition, angle-ply laminates $\left[(\pm\theta)_n\right]_s$ are considered a special type of orthotropic laminates, having equal number of equal thickness layers at $+\theta$ and $-\theta$ fibre orientation angles. Symmetric angle-ply laminates are characterized by

ISSN 1582-3024

INTERSECTII
INTERSECTIONS
http://www.intersections.ro

symmetry with respect to the middle plane of the laminate, in terms of material, thickness and fibre orientation angles [6].

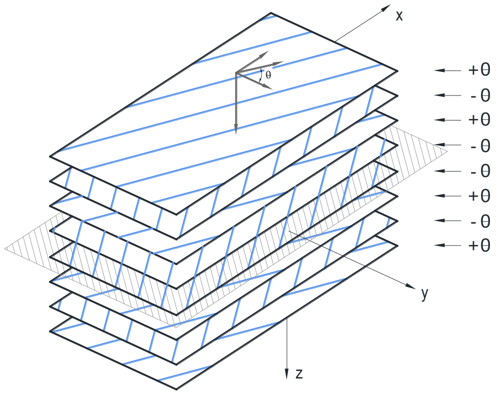


Figure 2. Angle-ply laminate $[(\pm\theta)_2]_s$

The computational equations which describe the stiffening behaviour of an angle-ply laminate are related to the elements of the laminate compliance $[a^*]$ and of the transformed reduced stiffness matrix $[\overline{Q}]$, as follows [4]:

$$\begin{aligned} E_x &= \frac{1}{a_{11}^*} = \frac{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}{\overline{Q}_{22}} \\ E_y &= \frac{1}{a_{22}^*} = \frac{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}{\overline{Q}_{11}} \\ G_{xy} &= \frac{1}{a_{66}^*} = \overline{Q}_{66} \\ \nu_{xy} &= -\frac{a_{12}^*}{a_{11}^*} = \frac{\overline{Q}_{12}}{\overline{Q}_{22}} \\ \nu_{yx} &= -\frac{a_{12}^*}{a_{22}^*} = \frac{\overline{Q}_{12}}{\overline{Q}_{11}} \end{aligned} \tag{3}$$

The elements of the transformed reduced stiffness matrix $\overline{Q}_{ij}$ are given by Equation (4) [4, 7]:

$$\begin{aligned} \overline{Q}_{11} &= Q_{11}c^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}s^4 \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})c^2s^2 + Q_{12}(s^4 + c^4) \\ \overline{Q}_{22} &= Q_{11}s^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}c^4 \end{aligned} \tag{4}$$

ISSN 1582-3024

$$\overline{Q}_{16} = \left[(Q_{11} - Q_{12} - 2Q_{66})c^2 + (Q_{12} - Q_{22} + 2Q_{66})s^2 \right] cs$$
$$\overline{Q}_{26} = \left[(Q_{11} - Q_{12} - 2Q_{66})s^2 + (Q_{12} - Q_{22} + 2Q_{66})c^2 \right] cs$$
$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})c^2 s^2 + Q_{66}(c^4 + s^4)$$

where: Q_{ij} represent the coefficients of the stiffness matrix, related to the in-plane engineering constants of the lamina, as follows:

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \cdot \nu_{21}}$$
$$Q_{12} = \frac{\nu_{21} \cdot E_1}{1 - \nu_{12} \cdot \nu_{21}} = \frac{\nu_{12} \cdot E_2}{1 - \nu_{12} \cdot \nu_{21}}$$
$$Q_{22} = \frac{E_2}{1 - \nu_{12} \cdot \nu_{21}} \tag{5}$$
$$Q_{66} = G_{12}$$

3. CASE STUDY

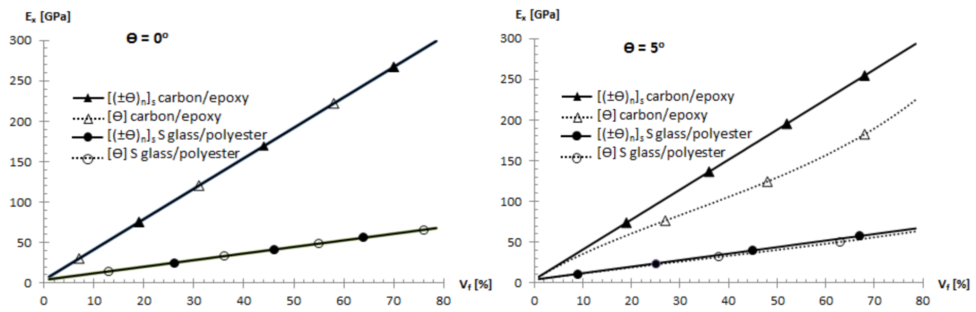
The prediction of the elastic engineering constants for orthotropic laminas and symmetric angle-ply laminates is demonstrated based on a case study. Two different types of composite materials are chosen for the analysis, such as carbon fibre/epoxy resin and S glass/polyester resin, given in Table 1.

Table 1. Properties of materials [7]

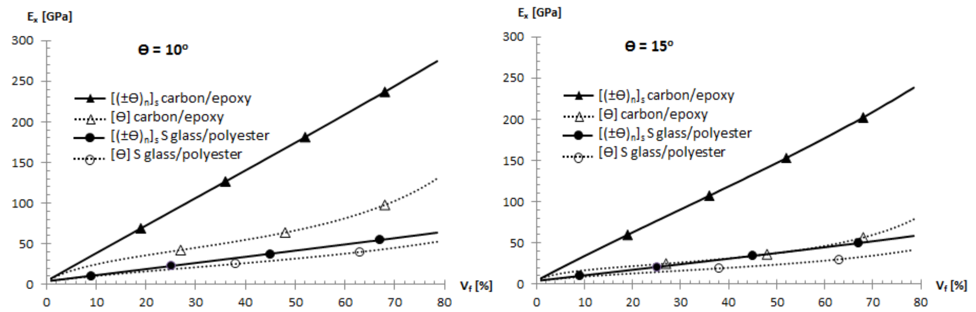
Composite materials		Longitudinal Young's modulus [GPa]		Poisson's ratios		Shear modulus [GPa]	
Fibre	Matrix	E_f	E_m	ν_f	ν_m	G_f	G_m
carbon	epoxy	380	4.1	0.2	0.4	118.75	1.46
S glass	polyester	85.5	4	0.22	0.39	26.72	1.44

The variations of the elastic engineering constants with respect to the fibre volume fractions are shown in Figs. 3-7. Suggestive values for the fibre orientation angles were selected as to illustrate the stacking stiffening effect, but also the highest engineering constants obtained values for angle-ply laminates.

INTERSECTII
http://www.intersections.ro
INTERSECTIONS



a). highest values for E_x



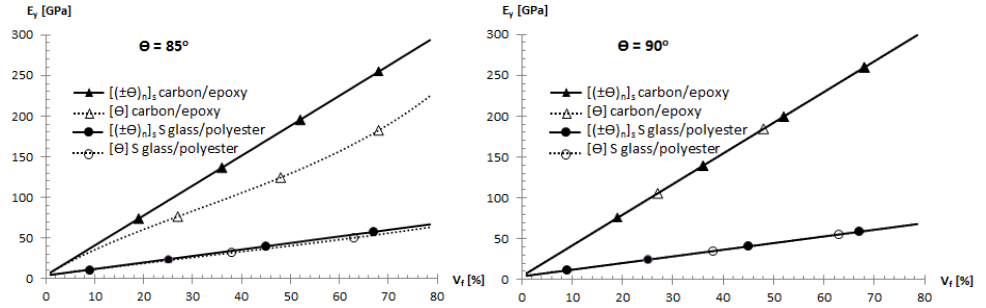
b). stacking stiffening effect

Figure 3. Variation of longitudinal modulus E_x

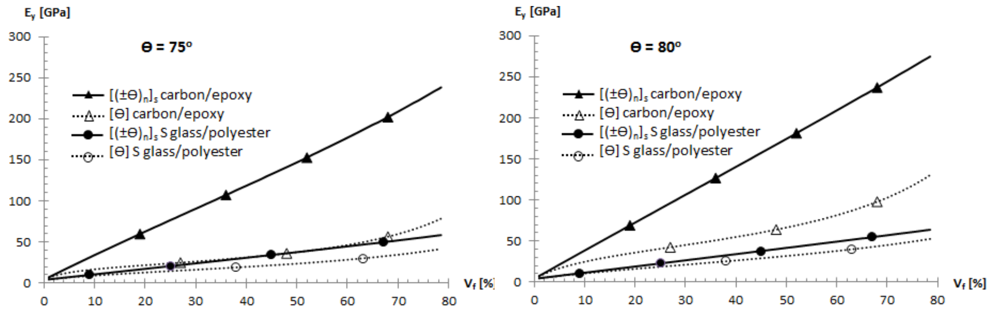
The longitudinal elastic modulus E_x reaches maximum values at $\theta = 0 - 5^o$, as shown in Fig. 3a. While increasing the fibre orientation angles, a gradual and continuous loss of stiffness appears. Moreover, the elastic properties for this interval are difficult to be distinguished for laminas compared with laminates, so the concept of an angle-ply laminate with low values of orientation angles is not feasible for design.

The stiffening effect is most clearly illustrated in Fig. 3b, for fibre orientation angles of $\theta = 10 - 15^o$, where significant differences can be observed between the orthotropic laminas and angle-ply laminates.

As expected, the graphical distributions shown in Fig. 4 for the transverse modulus E_y are identically with E_x, for complementary fibre orientation angles.



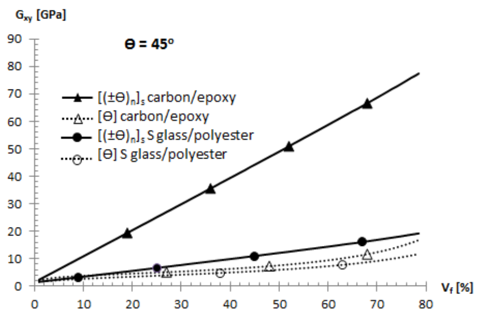
a). highest values for E_y



b). stacking stiffening effect

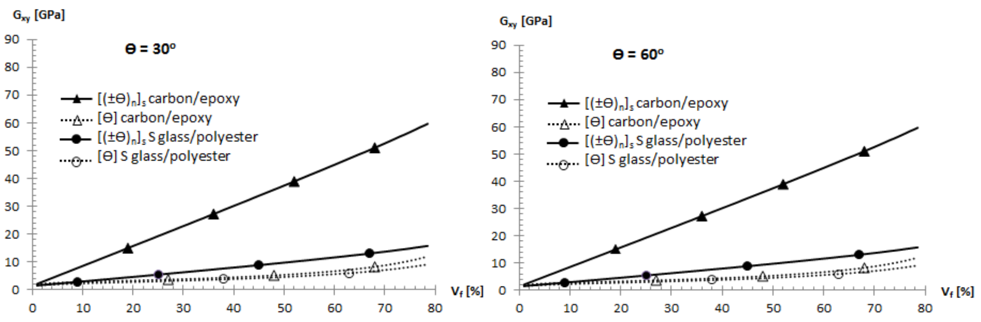
Figure 4. Variation of transverse modulus E_y

Comparison of the predictions for longitudinal modulus E_x and transverse modulus E_y shows that the fibre volume fractions influence the increasing of the analysed engineering constants.



a). highest values for G_{xy} and stacking stiffening effect

INTERSECTII INTERSECTIONS
http://www.intersections.ro

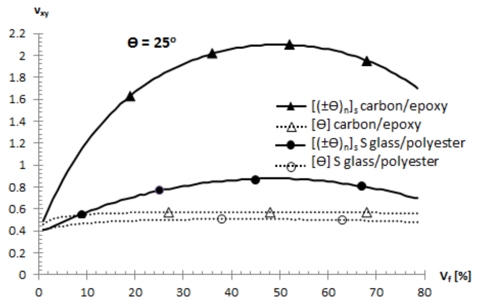


b). symmetry with respect to $\theta = 45^o$

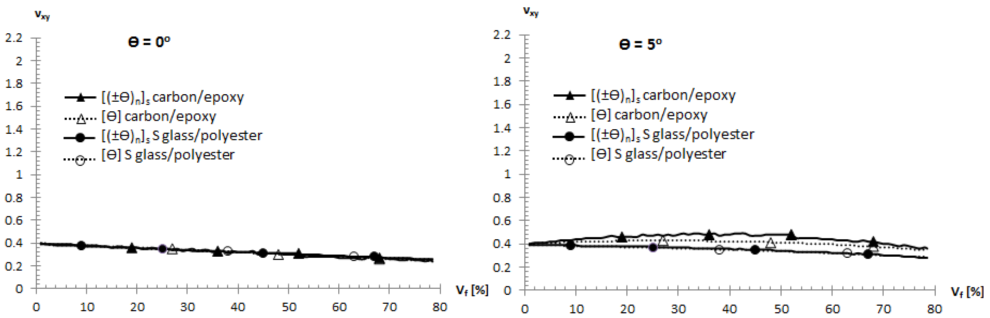
Figure 5. Variation of shear modulus G_{xy}

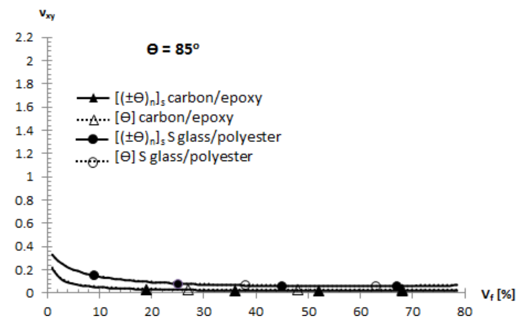
The shear modulus G_{xy} reaches highest values and the stacking stiffening effect is the most clearly illustrated at a fibre orientation angle of $\theta = 45^o$ (Fig. 5a). The G_{xy} graphical distributions indicate a symmetry with respect to $\theta = 45^o$, showing increasing values for $\theta = 0 - 45^o$ and decreasing values for $\theta = 45 - 90^o$ (Fig. 5b).

The variations of the Poisson's Ratios ν_{xy} and ν_{yx} are illustrated in Figs. 6-7.



a). highest values for ν_{xy} and stacking stiffening effect

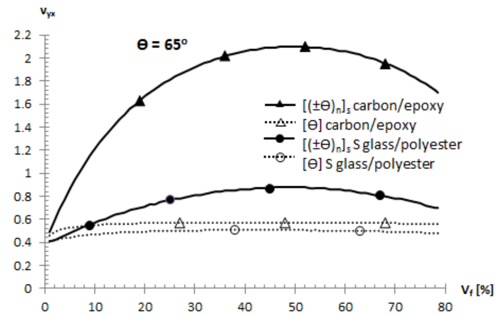




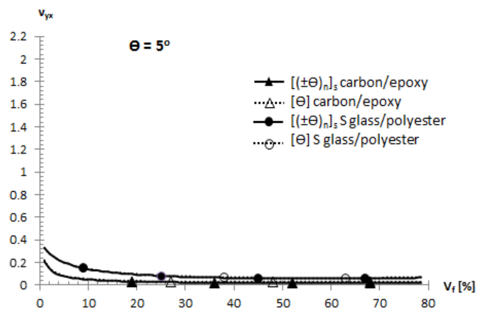
b). different slope distributions for ν_{xy}

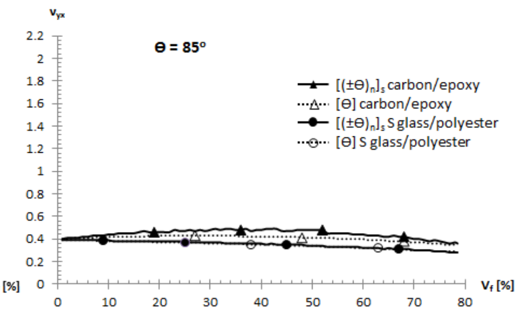
Figure 6. Variation of Poisson's ratios ν_{xy}

The stacking stiffening effect and the highest reached values for the Poisson's ratios ν_{xy} and ν_{yx} are shown in Fig. 6a and Fig. 7b respectively. As expected again, the fibre orientation angles for the Poisson's ratios are complementary, so the graphical distributions of the engineering constants appear as in a mirror one to the other.

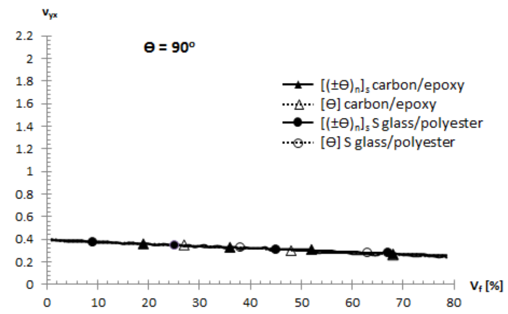


a). highest values for ν_{yx} and stiffening effect





ISSN 1582-3024



b). different slope distributions for ν_{yx}

Figure 7. Variation of Poisson's ratios ν_{yx}

The variation of Poisson's ratios is shown in Fig. 6b and Fig. 6a, for different fibre orientation angles. Different slope distributions are illustrated, suggesting a gradual loss or gain of stiffness once the fibre volume fractions or the fibre orientation angles are changed.

4. CONCLUSIONS

The influence of the fibre volume fractions on the elastic engineering constants of multi-layered composite was demonstrated. The presented graphical distributions indicate an increase on the elastic characteristics when higher percentages of fibre volume fractions are used. Even so, in the case of the Poisson's ratios, the increase is not proportional to the fibre volume fractions, neither to the fibre orientation angles.

Significant differences regarding the elastic characteristics and stiffening behaviour was observed between the two types of composite materials adopted for the case study. Therefore, when designing a composite laminate, all the previous discussed parameters have to be taken into account to fulfil the needed requirements.

References

1. Love, A.E.H., *On the small free vibrations and deformations of elastic shells*, Philosophical transactions, The Royal Society, London, Vol. serie A, No. 7, p. 491–549, 1888
2. Herakovich, C.T., *Mechanics of Fibrous Composites*, John Wiley & Sons, Inc., University of Virginia, 1998.
3. Vasiliev. V.V., Morozov E., *Advanced Mechanics of Composite Materials and Structural Elements, Third Edition*, Elsevier, 2013.
4. Ţăranu, N., Isopescu, D., *Structures made of composite materials*, Ed. Vesper, Iaşi, 1996.
5. Kaw, A.K., *Mechanics of Composite Materials. 2nd ed.*, CRC Press, Taylor & Francis Group, 2006.
6. Barbero, E.J., *Introduction to Composite Materials Design. 2nd ed*, CRC Press, 2011.
7. Ţăranu, N., Bejan, L., Cozmanciuc, R., Hohan, R., *Materiale şi elemente composite I. Prelegeri şi aplicaţii*, Ed. Politehnium, Iaşi, 2013.

ISSN 1582-3024