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Numerical Evaluation of Reinforced Concrete Beams Loaded in Bending

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Summary

In this paper, the behaviour of reinforced concrete beams is estimated with the Finite Element Method and improvements to the method are proposed. Because the reinforced concrete elements work in conjunction with the material cracking behaviour, the behaviour is very complex and depends on the shape of the reinforcement, mechanical characteristics of the materials and the intensity of loads.

In general, the Finite Element Method cannot emulate this complex process because the connection between elements at the node is fixed and slip behaviour of reinforcement is not allowed. For this reason, the results obtained with the Finite Element Method do not correctly describe the phenomenon. To address this, the authors propose an improvement on the Finite Element Method to eliminate the deficiency and describe the process with more accuracy.

KEYWORDS: concrete, reinforcement, finite element, numerical analysis.

1. INTRODUCTION

The numerical analysis is presented in the majority of engineering sciences, from aero spatial industry to electronics. Many programs are developed for civil engineering and almost all use Finite Element Method. The main goal consist in establishing of the structural behaviour under a various types of loads [1].

Reinforced concrete is a particularly material in civil engineering because it is a double composite material. First concrete is a composite material formed by cement and aggregates and with reinforcement is obtained a second-degree composite material. From this reason, the behaviour of reinforced concrete element is very complex and difficult to describe with simple mathematical relations [2].

Mathematical algorithms were developed in civil engineering for composite materials and the behaviour of such kind of materials can be numerically established [3]. These relations cannot be used for reinforced concrete elements because the reinforcement is not scattered on the entire volume of the element and is provided only in tensioned zone. This condition makes the analysis process very difficult especially when two different materials are combined together.

First programs that can study the behaviour of reinforced concrete elements was applied with limited success. The studies revealed important differences between numerical and experimental results as follow:

a) in numerical analysis the cracking are presented in all tensioned nodes; if the mesh is refined to 10 mm, the cracks appear at this distance and the correspondence with experimental results is no longer achieved;

b) in numerical analysis the stresses for the reinforced concrete element appear like in a plain concrete one and the tensile stresses are similar in and between the cracks; experimental results revealed large differences between tensile stresses, according with the crack pattern [4];

c) the reinforcement stresses have a continuous development in numerical analysis (according with the bending moment diagram) but in the experimental tests the reinforcement stresses have a number of relative maximum values (according with the crack pattern) [5].

In conclusion, the Finite Element Method can be successfully used for steel structures but in case of reinforced concrete elements this method can’t provide accurate results, mainly due to poor bond implementation [6]. The bond between concrete and reinforcement is a very complex phenomenon and a simple combination of concrete and reinforcement in meshing process is not enough.

2. PROPOSED MODEL

A regular simply supported beam was considered to evaluate the behaviour of reinforced concrete elements using the Finite Element Method (Figure 1). The used concrete grade is C30/37 and the longitudinal reinforcement is made from steel BSt500 class C. Stirrups are provided at the ends of the beam to carry shear but in the middle area no stirrups were provided to avoid influences from confining. Steel grade 250 are used for transversal reinforcements.

The test is a classical 4 point test, using two concentrated forces to eliminate the shear force effect in the middle part of the element.

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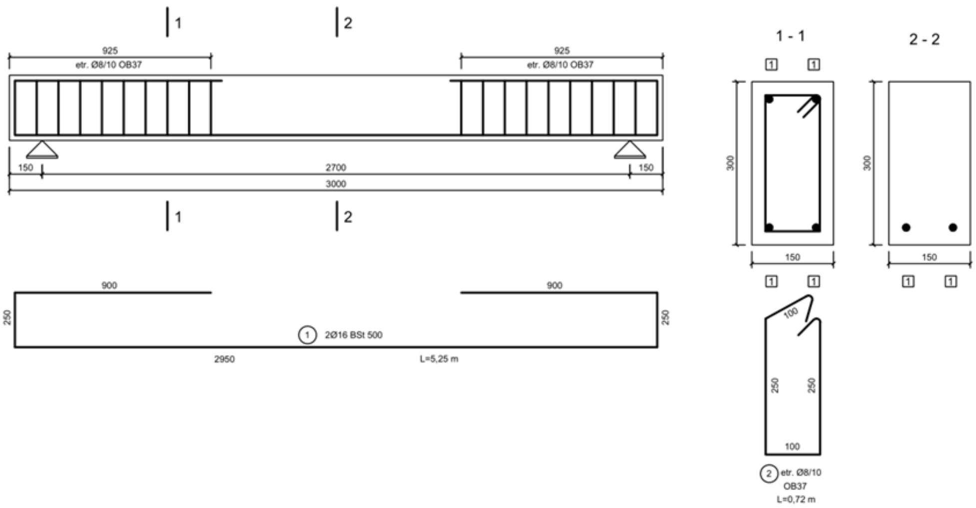

Figure 1. Making-up of reinforced concrete beam

To calibrate the Finite Element Method, the real bond between concrete and reinforcements must be included in the analytical model, according with the stress-slip diagram obtained from experimental tests (Figure 2).

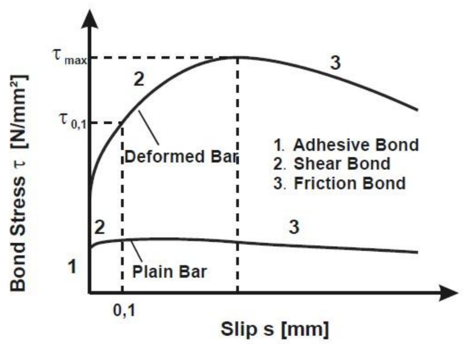

Figure 2. Making-up of reinforced concrete beam

In the Finite Element Method, the influence of the bond between concrete and reinforcement must be included. To do this, the external load must be applied in individual steps and the numerical model will be adapted according with intermediate results.

The initial loading step will determine the stresses and displacement for the perfect bond case with the finite element Equation (1):

$$\{F(t)\} = [k]^e \cdot \{u(t)\} \tag{1}$$

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where $\{F(t)\}$ is the vector of external forces from the finite element nodes, $[k]^e$ is the stiffness matrix for the elastic stage and $\{u(t)\}$ is the nodal displacement vector

The algorithm will check for crack appearance in all nodes. If a crack appears, the bond between concrete and reinforcement must be re-evaluated around the crack area. Thus, the stiffness matrix for all of the concrete elements near the crack will be modified according with the Equation (2):

$$[k]=[k]^e-[k]^d \tag{2}$$

where $[k]$ is the revised stiffness matrix and $[k]^d$ is a damaged stiffness matrix. In the $[k]^d$ matrix, all of the concrete elements located inside of the broken bond area will have a reduced stiffness value. All of the finite elements unaffected by cracks will have a zero value in the $[k]^d$ matrix.

Next, the new stresses and displacements are established with the Equation (3):

$$\{F(t)\}=[k]\cdot\{u(t)\} \tag{3}$$

If new cracks appear in this stage, the stiffness matrix will be revised again and the process will be repeated until no new cracks appear. Finally, if failure does not occur, then external loads can be increased to the next step and the process will continue until failure (Figure 3).

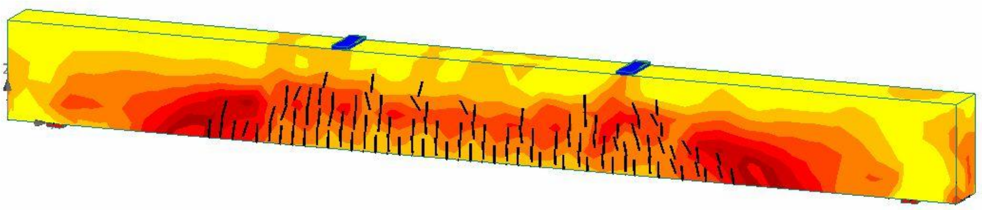

Figure 3. Numerical results on RC beam: stresses and crack pattern

The main problem of the proposed algorithm is the value of the loading step. If the loading step is very large, too many cracks appear at the same time and the results will be affected by errors.

To avoid this situation, a satisfactory solution is to diminish the loading step. In this case, accurate results will be obtained, but the computation time will increase substantially and many small useless steps will be processed.

The bisection algorithm can also be used to avoid this situation, but the convergence of the solution is still achieved quite slowly. Another faster method that establishes the optimum loading step and obtains accurate results in less time is proposed in [7].

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3. EXPERIMENTAL RESULTS

Checking the accuracy of the proposed numerical algorithm was realised on three experimental beams (Figure 4) with characteristics described in Figure 1. The beams were equipped with one load cell and six resistive transducers for measurement of displacements on bottom side.

Figure 4. Experimental test

The force-displacement diagram was established for each tested element and the final results obtained with numerical analysis and experimental tests are presented in Figure 5.

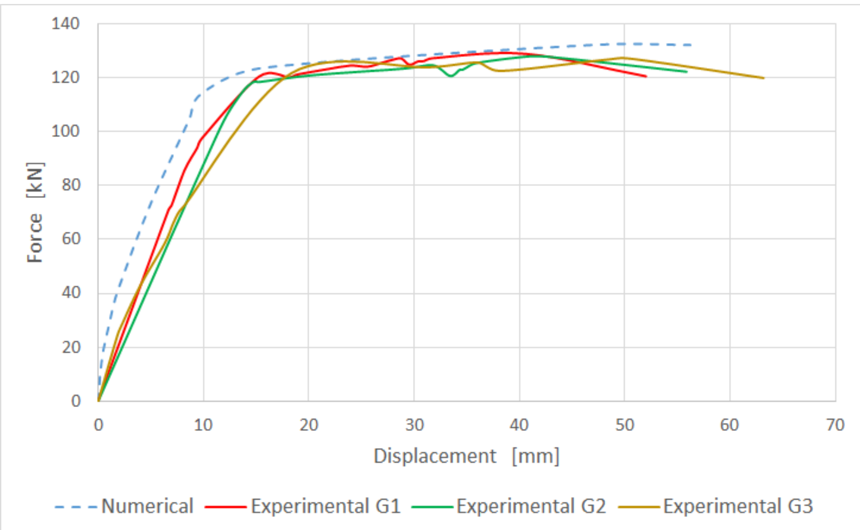

Figure 5. Force-displacement diagram

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The comparison between numerical and experimental results revealed a good approach of the proposed method. The bearing capacity and the maximum displacement estimated with numerical analysis are very close to those obtained experimentally.

3. CONCLUSIONS

The non-elastic behaviour of the concrete and the cracking stage developed in service for the reinforced concrete elements impose a new approach of the finite element method. Because the regular Finite Element Method cannot describe the behaviour of the reinforced concrete structures, a new method was proposed by the authors.

The modified algorithm can be successfully applied to evaluate bond characteristics between concrete and reinforcement in reinforced concrete elements.

The main disadvantage of the proposed model is the requirement of a fine mesh. Thus, analysis time can increase significantly for large elements and structures. To overcome this inconvenient, a coarse mesh can be used to decrease the number of elements, but in this case, the results can be affected by bigger errors.

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