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Numerical modeling method for dynamic behaviour of masonry towers

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Summary

Dynamic behaviour model is essential to the reliability evaluation and restoration structure of historical masonry towers. In this paper author developed the identification techniques and the main influence factors on the dynamic behaviour of historical masonry towers. Besides sensitivity analysis and model updating criteria are discussed.

KEYWORDS: Historical masonry towers, dynamic behaviour, computational simulations.

1. INTRODUCTION

Historical masonry towers were built long time ago, often suffered various natural disasters and damages, and most of them had been repaired. Therefore most towers have following common characteristics: construction type, non-uniform materials and coexistence of various damage conditions. Since the structural parameters affect directly the dynamic behaviour of historical masonry towers, the dimension, construction detail, damage condition and variation degree should be surveyed carefully and defined reasonably for a good analytical model to be developed.

The techniques of structural identification, and in particular those of modal parameters of linear or linearised models, represent an important tool for analysing existing structures. In fact, synthetic information on damage occurred to towers can be obtained comparing the value of such modal parameters. First of all natural frequencies and damping coefficients, before and after the seismic event.

According to the principle of dynamics, the motion equation of an historical tower can be expressed as follows:

$$[K]\{x\}+[C]\{\dot{x}\}+[M]\{\ddot{x}\}=-[M]\{1\}\ddot{x}_0, \tag{1}$$

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where $\{x\},\{\dot{x}\},\{\ddot{x}\}$ are the structural nodal displacement, velocity and acceleration vectors, respectively. $[K],[C],[M]$ are respectively, structural stiffness, damping, and mass matrices, $\ddot{x}$ is the ground acceleration.

Taking:

$$[C]=\alpha[M]+\beta[K], \tag{2}$$

if Rayleigh damping is adopted, the mass [M] and stiffness [K] are the main factors to influence dynamic behaviour of the structure.

2. IDENTIFICATION OF MODEL PARAMETERS

The extraction of model parameters from ambient vibration data was carried out by using two different procedures: Peak picking method and Frequency domain decomposition. Both methods are based on the evaluation of the spectral matrix in the frequency domain:

$$G(f)=E\left[A(f)A^{H}(f)\right]. \tag{3}$$

The Peak picking method leads to reliable results provided that the basic assumptions of low damping and well-separated modes are satisfied. For a lightly damped structure subjected to a white-noise random excitation, both auto-spectral densities and cross-spectral densities reach a local maximum at the frequencies corresponding to the system normal modes. For well-separated modes, the spectral matrix can be approximated in the neighborhood of a resonant frequency f_r by:

$$G(f_r)\approx \alpha_r \varphi_r \varphi_r^{H}, \tag{4}$$

where: α_r depends on the damping ratio, the natural frequency, the modal participation factor and the excitation spectra. In the present application of the Peak picking method, natural frequencies were identified from resonant peaks in the auto-spectral densities and in the amplitude of cross-spectral densities, for which the cross-spectral phases are 0 or π. The mode shapes were obtained from the amplitude of square-root auto-spectral densities curves while cross spectral densities phases were used to determine directions of relative motion.

The Frequency domain decomposition approach is based on the singular value decomposition of the spectral matrix at each frequency:

$$G(f)=U(f)\sum(f)U^{H}(f), \tag{5}$$

where: the diagonal matrix $\sum$ collects the real positive singular values in descending order and U is a complex matrix containing the singular vectors as columns. If only one mode is important at a certain frequency f_r, the spectral matrix can be approximated by a rank-one matrix and can be decomposed as:

$$G(f_r) \approx \sigma_1(f_r) u(f_r) u_1^H(f_r). \quad (6)$$

The first singular value $\sigma_1(f)$ at each frequency represents the strength of the dominating vibration mode at that frequency while the corresponding singular vector $u_1(f)$ contains the mode shape.

3. MODEL UPDATING

Due to the uncertainty of structural parameters of the historical masonry tower, dynamic characteristics predicted by the analytical model often differ from field measurements. The proper reference criteria should be provided for the structural parameters identification in the model updating. From the field testing we can obtain the first n order modal parameters of a structure with N degrees of freedom.

For example, natural frequencies:

$$\left\lfloor \omega_T^2 \right\rfloor = diag\left(\omega_1^2, \omega_2^2, ..., \omega_n^2\right), \quad (7)$$

and mode shape:

$$[\phi_T] = diag[\{\phi_1\}, \{\phi_2\}, ... \{\phi_n\}]. \quad (8)$$

Generally, $\left\lfloor \omega_T^2 \right\rfloor$ measured from the field test is accurate. On the other hand, $[M_A]$ obtained from structural analysis is comparatively accurate but $[K_A]$ is less close to the actual values. To improve the effect of the model updating, the sensitivity system should be constructed for selection of the structural parameters firstly. Besides, taking the fast analysis advantage of LUSAS program, the conventional trial-error method also can be used to simplify the model updating procedure.

4. SENSITIVITY OF DYNAMIC BEHAVIOUR FOR STRUCTURAL PARAMETER ADJUSTMENT

The sensitivity-based model updating procedure and the correlative researches are helpful for the construction of the sensitivity system of historical masonry towers.

Suppose the structural parameters such as mass, stiffness, geometric dimensions and material characteristics described as p_i (i=1, 2,…,n) and the eigenvalues are considered as derivative functions of structural parameters, the dynamic characteristics of the towers can be expressed as $F=F(p_1,p_2,\dots,p_n)$. Then the sensitivity of F to structural parameter p_i is $S_{F_{p_i}} = \partial F(p_1, p_2, \dots p_n)/\partial p_i$, and the bigger the absolute value of $S_{F_{p_i}}$, the more the sensitivity of model characteristic to structural parameter p_i.

According to the structural dynamics, the γ-order eigenvalue λ_γ and eigenvector $\phi^{(\gamma)}$, should satisfy:

$$([k]-\lambda_r[M])\varphi^{(r)} = 0. \tag{9}$$

By solving the above formula's partial derivative to the i-parameter, the sensitivity of eigenvalue can be obtained:

$$\lambda_{ri} = \varphi^{(r)T}([K]_i - \lambda_r[M]_i)\varphi^{(r)}. \tag{10}$$

And the sensitivity of the eigenvector is:

$$\varphi_j^{(r)T} = \sum_{\substack{k=1 \\ k\neq r}}^{n} \frac{-\varphi^{(r)T}([k]_j - \lambda_r[M]_j)\varphi^{(r)}}{\lambda_k - \lambda_r}\varphi^{(k)} - \frac{1}{2}\varphi^{(r)T}[M]_j\varphi^{(r)}\varphi^{(r)}. \tag{11}$$

Taking the stiffness parameter of structure as main study object, the sensitivity of the γ-order eigenvalue and eigenvector of stiffness k_{ij}, are respectively:

$$\frac{\partial \lambda_r}{\partial k_{ij}} = \varphi_{ir}\varphi_{jr}, \tag{12}$$

$$\frac{\partial \varphi^{(r)}}{\partial k_{ij}} = \sum_{\substack{k=1 \\ k\neq r}}^{n} \frac{-\varphi_{ik}\varphi_{jr}}{\lambda_k - \lambda_r}\varphi^{(k)} - \frac{1}{2}\varphi_{ir}\varphi_{jr}\varphi^{(r)}. \tag{13}$$

5. CONCLUSIONS

The integrated modeling method for the dynamic behaviour takes advantage of ambient vibration technique and finite element analysis, which can obtain not only

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the synthesis dynamic response of the whole structure but also the contributions of individual factors such as connections, restraints and damage conditions.

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