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Finite Element Analysis of RC beams Reinforced with Fiber Reinforced Polymers Bars

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Summary

Fiber reinforced polymers (FRP) have been used for many years in the aerospace and automotive industries. In the construction industry they can be used for cladding or for structural elements in a highly aggressive environment. These materials are now becoming popular mostly for the strengthening of existing structures. Fiber reinforced polymers can be convenient compared to steel for a number of reasons. There are a number of advantages in using fiber reinforced polymers. These materials have higher ultimate strength and lower density than steel.

The subject of this paper is the numerical analyses of RC structural elements. The finite element method has been chosen as a basic framework for the analyses. The main aim was to make the most effective use of the algorithms currently available for the numerical non linear analysis and to improve them, where possible, in order to reduce the number of hypothesis conditioning the results. Such results can then support the interpretation of experimental data and can be used to determine quantities that cannot be easily measured in laboratory tests.

The analysis have been carried out by using the finite element code LUSAS, widely used in both the scientific research and the design industry.

KEYWORDS: FRP materials reinforced concrete, internal reinforcement, finite element.

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1. INTRODUCTION

The finite element analysis of reinforced concrete structures can be carried out using several models according to the purpose of the research and the size of the control volume relevant for the specific application. For example the analysis could be either used to calculate the deflections on the whole structure under a given loading condition or to investigate the local effects in a particular area of the structure. In the first case we can adopt a model that describes the overall stiffness of the reinforced concrete, either cracked or not, while in the second case we may find convenient to understand where the cracking will occur, how it will develop and to compute the distribution of stresses between concrete and steel and concrete and FRP. In general the overall behavior of a structure can be successfully investigated using structural elements such as beams, shells, trusses.

Their use will limit the computational onus and simplify the definition of the structure. When it comes to investigate a reduced volume from a bigger structure, solid elements combined if necessary with structural elements are more appropriate. This is the case for this analysis as the focus is on what happens within a single structural element.

The main structural material for the systems under investigation is concrete. Concrete gives a defined shape to the structural elements and the loads are, in fact, applied directly to the concrete. The standard and FRP reinforcement, although essential, are auxiliary components. Correct modeling of the nonlinear behavior of concrete is therefore essential. The mechanical behavior of concrete has been investigated worldwide and today there is a general agreement among researchers on its characteristic properties [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. There are models to describe almost every single mechanical property of the concrete along every kind of load path. The most sophisticated ones include elasto-plastic constitutive laws with complex hardening laws, non-associative flow-rules, post yielding softening.

However these models cannot be easily used within a finite element code and simplified models have been developed to take into account only the particular aspects relevant to each specific application.

A very important aspect in the modeling of RC beams retrofitted with FRP is the representation of the interfacial behavior between the different materials. Correct modeling of the interface FRP/concrete is necessary as slippage occurs between reinforcement and concrete. Moreover slippage is affected by cracking of concrete. To allow the cracks to open it is necessary to model relative displacements between concrete and reinforcement. This is possible by mean of special interface elements or joint elements.

2. FINTE ELEMENT MODELING OF RC BEAMS REINFORCED WITH FRP REINFORCING BARS

For a rational and safe design of any strengthening work an appropriate analysis method is required. The choice of such a method is not uniquely determined and depends largely on the purpose of the analysis. Usually in engineering simple and conservative models are sought. Simple models have two main advantages: the first one is obviously the ease of use, but engineers are also interested in having models not very sensitive to parameters difficult to determine with the required accuracy and reliability.

The intrinsic complexity of structural problems implies that simple models are possible only if strong assumptions are made. This can be done only if there are sufficiently wide experimental grounds to prove that they are acceptable. Also assuming something arbitrarily implies that the model is stripped off of all the features that are deemed not to be relevant in the calculation of the quantities of interest. This means that even though the results calculated are sufficiently accurate the model is not encompassing all the aspects of the physics of the problem and some aspects are missed out or included together with others on an empirical basis. Besides, different models are usually used to calculate different quantities belonging to the same structural element.

As an example, when we calculate the ultimate bending capacity of a section of reinforced concrete we do not bother modeling the behavior of the interface between the reinforcement bars and the concrete assuming perfect adherence. The consequences of this assumption are only taken into account limiting the failure strain of the reinforcement. If we want to calculate spacing and width of cracks we must resort to models including the bond slip behavior at the interface.

If the model is to be used for a more thorough understanding of the structural behavior of the element being analyzed or to carry out a design outside the boundaries of the experimentation validating the simplified models, some of the assumption must be removed and consequently the related aspects included realistically.

As the objective of this work is not the determination of a specific quantity but rather the understanding of how RC structures reinforced with FRP work and what should be included and what not in their analysis, complex and comprehensive models are sought.

In this paper, the modeling of FRP reinforced structures is discussed with a view to defining a model as close as possible to reality, capable of replacing or integrating laboratory testing for the investigation of the structural behavior.

In order to do so, the physical problem is described first. Subsequently the models developed are described. Details of the different features included are given.

2.1 Finite Element Modeling of Reinforced Concrete

Suitable Finite Element models are required for reinforced concrete structures. Herein an overview of typical approaches, their motivations and range of applicability is given to provide background for the adopted models. Within the framework of the finite element method reinforced concrete can be represented either by superimposition of the material models for the constituent parts (i.e., for concrete and for reinforcement), or by a constitutive law for the composite concrete and embedded reinforcement considered as a continuum at the macro level.

Because of their wider range of applicability, models of the first type are more popular.

The finite element method is well suited for superimposition of the material models for the constituent parts of a composite material. Material models of this type can be employed for virtually all kinds of reinforced concrete structures. Depending on the type of problem to be solved, concrete can be represented by solid elements, shell or plate elements, or beam elements.

The reinforcement is modeled either by separate truss or beam elements (discrete representation) or by separate elements of the same type as the concrete elements, which are superimposed on the latter (embedded representation) or by distribution of reinforcement to thin layers of equivalent thickness (distributed representation).

Superimposition of concrete and reinforcement to model reinforced concrete requires constitutive models to account for bond and dowel action on the concrete-reinforcement interface.

Discrete representation of reinforcement allows modeling of bond and dowel action by means of special elements connecting adjacent nodes of concrete and reinforcement elements. The distributed representation and the embedded representation of the reinforcement, however, do not permit the use of bond elements, because the displacements of concrete and reinforcement at the interface are presumed to be the same. Consequently, the effect of bond slip can only be accounted for implicitly by modifying the constitutive relations for concrete or reinforcement.

If reinforced concrete is modeled by a constitutive law for the composite concrete and embedded reinforcement considered as a continuum, the material behavior of reinforced concrete on the macro level is described such as if this composite material was a single material. Constitutive models of this type are essentially based on the results of experimentation on reinforced concrete panels [9], [10], [11], [5]. Since reinforced concrete is treated as a single material, neither the

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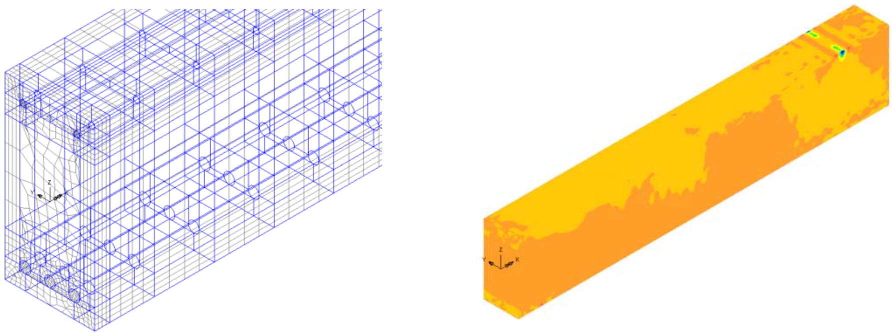
reinforcement nor the reinforcement-concrete interaction needs to be modeled separately. Models of this type are appropriate only if reinforcement is distributed uniformly.

2.1.1 Finite elements for concrete



a) b)

Figure 1. Concrete representation: a) concrete and reinforcement meshing, b) strain distribution in the concrete

Depending on the application a number of finite element types can be used for concrete. These elements can be continuum elements (solids) or structural elements (shells, beams). The above elements are generally of the same type used for any other material. Special mention can be made of multilayered shells or fibber beams in which nonlinear behavior of the main material and inhomogeneities are dealt with by subdividing an element into layers or fibers. Multilayered and fiber elements are not used in this work and therefore are not discussed but provide yet an alternative approach for modeling of reinforcement.

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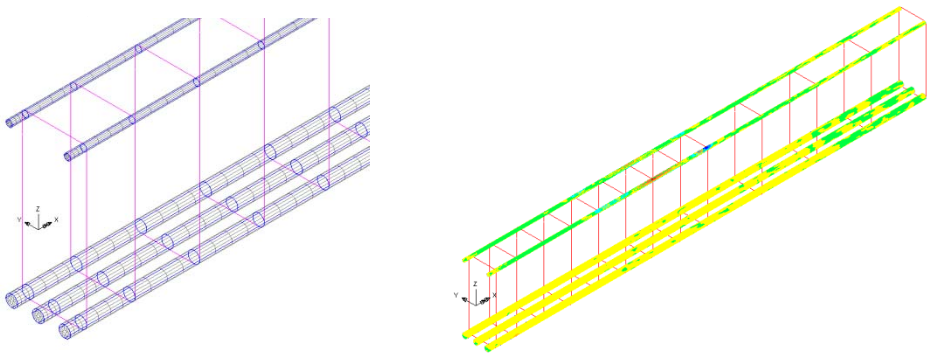
2.1.2 Representation of reinforcement



a) b)

Figure 2. Reinforcement representation: a) reinforcement meshing, b) strain distribution in the reinforcement

2.1.2.1 Discrete modeling

Discrete representation of the reinforcement is based on modeling the reinforcing bars as separate elements. Commonly, truss or cable elements are used for this purpose. However, for the investigation of structural details, occasionally two-dimensional or even three-dimensional elements are used. Truss and cable elements do not have rotational degrees of freedom and carry only axial forces.

The material behavior of truss and cable elements is described by means of the one-dimensional constitutive relations. In order to guarantee compatibility of the displacements of the concrete and reinforcement, truss and cable elements must coincide with the boundaries of the concrete elements. The node points of both types of elements must also coincide. Hence, the shape functions for the concrete elements and the truss or cable elements must be of the same order.

For instance, three-dimensional isoparametric trilinear 8-node elements and two dimensional isoparametric bilinear 4-node elements for the representation of concrete are compatible with linear 2-node truss elements for reinforcing steel. Three-dimensional isoparametric quadratic 20-node elements and two-dimensional isoparametric quadratic 8-node elements for the representation of concrete are compatible with quadratic 3- node cable elements for the reinforcing bars.

The location of the reinforcement elements is obviously determined by the layout of the reinforcement. Consequently, the boundaries of the concrete elements must follow the reinforcing bars. Thus, the layout of the reinforcement has a strong influence on the generation of the finite element mesh for a concrete structure.

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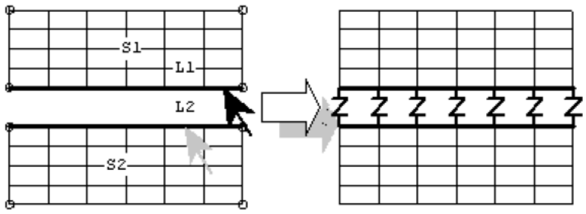
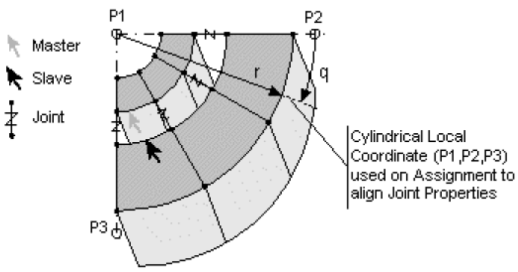
Commonly, when the overall structural behavior is investigated, coinciding nodes of concrete and reinforcement elements are assigned the same degrees of freedom. Bond slip and dowel action are either disregarded or considered implicitly by modifying the constitutive relations of concrete or reinforcement. However, especially for the investigation of the behavior of structural details, it may be necessary to model bond slip and dowel action more accurately.

For this purpose, different degrees of freedom are assigned to the coinciding nodes of concrete and reinforcement elements. Special interface elements, referred to as bond or contact elements, are employed to connect the different degrees of freedom of coinciding nodes and concrete elements. Simple interface elements connect a single node of a concrete element with a single node of a reinforcement element and are often referred to as joint elements. Such elements are basically nonlinear springs.



a)



b)

Figure 3. Interface elements: a) 2D interface element (mesh), b) 3D interface element mesh

An alternative to nodal interface elements are continuous interface elements [13]. Such elements are characterized by a continuous concrete-reinforcement interface along the entire length of the reinforcing bars. Compared with nodal interface elements, their performance is better [Keuser 1987, [14]]. Obviously discrete reinforcement elements and continuous interface elements can be combined to reinforcement-interface elements. Such elements allow modeling of the behavior of both the reinforcing bar and the interface. Moreover, if a discrete crack model is used, then the concrete to concrete interface behavior at cracks, governed by aggregate interlock can be modeled by interface elements. Interface elements are also used in this work to model the interface between FRP and concrete.

The main advantage of modeling reinforced concrete by superimposition of concrete and reinforcement elements is the relatively accurate representation of the mechanical behavior of the reinforcement and the interface. The discrete representation is the only way of accounting for bond slips and dowel action directly. Disadvantages of this approach are the great effort required for the discretization of a structure and the significant increase of the number of degrees of freedom. These disadvantages are the consequence of having to consider each reinforcing bar in the finite element mesh. Therefore, discrete modeling of the reinforcement is generally restricted to the analysis of structural details or single structural elements as beams taken in isolation from the remainder of the structure.

It is important to note, as will be recalled later on, that opening of localized cracks can be appropriately modeled only by this approach.

2.1.2.2 Embedded modeling

Separate elements for concrete and reinforcement are also used for the embedded representation. However, this representation of the reinforcement, the same type of elements with the same number of nodes and degrees of freedom and, consequently, the same shape functions are used for the concrete and reinforcement.

Hence, the embedded approach is characterized by incorporating the one-dimensional reinforcing bar into two- or three-dimensional elements Figure 4. The stiffness matrix and the internal force vector of embedded reinforcement elements only contain the contribution of reinforcement bars. They are computed by integration along the curves representing the segments of the reinforcing bars within the respective element. The embedded reinforcement elements are then superimposed on the respective concrete elements. The reinforcement bars do not have to follow the boundaries of the concrete elements. Hence, the embedded representation of the reinforcement allows generating a finite element mesh without taking much care about the layout of the reinforcement. Rather, the reinforcing bars may pass through the concrete elements in an arbitrary manner. Since the reinforcement elements and the concrete elements must be assigned the same degrees of freedom, perfect bond between concrete and reinforcement is obtained. Hence, bond slip and dowel action can only be modeled implicitly by modifying the constitutive relations for concrete or reinforcement. A disadvantage of this type of approach is that special reinforcement elements are required. Such elements may not exist in the available finite element program. Moreover, similar to the discrete approach, each reinforcing bar must be considered when preparing the input for the analysis.

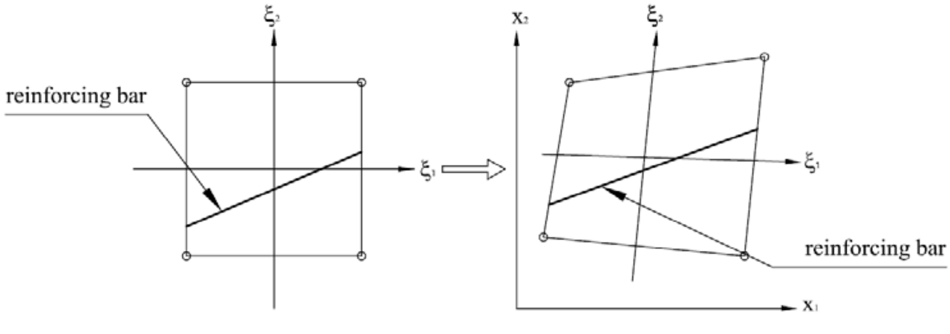


Figure 4. Embedded reinforcement element: a) in the local coordinate system, b) in the global Cartesian coordinate system.

2.1.2.3 Distributed modeling

The distributed modeling of the reinforcement is characterized by smearing reinforcing bars over an element that is superimposed onto the main concrete element. Accordingly, for instance, membrane elements with an eccentricity can be superimposed onto shell elements to model a layer of reinforcement.

The correct area of reinforcement along a unit length section of the structure is obtained assuming an equivalent thickness for the elements. The constitutive equation for such an element with a unidirectional layer of smeared reinforcement is generally referred to the local directions of the element which are parallel and normal to the reinforcing bars.

A combination of the distributed and the embedded representation of the reinforcement is obtained by smearing the reinforcement to thin layers, embedding the smeared layers into elements of the same type as the concrete elements and superimposing these elements on the concrete elements. This approach is convenient, for three-dimensional concrete structures with arbitrarily oriented layers of reinforcement.

Combining concrete and reinforcement within an element requires the assumption of perfect bond between the concrete and the reinforcement layers. Hence, bond slip and dowel action can only be modeled implicitly by modifying the constitutive relations of concrete or reinforcement.

Note that this approach is only appropriate for uniformly distributed reinforcing bars.

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Models for consideration of interface behavior were emphasized that the discrete representation of the reinforcing bars allows explicit consideration of bond slip and dowel action by means of special interface elements.

If, on the other hand, the embedded or distributed representation is chosen for the reinforcement, then the interface behavior can only be modeled implicitly by means of appropriate modifications of the constitutive relations for concrete or reinforcement.

The implicit representation of the interface behavior is characterized by an appropriate empirical or theoretical modification of the constitutive laws for the concrete and/or the reinforcement. Especially for the analysis of relatively large structures, where the reinforcement is modeled by the embedded or distributed approach and cracking is taken into account by a smeared crack model, the implicit approach is the only possibility to model the interface behavior.

Aggregate interlock at cracks is considered implicitly by introducing a modified shear modulus into the constitutive relations for concrete. The interface behavior at concrete to reinforcement interfaces, caused by bond slip, is modeled implicitly by relating the tension stiffening effect either to concrete or to reinforcement. Hence, either the constitutive law for the concrete or the one for the reinforcement is modified appropriately.

Concrete related models for consideration of tension stiffening are more popular than reinforcement related models. In concrete related models, tension stiffening is accounted for by replacing the softening branch of the tensile stress-strain diagram for plain concrete, by the respective average stress-average strain diagram for the concrete component of reinforced concrete.

The difference between plain concrete and reinforced concrete is given by the magnitude of the ultimate strain. The values for the ultimate tensile strain of reinforced concrete reported in the literature are characterized by a large scatter. However, as a rule of thumb, the ultimate tensile strain of reinforced concrete can be taken as one order of magnitude larger than the ultimate strain of plain concrete.

If modified constitutive relations for concrete are obtained from the examination (experimental or analytical) of the behavior of a specimen reinforced only in the longitudinal direction and subjected to uniaxial tension the constitutive model must be extended to multiaxial case where cracks are not necessarily orthogonal to the reinforcement.

The simplest possible approach is to apply the modified uniaxial tensile post-peak constitutive law for the concrete to the principal directions of strain without consideration of the layout of the reinforcement.

However, since tension stiffening is caused by bond stresses between the concrete and the reinforcing bars, it is preferable that concrete-related tension stiffening

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models are referred to the directions of the reinforcement. This can be done considering tension stiffening as a function of the concrete strains in the direction of the reinforcing bars.

Alternatively, a reinforcement-related tension stiffening model can be employed. In this approach the residual tensile load-carrying capacity of cracked concrete is accounted for by a modified stress strain curve for the reinforcement. Consequently, the reinforcement-related tension stiffening model is a priori referred to the directions of the reinforcing bars.

In Figure 5 basic concrete-related (a) and reinforcement-related (b) tension stiffening models are reported. The model was derived on an experimental base [15, 16].

In the figure $\bar{\varepsilon}$ denotes the average tensile strain, σ_{ts}^{C} is the average residual tensile stress carried by the concrete and ρ denotes the reinforcement ratio.

If the residual tensile load-carrying capacity of the cracked concrete is related to the reinforcing reinforcement, the additional stresses in the reinforcement $\Delta\sigma_{ts}^{S}$ is computed from σ_{ts}^{C} and ρ, i.e., $\Delta\sigma_{ts}^{S} = \sigma_{ts}^{C}/\rho$. Hence, the material parameters for the tension stiffening models shown in Figure 5 are σ_{ts}^{C}, $\bar{\varepsilon}_{ck}$, $\bar{\varepsilon}_{ts,A}$, $\bar{\varepsilon}_{ts,B}$ and $\bar{\varepsilon}_{ts,C}$.

The experimental results, on which this model is based on, indicated that the tension stiffening mainly depends on the reinforcement ratio and that is practically independent of the angle enclosed by the reinforcement and the cracks (provided the tension stiffening is evaluated in the direction of the reinforcement).

The constitutive law represented in Figure 5 relates to the uniaxial case. When it is generalized to the multiaxial case the direction of the cracks may not be orthogonal to the reinforcement. In this case the strain in the direction of the reinforcement at cracking in lower than $\bar{\varepsilon}_{ck}$ and can even be negative. For this reason, if the tension stiffening relation above is to be applied in the direction of the reinforcement, it has been proposed to assume that after cracking the initial stress in the concrete, in the direction of the reinforcement, is σ_{t}^{C} and the linear branch is omitted. In this approach there is, therefore, a discontinuity of the stresses at the initiation of cracking.

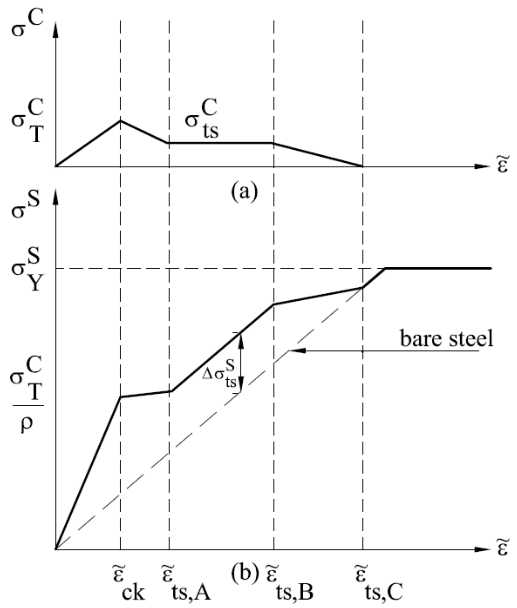

Figure 5: Modeling of tension stiffening by modifying the constitutive laws: a) for concrete, b) for reinforcement

The concrete-related model of Figure 5(a) yields almost the same structural response as the reinforcement-related model (Figure 5(b)), provided it is formulated in terms of the concrete strain in the direction of the reinforcement.

However, if the concrete related model is formulated in terms of the principal tensile strains of the concrete, the stiffening effect vanishes too early. It is also noted that in reinforcement related tension stiffening models, the compressive stresses in the concrete are somewhat overestimated. The reason for this is the neglect of the stress-carrying capacity of the cracked concrete.

The above simple model has been described to introduce the basic principles of the implicit representation of the bond slip behavior. There exist a large number of very refined models for implicit representation of bond slip that are derived on theoretical considerations and incorporated in a number of constitutive laws, proposed for reinforced concrete, featuring the smeared crack concept [5]. These refined models include also other aspects of concrete behavior like, dilatancy; aggregate interlocking, reinforcement dowel action and damage accumulation under cyclic loading. This class of models will be examined in the chapter on crack modeling but the focus will be on aspects other than bond slip behavior.

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In the context of the present work, the implicit representation of the bond slip behavior has the relevant shortcoming that it cannot be effectively combined with a discrete cracking approach and it is therefore not used in the applications.

However, the method is appealing for applications to macroscale problems because interface elements do not need to be specified.

3. CONCLUSIONS

From the inspection of the results presented in this section the following conclusive remarks can be drawn:

• the proposed nonlinear numerical model of RC beams reinforced with FRP in flexure is capable of capturing many important aspects of the behavior of these structural systems, up to failure;

• cracking plays a key role in the development of the stresses at the FRP/concrete interface;

• bond-slip between concrete and reinforcement rebars is to be taken into account in order to get accurate results;

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