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Numerical Simulation of Wave Transformation in the Fars Gulf

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Summary

A numerical model has been developed for the simulation of wave transformations that is applicable to irregular bottom topographies. Model is based on nonlinear parabolic mild slope equation and could simulate wave shoaling, refraction, diffraction together. The numerical model has been solved by Mac Cormack Method with using Point Gauss Seidel Iteration Method. Wave phase gradient of Ebersole (1985) has been used to determine local wave number in the model. The model is applicable to arbitrary varying bottom topographies. Unidirectional waves are considered for the numerical model. It is a reliable tool to simulate wave shoaling, refraction and diffraction. Model predictions are compared with the physical experiment over semicircular shoaling area. Model has been applied to Fars Gulf located on the Mediterranean Sea Coast of Iran which has an industrially important role for Iran since a great industrial harbour and oil pipelines are located.

KEYWORD: Sea wave, refraction, diffraction, wave shoaling, Numerical Simulation

1. INTRODUCTION

Berkhoff (1972) solved the wave propagation from deep water to shallow water under combined refraction and diffraction effect with an elliptical equation. This equation is called as mild slope equation in the literature.

$$\frac{\partial}{\partial x}(CCg\frac{\partial\tilde{\phi}}{\partial x})+\frac{\partial}{\partial y}(CCg\frac{\partial\tilde{\phi}}{\partial y})+w^2\frac{C_g}{C}\tilde{\phi}=0 \qquad (1.1)$$

C is the wave celerity, C_g is the group velocity, w is angular frequency, $\tilde{\phi}$ is two dimensional complex potential function. It is assumed that the bottom topography has a mild slope for this equation. It is to say $|\nabla h|/kh << 1$ where h, k, ∇ are water depth, wave number, horizontal gradient operator, respectively. This elliptical mild slope equation includes combined refraction-diffraction, shoaling and reflection effects.

Radder (1979) recommended parabolic approach because of complexity of solving elliptical mild slope equation. Elliptical mild slope equation is reduced to Helmholtz equation and parabolic equation is obtained after this reduction. Parabolic equation is applicable to the short waves over irregular bathymetries in large coastal areas if reflection is negligible.

Kirby and Dalrymple (1983) solved parabolic equation for the combined refraction-diffraction of Stokes waves by mildly varying topography. In this approach, only unidirectional waves and forward scattered components arising from interaction with structures or inhomogenities of the domain are considered. This parabolic equation is obtained by a WKB- type expansion for the velocity potential. That represents a wave travelling in a prespecified direction.

Dalrymple et al. (1984) developed a parabolic model to simulate combined refraction- diffraction phenomena including dissipation of wave energy. They focused on the nature of the localized energy dissipation. A shadow region of low wave energy exists due to the region of localized dissipation.

Kirby (1986) developed rational approximations based on minimax principles to overcome small angle incidence limits in the parabolic equation.

In this study, parabolic equation proposed by Kirby and Dalrymple (1983) is solved as the governing equation including both refraction and diffraction effects. This equation amplitude order, therefore it is simpler to solve than the potential order equations. This nonlinear equation gives approximate results with the general elliptic mild slope equation and it overcomes small angle incidence restriction of the linear parabolic equation. Furthermore, the definition of wave number recommended by Ebersole (1985) is used in this study. Ebersole (1985), developed an alternative equation that expresses the propagation of linear waves over mild sloping bathymetry and also defined wave number as a function wave phase derivative.

2. THEORY

Yue and Mei (1980) obtained a parabolic equation that described the propagation of weakly nonlinear Stokes waves in a specified direction over constant depth. This nonlinear Schrödinger equation is given below.

$$2iA_x + \frac{1}{k}A_{yy} - K'|A|^2 A = 0 \tag{2.1}$$

A is complex amplitude; x is the principal direction of propagation. k referred wave number.

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$$K' = k^3\left(\frac{C}{C_g}\right)D \tag{2.2}$$

$$D = \frac{\cosh 4kh + 8 - 2\tanh^2 kh}{8\sinh^4 kh} \tag{2.3}$$

Equation (2.1) is applicable to the bathymetry with a constant depth. Kirby and Dalrymple (1983) proposed a more general formulation suitable for slow but arbitrary depth variations. Nonlinear effects are important when waves are focused by topographic variations. The inclusion on nonlinearity enhances the lateral spread of energy away from regions with high waves.

$$\begin{aligned} &2ikCC_gA_x + 2k(k-k_0)(CC_g)A + i(kCC_g)_x A + \\ &(CC_gA_y)_y - k(CC_g)K'|A|^2 A = 0 \end{aligned} \tag{2.4}$$

k	**: Wave number (1/m)**
C	**: Wave celerity (m/sec)**
C_g	**: Group velocity (m/sec)**
k_0	**: Wave number in deep water (1/m)**
A	**: Wave amplitude (m)**

Subscripts x and y define the first derivatives in x and y direction, respectively.

Derivation of governing equation that is solved in this study can be found below where k_0 is the initial wave number at deep water.

Ebersole (1985) showed the relationship between wave number and phase function with the equation below where s is wave phase function and H is wave amplitude in the equation (2.5).

$$|\nabla s|^2 = k^2 + \frac{1}{H}\left[\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{1}{CC_g}\left(\frac{\partial H}{\partial x}\frac{\partial CC_g}{\partial x} + \frac{\partial H}{\partial y}\frac{\partial CC_g}{\partial y}\right)\right] \tag{2.5}$$

In this study, equations (2.4) and (2.5) are used to solve the propagation of travelling waves from deep water to shallow water in a prespecified direction over slowly arbitrary bottom topography under combined refraction-diffraction effects.

3. NUMERICAL MODEL

MacCormack method is applied to equation (2.4) using Point Gauss Seidel iteration method. MacCormack method is a multistep method. Firstly, forward finite difference approximations are used to obtain the predictor and then backward finite difference approximations are applied to the governing equation to find the

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corrector. This method gives more realistic results then the other methods since it assures static stability. Point Gauss Seidel Iteration provides to reach the convergence more rapidly because the current variables of dependent variable are used to compute the neighbouring points as soon as they are available.

4. APPLICATION TO SEMICIRCULAR SHOALING AREA

Whalin (1971) tested wave refraction and diffraction phenomena over a semicircular shoaling area. The model topography was determined with the equations below and shown in Figure 1.

$$h(x,y) = 0.4572 \qquad (0 \le x \le 10.67 - G(y)) \tag{4.1}$$

$$h(x,y) = 0.4572 + \frac{1}{25}\left(10.67 - G(y) - x\right) \qquad (10.67 - G(y) \le x \le 18.29 - G(y)) \tag{4.2}$$

$$h(x,y) = 0.1524 \qquad (18.29 - G(y) \le x \le 21.34) \tag{4.3}$$

$$G(y) = \left[y(6.096 - y)\right]^{1/2} \qquad (0 \le y \le 6.096) \tag{4.4}$$

In these equations, x and y are in meters.

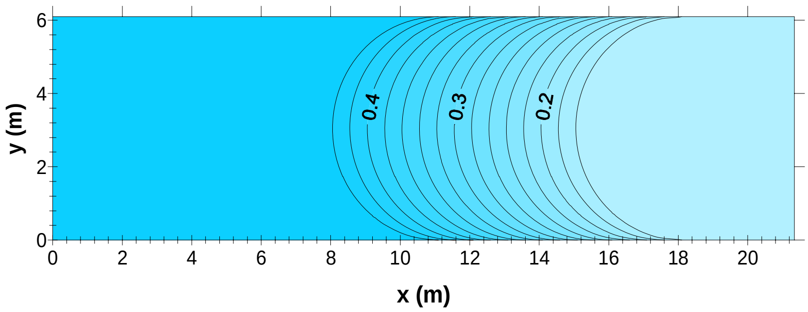

Figure 1: Tank bathymetry (h(m))

The bathymetry is symmetric to the centerline y=3.048m. Model topography shoals from 0.4572m to 0.1524m.

The wave amplitude distribution along the centerline for the wave approaching normally with the wave period T=2sec has been shown in Figure 2 where the consistency between numerical and physical results can be observed. Diffraction effects become an important role after the distance of 15m in x direction and a caustic region occurs so the linear theory fails in that region. With the use of parabolic mild slope equation, this problem has been overcome.

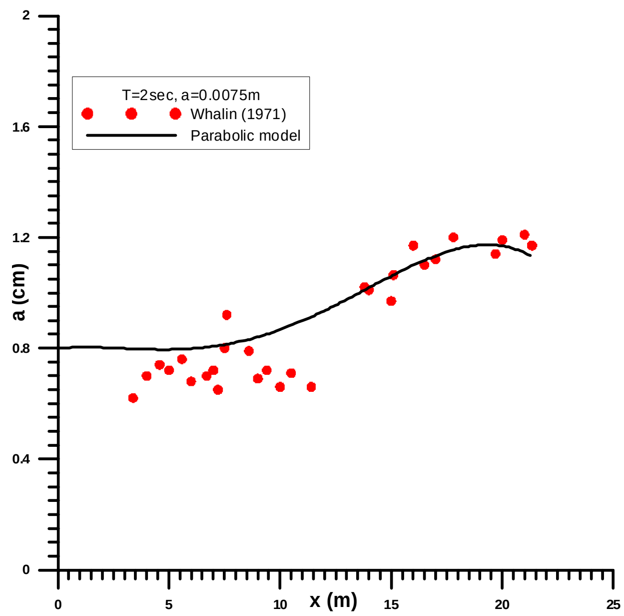

Figure 2: Wave amplitude (cm) along x-axis (y=3.048m, T=2 sec., a=0.0075m, $\theta = 0^0$)

5. APPLICATION TO FARS GULF

Fars Gulf is located in the Mediterranean Coasts of Iran. Fars Gulf coast has a great harbour and oil pipelines therefore it is important from the coastal engineering point of view. The dominant wave direction is SSW. Significant wave period T=9.02sec and significant wave height Hs=5.46m for the return period t=50 years. In this study, a part of Fars Gulf coast has been numerically modeled. In Figure 3, the bathymetry has been given. Depths are in meters. In Figure 4, the predicted wave height distribution has been shown. The wave heights are in meters, too.

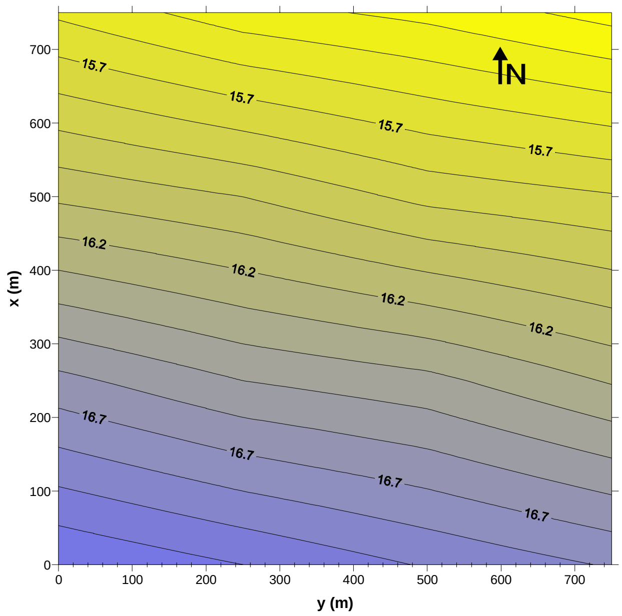

Figure 3: Bathymetry of Fars Gulf Coastal Region (Depths are in meters.)

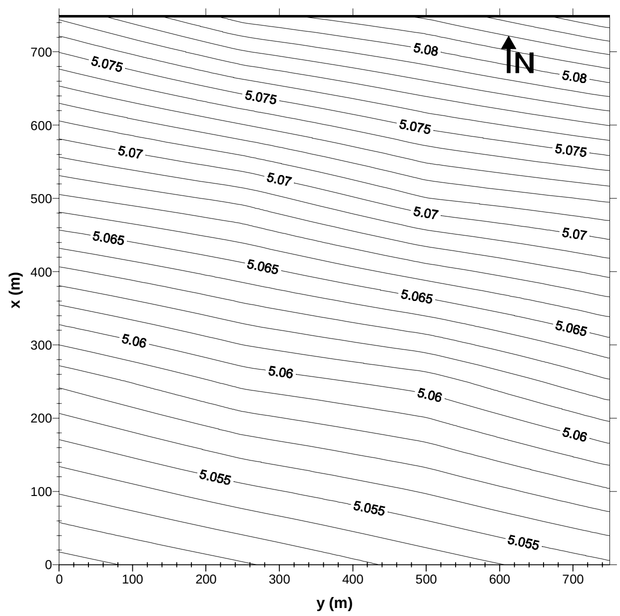

Figure 4: Wave height distribution (Wave heights are in meters.)

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6. CONCLUSIONS

A numerical model that simulates the propagation of waves from deep water to shallow water under shoaling, refraction and diffraction effects has been presented. Nonlinear effects have been included in the solution. Mac Cormack method is applied to the governing model equation using Point Gauss Seidel Iteration method. The nonlinear parabolic model has been used in this study so as to overcome caustics problems of the linear theory. The use of wave phase gradient provides more accurate results to obtain the local wave number. The model can be successfully used over arbitrary bathymetries in the prespecified direction.

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