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Analysis of Active Seismic Response for Controlled Structures Using Stochastic Models

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Summary

This paper investigates the seismic response of active controlled structures using a stochastic approach. The study is at a starting point for the author's direction of research and it is intended to focus on principles and methodologies for simple structures and simple control strategies. The research is envisioning validating stochastic domain strategies for controlled structures that have been already validated in time and frequency domains.

In a relative recent past the author has been studied structural active control. Especially energy base optimal active control was used as the strategy of control. This strategy was used for large structures (bridges and buildings). Variants for full state and reduced order controllers were tested. Other variants were used for taking into account the noise and time lag. Overlapping and disjoint distributed controllers have been in the views, too. Time-history and frequency analysis were done. For showing the validity of the methodology, internationally proposed benchmark structures and conditions have been employed.

A stochastic approach for the seismic response of structures with control is proposed. The seismic action is described as a system with a white noise input and a seismic-like time-history response. A non-negative envelope function is modeling the input stationary random process.

Application of the methodology on very simple structures is in the views in order to begin a longer term investigation in the field. The main results show the differences between the controlled and the non-controlled cases.

From the stochastic point of view, results show that the controlled structures behave very well. It is confirmed that this type of analysis is very useful in judging a control methodology together with other analysis as time-history response and/or frequency response analysis.

KEYWORDS: stochastic response, passive control, seismic action, structural model.

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1. INTRODUCTION

Structural active control is seen as a direction of development for mitigating the effects of actions as natural disasters against civil engineering constructions. The author has been studied this broad field in a relative recent past [1-5]. Especially energy base optimal active control was used as the strategy of control. This strategy was used for large structures (bridges and buildings). Variants for full state and reduced order controllers were tested. Other variants were used for taking into account the noise and time lag. Overlapping and disjoint distributed controllers have been in the views, too. Time-history and frequency analysis were done. For showing the validity of the methodology, internationally proposed benchmark structures and conditions have been employed.

This paper investigates the seismic response of active controlled structures using a stochastic approach. The study is at a starting point for the author's direction of research and it is intended to focus on principles and methodologies for simple structures and simple control strategies. The research is envisioning validating stochastic domain strategies for controlled structures that have been already validated in time and frequency domains.

2. FREQUENCY RESPONSE

The frequency response can be seen as the degree of amplification of the input or the ratio between the output and the input of a structure under harmonic excitation. In this case, the excitation is usually written:

$$f(t) = e^{iwt} \tag{1}$$

i.e. a complex harmonic function with amplitude equal to unity and with a circular frequency ω.

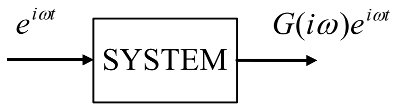

Figure 1. The frequency response principle

Graphically, the frequency response principle is shown in Figure 1.

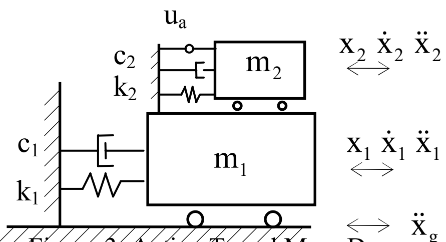

Figure 3. Active Tuned Mass Damper

As an example, in Figure 4, the effect of the TMD over the acceleration response in frequency domain for the problem shown Figure 3 is presented. It is obvious that, for this kind of action, the passive control is very effective.

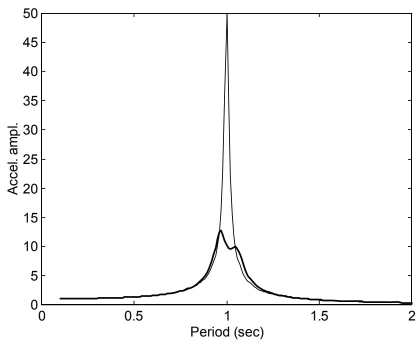

Figure 4. Acceleration amplification

3. MODELING THE SEISMIC ACTION

Artificial accelerograms are extremely useful in structural analysis because of the possibility to have a clearer image of the action and of the response of a structure under the earthquake excitation.

Figure 5 shows a general concept in time-history analysis using generated accelerograms.

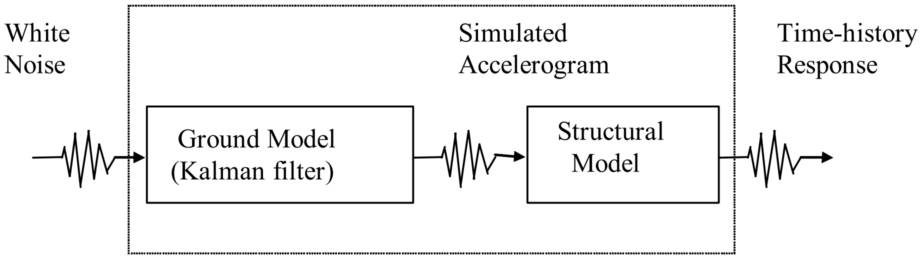

Figure 5. Time-history response using generated accelerogram

In mathematical terms, the earthquake acting the structure is modeled as a nonstationary random process, as it is given in Equation (2), [6]:

$$\ddot{x}_g(t) = \psi(t)\ddot{x}(t) \tag{2}$$

where $\psi(t)$ is a deterministic nonnegative *envelope function* and $\ddot{x}(t)$ is a *stationary random process*.

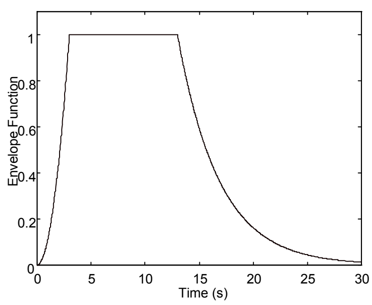

Figure 6. Envelope function

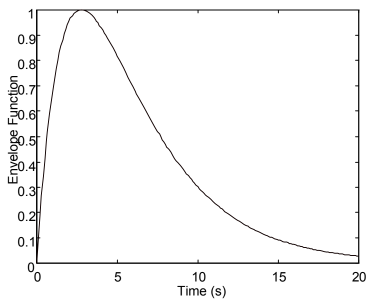

Figure 7. Envelope function

Based on statistical observations there were proposed many formulae for the envelope function, [6,7]. One example is expressed by Equation (3):

$$\psi(t) = \frac{e^{-at} - e^{-bt}}{\max(e^{-at} - e^{-bt})} \tag{3}$$

Letting *a=.25* and *b=0.5*, this function is drawn in Figure 6.

Another very used envelope function is that from Equation (4):

$$\psi(t) = \begin{cases} 0 & t < 0 \\ \left(\frac{t}{t_1}\right)^2 & 0 \le t \le t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-c(t-t_2)} & t > t_2 \end{cases} \tag{4}$$

and, for *t1=3, t2=13, c=0.26*, this function is plotted in Figure 7.

For computing this, a MATLAB function, **psifc**, is shown below:

```
function [psi,t]=psifc(t1,t2,tf,csp,np);
t=linspace(0,tf,np);
for i=1:np
if t(i)<t1;
psi(i)=(t(i)/t1)^2;
else
if t(i)<t2;
psi(i)=1;
else
psi(i)=exp(-csp*(t(i)-t2));
end
end
end
```

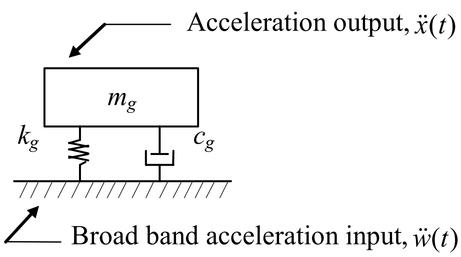

Figure 8. Kanai-Tajimi model for the ground

In order to generate the stationary random process, $\ddot{x}(t)$, from Equation (2), it is widely accepted the *Kanai-Tajimi model* for the ground, shown in Figure 8, [7]. Even if this model is one of the simplest models, it is very practical and gives good results in applications to structural control problems.

The equation describing this model is (5):

$$\ddot{x}(t) + 2\varsigma_g \omega_g \dot{x}(t) + \omega_g^2 x(t) = 2\varsigma_g \omega_g \dot{w}(t) + \omega_g^2 w(t) \tag{5}$$

where:

$\ddot{x}(t)$ is the absolute acceleration of the mass;

$\ddot{w}(t)$ is the acceleration of the base (white noise);

$\omega_g = \sqrt{\dfrac{k_g}{m_g}}$ is the ground circular frequency;

$\varsigma_g = \dfrac{c_g}{2m_g\omega_g}$ is the ground damping coefficient.

For the system (5) the *frequency response function* is (6)

$$H(\omega) = \frac{\omega_g^2 + 2i\varsigma_g\omega_g\omega}{\omega_g^2 - \omega^2 + 2i\varsigma_g\omega_g\omega} \tag{6}$$

and the *power spectral density* is shown by Equation (7):

$$\Phi(\omega) = \frac{\omega_g^4 + (2\varsigma_g\omega_g\omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\varsigma_g\omega_g\omega)^2} S = \frac{1 + 4\varsigma_g^2 \frac{\omega^2}{\omega_g^2}}{\left(1 - \frac{\omega^2}{\omega_g^2}\right)^2 + 4\varsigma_g^2 \frac{\omega^2}{\omega_g^2}} S \tag{7}$$

with S being the *intensity* of the white noise. S is actually the power spectral density of the white noise.

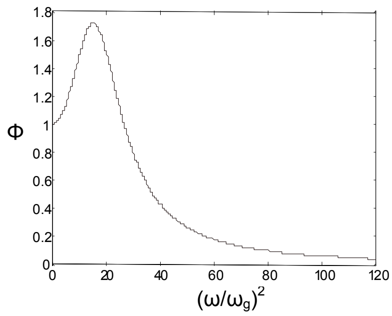

Figure 9. Function from Equation (7)

Figure 9 shows the power spectral density function described by Equation (7), when $S = 1\text{m}^2/\text{sec}^4$ and $\varsigma_g = 0.65$.

Note that the model shown in this section, Equation (5), can be also seen as a Kalman filter and it can be easily employed using a state equation (please see the next section).

4. STOCHASTIC RESPONSE

The Equations (2) and (5) can be transformed in a state form (3.19):

$$\begin{cases} \dot{\mathbf{x}}_1(t) = \mathbf{A}_g \mathbf{x}_1(t) + \mathbf{B}_g w(t) \\ \ddot{x}_g(\mathbf{t}) = \psi(t)\mathbf{C}_g \mathbf{x}_1(t) \end{cases} \tag{8}$$

where: $\mathbf{x}_1(t) = \begin{Bmatrix} x(t) - w(t) \\ \dot{x}(t) - \dot{w}(t) \end{Bmatrix}$ is the state vector and

$$\mathbf{A}_g = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\varsigma_g\omega_g \end{bmatrix}, \quad \mathbf{B}_g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \text{and } \mathbf{C}_g = \begin{bmatrix} -\omega_g^2 & -2\varsigma_g\omega_g \end{bmatrix}$$

Above, the state equation for the action was written. Now, supposing the structure's equation of motion is (9)

$$\dot{\mathbf{x}}_s(t) = \mathbf{A}\mathbf{x}_s(t) + \mathbf{B}\ddot{\mathbf{x}}_g(t) \tag{9}$$

then the whole system (earthquake acceleration generation plus structure) will be written in a new state equation (10)

$$\dot{\mathbf{y}}(t) = \mathbf{A}_e(t)\mathbf{y}(t) + \mathbf{B}_e w(t) \tag{10}$$

because a new state vector $\mathbf{y}(t) = \begin{Bmatrix} \mathbf{x}_s(t) \\ x_g(t) \end{Bmatrix}$ have been introduced and, therefore, $\mathbf{A}_e(t) = \begin{bmatrix} \mathbf{A} & \psi(t)\mathbf{B}\mathbf{C}_g \\ \mathbf{O} & \mathbf{A}_g \end{bmatrix}$, $\mathbf{B}_e = \begin{bmatrix} \mathbf{O} \\ \mathbf{B}_g \end{bmatrix}$.

Looking to the equation (10), it is seen that the input is the white noise. Also it should be observed that the system matrix is time dependent.

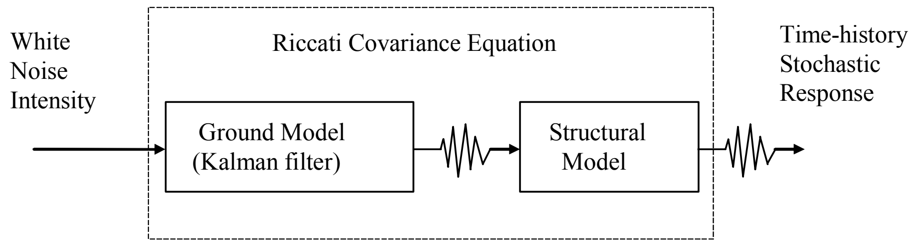

Figure 10. Stochastic response principle

Using the Equation (10), one can compute the stochastic response shown in the Figure 10, [9,10]. The *covariance*, time dependent, matrix of the response is the solution of the Riccati equation (11):

$$\dot{\mathbf{R}}(t) = \mathbf{A}_e(t)\mathbf{R}(t) + \mathbf{R}(t)\mathbf{A}'_e(t) + \mathbf{B}_e W \mathbf{B}'_e \tag{11}$$

where the prime sign means the transpose of a matrix and $W=2\pi S$. The matrix $\boldsymbol{R}(t)$ is defined as follows, Equation (12), [11]:

$$\mathbf{R}(t) = E[(\mathbf{y}(t) - \mathrm{m}_{\mathbf{y}})(\mathbf{y}(t) - \mathrm{m}_{\mathbf{y}})'] = E[\mathbf{y}(t)\mathbf{y}'(t)] \quad (12)$$

where $E[.]$ means expectation and $m_{\mathbf{y}}=E[\mathbf{y}(t)]=\mathbf{0}$ is the mean value of $\boldsymbol{y}$.

5. EXAMPLE

For the same example of a TMD from Figure 3, the envelope function (3) with $a=0.25$, $b=0.5$, the ground coefficients $\omega_g = 6.28$, $\varsigma_g = 0.2$, and $S=9cm^2/sec^4$ was chosen.

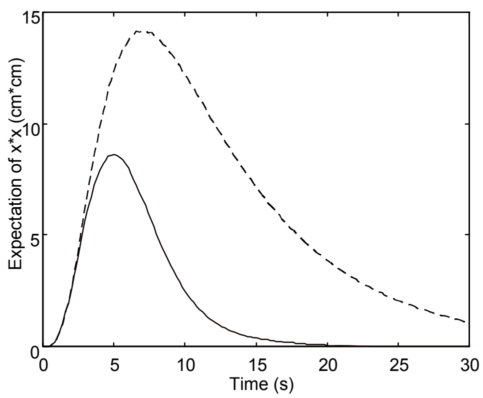

Figure 11. Displacement covariance

Figure 11 presents a comparison between the covariance of the displacement in the case with TMD and in the case in which the TMD is not employed. The effectiveness of the passive control is seen again.

6. CONCLUSIONS

A stochastic approach for the seismic response of structures with control is proposed. The seismic action is described as a system with a white noise input and a seismic-like time-history response. A non-negative envelope function is modeling the input stationary random process.

From the stochastic point of view, results show that the controlled structures behave very well. It is confirmed that this type of analysis is very useful in judging a control methodology together with other analysis as time-history response and/or frequency response analysis. The main results show the differences between the controlled and the non-controlled cases.

Application of the methodology on very simple structures is in the views in order to begin a longer term investigation in the field.

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