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The Seismic Soil-Structure Interaction Effect for Pile-Raft Systems

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Summary

The horizontal displacement and the base rotation for a structural system resting on general vertical pile-raft foundation is evaluated. Two different cases for the boundary condition at the top of the pile are considered: pinned and fixed head. The effect of these displacements about the shear force in the seismic design of the structural system is analysed. The influence on the twisting of the structure is discussed too.

KEYWORDS: pile-raft system, soil-structure interaction, vibration period, base displacement

1. INTRODUCTION

The analysis of the structural seismic response includes, in a more generalised case, the influence of the Soil Structure Interaction (SSI). The equivalent fixed-base models for SSI systems can be accepted only for those sufficiently rightful cases. In fact, a more developed analysis is necessary. The analysis must take into account the displacements of the base because of the surrounding soil flexibility. A practical case is the structures standing on a pile-raft system, wich following are analysed.

The piled rafts, subjected at strong horizontal forces, such as the base shear force from the seismic action, moves in horizontal direction because of the piles deformation. The raft is very stiff (with respect to the piles) and all the piles have the same displacement by translation. The action of the moment transmitted by the structure to the foundation system leads to its rotation. It is considered the case when the moment is placed in the vertical principal plane of inertia. Because of the piled raft stiff, the ends of the piles after the foundation rotation will be situated in a plane, which can be adequately fixed.

By the translation of the piled raft, the same displacement is transmitted on the height of the building and by its rotation; the displacement on height will be proportional with the distance from the base. These displacements correct the displacements from the fixed-base model and accordingly the dynamic response of the structural system is modified.

2. THE INFLUENCE OF THE BASE DISPLACEMENTS ON THE VIBRATION PERIOD

If we refer to the vibration periods, these depend of the real displacements which take place in the system. Thus, at a certain level, at one point, the displacement can have the following components:

- the displacement because of the structure bending considered with a fixed base (x_M);
- the displacement because of the shear force of the structure with fixed base (x_Q);
- the displacement generated by the translation (sliding) of the piled raft (x_R);
- the displacement caused by the rotation of the piled raft (x_φ).

If it is assumed that the analysis for the determination of the fundamental period T_1 is make in the principal plane of inertia xOz, we can use, for example, the Rayleigh method to obtain the expression [1]:

$$T_1 = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{x_{ST,\max}}{gA_{n1}}} \tag{1}$$

where:

T_1 is the fundamental period of vibration, in seconds;

ω_1 is the fundamental circular frequency, in radians per second;

$x_{ST,max}$ is the maximum static displacement of structure, which is produced by the gravitational forces applied in the degrees of freedom directions;

g is the gravity acceleration (= 981 cm/s^2);

A_{n1} is the shape coefficient corresponding to the degree of freedom.

Particularly, for a multistory structure can be shown that the medium value of the shape coefficient is approximate 1,41 [1].

In this conditions the fundamental period T_1 may be expressed as:

$$T_1 = \xi_n \sqrt{x_{ST,\max}} \tag{2}$$

where ξ_n is a dimensionless coefficient which depends on the gravitational loads and number of levels.

For instance, if the gravitational loads are the same for each floor, ξ_n is between 0,2 for $n = 1$ and 0,165 for $n \geq 12$ [1].

From (2) we find out directly that the period T_1 depends on maximum static displacement and, implicitly on the flexibility degree of the structure, respectively on its stiffness.

The displacement $x_{ST,max}$, as well as the displacemnt $x_{n,max}$ at the n level, consists of those foregoing displacements:

$$x_{ST,\max} = x_M + x_Q + x_R + x_\varphi \tag{3}$$

The displacement x_M and $x_Q = x_V$ are produced by the gravitational forces applied to horizontal direction on the structure with fixed base.

The displacements x_R and x_φ depend on the boundary conditions because the piles may be considered with pinned or fixed head.

3. THE CALCULUS OF DISPLACEMENTS $x_R = x_0$ AND $x_\varphi = \varphi_0$

3.1 Vertical pinned-head pile

3.1.1. The displacement x_0 produced by the level horizontal forces

The pile is considered as a beam of semi-infinite length on elastic foundation, subjected at the upper end by a horizontal force q_i [2], [3], (Fig. 1.a.). From elementary beam theory, the deflection under the load, at the end ($z = 0$) is:

$$x_{0i} = \frac{2q_i\alpha_i}{b_i c_h} \tag{4}$$

where:

b_i is the width or diameter of the pile in the normal direction on the bending plane, in cm;

c_h is the coefficient of horizontal subgrade reaction, in N/cm^3, considered constant on depth; this coefficient can be determined by using the horizontal loading test, for which the displacement of the upper end acted by a horizontal force is measured;

α_i is a dimensionless parameter given by the expression:

$$\alpha_i = \sqrt[4]{\frac{c_h b_i}{4EI_{pi}}} = \left(c_h b_i / 4EI_{pi}\right)^{1/4} \quad (5)$$

where

E is the modulus of elasticity for constitutive material of the pile;

I_{pi} is moments of inertia with respect to the bending axis of the pile cross-section.

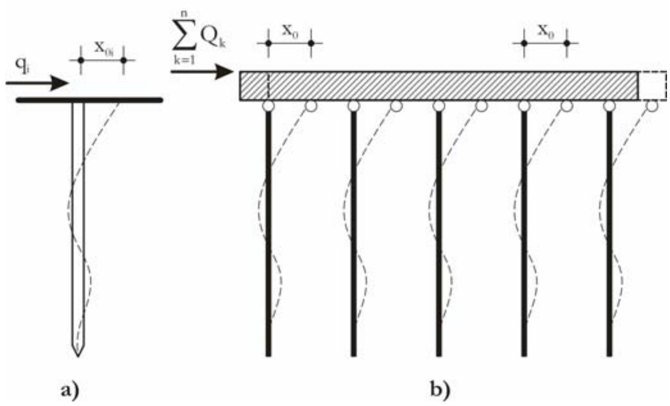

Figure 1. Vertical pinned-head pile: **a)** pile subjected by a horizontal force q_i; **b)** the horizontal displacement x_0 of the raft

For $q_i = 1$, the translation stiffness of the pile, can be expressed as [2], [4-5]:

$$k_i^* = \frac{1}{x_{oi}} = \frac{b_i c_h}{2\alpha_i} \quad (6)$$

The stiffness of a m piles group will be:

$$K^* = \sum_{i=1}^{m} k_i^* = \sum_{i=1}^{m} \frac{b_i c_h}{2\alpha_i} \quad (7)$$

The horizontal displacement x_0 of the raft (Fig. 1.b), produced by the shear force

$$V = Q = \sum_{k=1}^{n} Q_k \quad (8)$$

where n is the number of levels, can be expressed as:

$$x_0 = \frac{\sum_{k=1}^{n} Q_k}{\sum_{i=1}^{m} \frac{b_i c_h}{2\alpha_i}} = \frac{Q}{K^*} \tag{9}$$

If the piles have the same cross-section and orientation, the previous expression becomes:

$$x_0 = \frac{\sum_{k=1}^{n} Q_k}{mk^*} = \frac{2\alpha \sum_{k=1}^{n} Q_k}{mbc_h} \tag{10}$$

3.1.2. The raft rotation produced by the bending moment

Considering that the centroid of the building coincides, on the vertical direction, with the centroid of the piles cross-sections, the raft will be acted by the bending moment (Fig. 2)

$$M_0 = M_{Q_k} + \sum_{k=1}^{n} Q_k \cdot h_r \tag{11}$$

which is a vector in *y-y* direction (h_r is the height of the raft foundation). The axial force acted on the *i* pile, produced by the moments M_0 is:

$$N_{pi} = \frac{M_0 x_i}{\sum_{i=1}^{m} A_{pi} x_{pi}^2} A_{pi} \tag{12}$$

where:

A_{pi} is the area of the *i* pile cross-section;

$x_{pi} = x_i$ is the distance from the *i* pile to the y-y axis;

The endmost piles, with respect to the *y-y* axis, (Fig. 2), will be most loaded (identical piles); if the *m* piles in the group are identical, it results:

$$N_{pi} = N_{p\max} = \frac{M_0 x_{\max}}{\sum_{i=1}^{m} A_{pi} x_{pi}^2} A_{pi} = \frac{M_0 x_{\max}}{\sum_{i=1}^{m} x_{pi}^2} \tag{13}$$

where x_{max} is the distance from the *y-y* axis to the endmost piles.

In the relations (12) and (13), the denominator is the pile moment of inertia, if we ignore the moments of inertia with respect to own axis:

$$I_y = \sum_{i=1}^{m} A_{pi} x_{pi}^2 \tag{14}$$

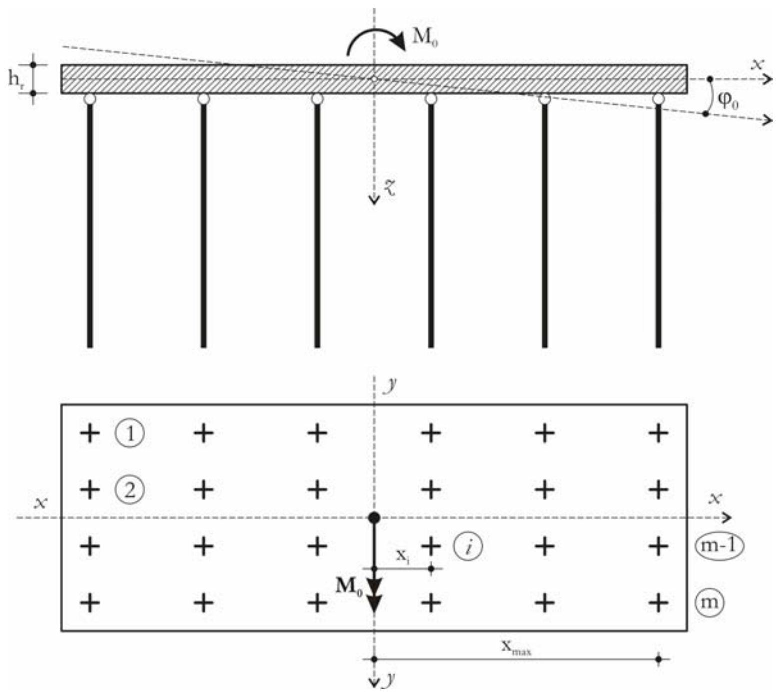

Figure 2. Raft subjected by a bending moment M_0

The raft rotation φ_0 can be expressed, from geometrical considerations, as:

$$\varphi_0 \cong tg\varphi_0 = \frac{\Delta l_{\max}}{x_{\max}} \tag{15}$$

where Δl_{max} is the shortening (or elongation) of the endmost pile, and it has the expression:

$$\Delta l_{\max} = \frac{N_{p\max} \cdot l_p \cdot s}{E_p A_p} \tag{16}$$

where:

l_p is the pile lengh;

s is a coefficient which takes into consideration the settlement of the pile tip and the skin friction effect; it is determined on testing piles;

E_p, A_p are the elasticity modulus of the constitutive material, respectively the cross-sectional area of the pile.

Therefore, the rotation of the raft results:

$$\varphi_0 = \frac{M_0 s l_p}{\left(\sum_1^m A_p l_p\right) E_p x_{\max}} = \frac{M_0 s l_p}{E_p x_{\max} \sum_{i=1}^m A_{pi} l_{pi}} \tag{17}$$

For a group of m identical piles results:

$$\varphi_0 = \frac{M_0 s}{m x_{\max} E A_p} \tag{18}$$

3.2. Vertical fixed-head pile

3.2.1. The displacement x_0

By the raft translation, at the pile head appear horizontal shear forces and bending moments. From the invariability condition of the angle in the fixed section of the pile (Fig. 3a), the value of the bending moment at the pile head can be expressed as:

$$m_i = \frac{q_i}{2\alpha_i} \tag{19}$$

The displacement of the pile head is obtained:

$$x_{0i} = \frac{2 q_i \alpha_i}{b_i c_h} - \frac{2 m_i \alpha_i^2}{b_i c_h} = \frac{q_i \alpha_i}{b_i c_h} \tag{20}$$

For $q_i = 1$ the translation stiffness of the pile can be written [2], [4-5]:

$$k_i^* = \frac{1}{x_{oi}} = \frac{b_i c_h}{\alpha_i} \tag{21}$$

and the translation stiffness of the piles group results:

$$K^* = \sum_{i=1}^m k_i^* = \sum_{i=1}^m \frac{b_i c_h}{\alpha_i} \tag{22}$$

Therefore, the raft displacement x_0 is (Fig. 3.b):

$$x_0 = \frac{\sum_{k=1}^{n} Q_k}{K^*} = \frac{\sum_{k=1}^{n} Q_k}{\sum_{i=1}^{m} \frac{b_i c_h}{\alpha_i}} \quad (23)$$

For the case of the identical piles, the raft displacement can be expressed as:

$$x_0 = \frac{\alpha}{mbc_h} \sum_{k=1}^{n} Q_k \quad (24)$$

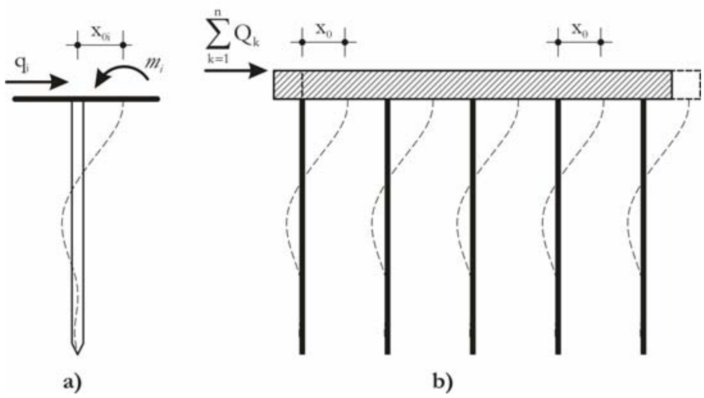

Figure 3. Vertical fixed-head pile: **a)** pile subjected by a horizontal force q_i; **b)** the horizontal displacement x_0 of the raft

3.2.2. The global rotation of the raft produced by the structure bending

The bending moment M_0 which acts on the raft (Fig. 4) is given by the horizontal forces Q_k at the floors level, and by eventualy eccentric gravitational forces ($M_{Q,P}$). Adding the effect of the thickness of the foundation raft, for Q_k forces, the M_0 bending moment can be expressed as:

$$M_0 = M_{Q,P} + h_r \sum_{k=1}^{n} Q_k \quad (25)$$

The rotation φ_0 of the raft induces axial forces in piles by the displacement

$$\Delta l_i = \varphi_0 x_i \quad (26)$$

The axial forces in piles will be:

$$N_i = \frac{E_p A_p}{s l_p} \Delta l_i = \frac{E_p A_p}{s l_p} x_i \varphi_0 \quad (27)$$

Also, the rotation φ_0 will give a bending moment at the pile head, expressed as

$$m_\varphi = \frac{c_h b}{4\alpha^3}\varphi_0 \tag{28}$$

This moment corresponds to a beam on elastic foundation acted by a φ_0 rotation at the upper end.

The moment's equation with respect to the *y-y* axis will give

$$\frac{EA_p}{sl_p}x_i\varphi_0 x_i + m\frac{c_h b}{4\alpha^3}\varphi_0 = M_0 \tag{29}$$

where *m* is the number of piles.

So the φ_0 rotation can be written as:

$$\varphi_0 = \frac{M_0}{\frac{EA_p}{sl_p}\sum_{i=1}^{m}x_i^2 + \frac{mc_h b}{4\alpha^3}} \tag{30}$$

Knowing the rotation of the piled raft we can compute the displacement because of it at a point at *h* height, being $\varphi_0 h$.

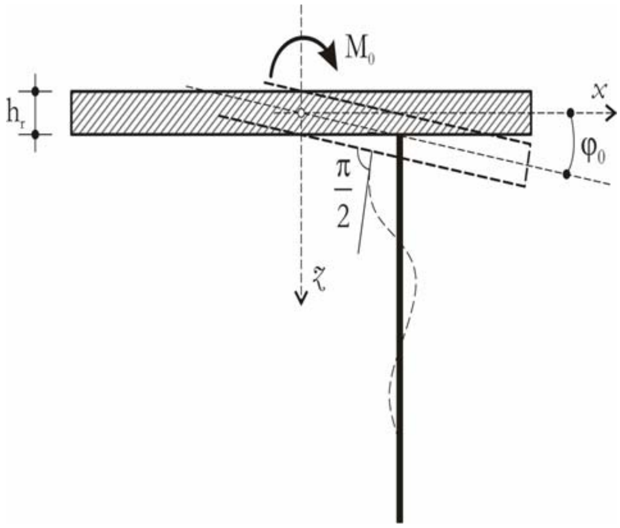

Figure 4. The global rotation of the raft

4. CONCLUSIONS

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The made analysis allows to take into account, in design, the interaction of structure with the foundation system (piled raft type) for the determination of the fundamental vibration period.

To extend the analysis in the spatial case, a similar calculus in *yOz* plane is needed at first, and then a vectorial superposition of the resultants, or according to some recommendations prescribed in the Design Codes, can be made.

If the mass centre of the building doesn't coincide with the centroid of the piles sections, at the inferior level of the piled raft, a torsional moment appears, that makes modifications in the structural seismic response and in the internal forces and moments of the piles.

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