Construction Materials

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Application of damage model for analysis of masonry structures

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Summary

The predictive modelling of the masonry structures behaviour, in the non-linear range represents a challenge due to their semi-discrete and composite nature. An adequate computational model should include the fundamental mechanisms that characterize the masonry behaviour at failure. This paper present the application of a damage model, based on the finite elements method, to simulate the ultimate response and the mode of failure of different masonry structures. To take into account the mechanical properties degradation, a damage variable is introduced in the constitutive law of the material.

KEYWORDS: Masonry structures, damage model, computational simulations

1. INTRODUCTION

The masonry structures are widely used in civil engineering. These structures are characterised by a softening response. Indeed, the non-linear behaviour of masonry is due to the damage and plastic micromechanical processes. From a microscopic point of view the damage is linked to the growth and coalescence of micro-cracks, leading to the formation of macro-cracks which can induce the collapse of the structure. The plasticity is due to intergranular displacements and accounts for inelastic deformations occurring during the loading process. Various nonlinear models have been proposed in the literature to describe the softening response of masonry structural elements. The available models adopted for structural computations are mainly based on macro-mechanical approaches using damage mechanics and plasticity theory.

As known, any macro-model of masonry structures always includes some approximations, since the different failure mechanisms of masonry (cracking of the mortar joints, sliding along bed and/or head mortar joints, cracking of the bricks under direct tension, masonry crushing) are not exactly reproduced, but are smeared out in the continuum.

This paper present the application of a damage model, based on the finite elements method, to simulate the ultimate response and the mode of failure of different masonry structures.

ISSN 1582-3024

2. THE CONCEPTION OF DAMAGE MODEL

The nonlinear behaviour of masonry can be modelled using concepts of damage theory. In this case an adequate damage function is defined for taking into account different response of masonry under tension and compression states. Cracking can, therefore, be interpreted as a local damage effect, defined by the evolution of known material parameters and by one or several functions which control the onset and evolution of damage. The model takes into account all the important aspects which should be considered in the nonlinear analysis of masonry structures such as the effect of stiffness degradation due to mechanical effects and the problem of objectivity of the results with respect to the finite element mesh.

A useful concept for understanding the effect of damage is that of effective stress. The damaged $\boldsymbol{\sigma}_d$ and effective undamaged $\boldsymbol{\sigma}$ stress tensors are correlated, according to continuum damage mechanics, by the relation:

$$\sigma_d = (1 - d)\, D\, \varepsilon = (1 - d)\sigma \qquad (1)$$

where d is a scalar value, ranging from 0 to 1 and representing the local damage parameter, **D** is the elastic stiffness matrix and **ε** is the strain tensor.

The damage function $g(\bar{\tau}, r)$ defines the limit of the region of undamaged response and is written at time t as:

$$\left(g(\bar{\tau}, r)\right)^t = (\bar{\tau})^t - (r)^t \le 0 \qquad (2)$$

where the undamaged complementary energy norm is defined as:

$$(\bar{\tau})^t = \gamma\sqrt{2\Lambda^0\left((\underline{\sigma})^t\right)} \qquad (3)$$

where $\Lambda^0(\underline{\sigma})$ is the elastic complementary energy.

For Simo's damage model $\gamma = 1$.

$(r)^t$ in the damage function of Equation (2) is the current damage strength measured with an energy norm and can be given as:

$$(r)^t = \max\left\{(r)^0, (\bar{\tau})^t\right\} \qquad (4)$$

where $(r)^0$ denotes the initial damage threshold of the material.

The initial damage threshold $(r)^0$, can be considered to carry out a similar function to the initial yield stress in an analysis involving an elasto-plastic material. However, in a damage analysis, the value of the damage threshold influences the degradation of the elastic modulus matrix. A value for $(r)^0$ may be obtained from:

$$(r)^0 = \frac{\sigma_d^t}{(E_0)^{1/2}} \tag{5}$$

where σ_d^t is the uniaxial tensile stress at which damage commences and E_0 is the undamaged Young's modulus. The damage criterion is enforced by computing the elastic complementary energy function as damage progresses:

$$\beta\left(\sigma^T D_e\, \sigma\right)^{1/2} - (r)^t \le 0\,. \tag{6}$$

The damage flow rule defines the damage softening and is given by

$$\dot{d} = \dot{\mu}^t \frac{\partial\left(G\left(\bar{\tau}^t, d\right)^t\right)}{\partial\tau} \tag{7}$$

where $\dot{\mu} \ge 0$ is the damage consistency parameter and defines damage loading/unloading conditions according to the Kuhn-Tucker relations

$$\dot{\mu} \ge 0,\; g(\bar{\tau}, r) \le 0,\; \dot{\mu} g(\bar{\tau}, r) = 0\,. \tag{8}$$

In addition, to simplify the calculations in damage analysis, the damage multiplier $\dot{\mu}$ is defined so that

$$\dot{\mu} = \dot{r}\,. \tag{9}$$

From the consistency of the damage condition in Equation (2) it is given that

$$\dot{\bar{\tau}} = \dot{r} = \dot{\mu}\,. \tag{10}$$

According to Equation (10), the definition (3) we have

$$\dot{\mu} = \frac{\gamma^2}{\bar{\tau}} \underline{\sigma}^T D_e^{-1} \underline{\dot{\sigma}}\,. \tag{11}$$

$(\partial G / \partial\bar{\tau})^t$ defines the damage rate with respect to the undamaged elastic complementary norm. If the damage potential function G is assumed to be independent of d, substitution of Equation (10) into Equation (7) will lead to:

$$d = G \tag{12}$$

with the undamaged condition being enforced so that

$$\left\{G\left(r^t\right)\Big|_{(r)^t = (r)^0}\right\} = 0\,. \tag{13}$$

Damage accumulation functions is given by:

$$G\left((r)^t\right) = 1 - \frac{(r)^0 (1 - A)}{(r)^t} - A \exp\left[B\left((r)^0 - (r)^t\right)\right]. \tag{14}$$

For no damage, $\mathbf{G}(r)^t = 0$. The characteristic material parameters, A and B, would generally be obtained from experimental data.

3. DAMAGE CRITERION

The damage criterion is defined as a function of the free energy $\boldsymbol{\Psi}_0$ of the undamaged material, expressed in terms of undamaged principal stresses $\boldsymbol{\sigma}_i^{p,0}$:

$$F = K\left(\sigma^{p,0}\right)\sqrt{2\rho_0\Psi_0} - 1 = \frac{K\left(\sigma^{p,0}\right)}{\sqrt{E^0}}\sqrt{\sum_{i=1}^{3}\left(\sigma^{p,0}\right)^2} - 1 \leq 0 \tag{15}$$

where: ρ_0 is the density in the material configuration.

The terms of the above equation have the following meaning:

$$K\left(\sigma^{p,0}\right) = \frac{r}{\sqrt{2\rho_0\left(\Psi_t^0\right)_L}} + \frac{1-r}{\sqrt{2\rho_0\left(\Psi_c^0\right)_L}}, \tag{16}$$

$$r = \frac{\sum_{i=1}^{3}\left\langle\sigma_i^{p,0}\right\rangle}{\sum_{i=1}^{3}\left|\sigma_i^{p,0}\right|}, \tag{17}$$

$$2\rho_0\left(\Psi_{t,c}^0\right)_L = \sum_{i=1}^{3}\left\langle\pm\sigma_i^{p,0}\right\rangle\varepsilon_i, \tag{18}$$

$$\left(\Psi_0\right)_L = \left(\Psi_t^0\right)_L + \left(\Psi_c^0\right)_L. \tag{19}$$

In these equations $\left(\Psi_{t,c}^0\right)_L$ represent the part of the free energy developed when the tension/compression limit is reached. Taking into account that the tension and compression strengths are $f_t = \sqrt{\left(2\rho_0\Psi_t^0E_0\right)_L}$ and $f_c = \sqrt{\left(2\rho_0\Psi_c^0E_0\right)_L}$ respectively, and substituting the last definition in the Equation (16), the damage function can be written as:

$$F = \bar{\sigma} - f_c \leq 0 \tag{20}$$

where

$$\bar{\sigma} = \left[1 + r(n-1)\right]\sqrt{\sum_{i=1}^{3}\left(\sigma_i^{p,0}\right)^2} \tag{21}$$

with $n = \frac{f_c}{f_t}$. The advantage of the yield criterion written in Equation (21) is that any yield function $\mathbf{F}$ can be used always as long as it is homogeneous and of first order in stresses (Mohr-Coulomb, Drucker-Prager).

4. GLOBAL DAMAGE IMPLEMENTATION

The idea for global damage indices definition stemmed from a macroscale analogy with the microscale local damage index definition. Thus, the starting point for deducing a global structural damage index is Equation (22), which defines local damage as a relation between the actual free energy $\mathbf{\Psi}$ of the damaged material and the elastic free energy $\mathbf{\Psi}_0$ of a fictitious undamaged material.

$$\Psi(\varepsilon, d) = (1-d)\Psi_0(\varepsilon) = (1-d)\left(\frac{1}{2\rho_0}\varepsilon^T\sigma^0\right). \tag{22}$$

It seemed natural to reach this objective by integrating the pointwise Equation (22) over a finite mass, as follows:

$$\Psi = (1-d)\Psi_0 \Rightarrow W_p = \int_V \rho_0 \Psi dV = \int_V (1-d)\rho_0\Psi_0 dV = (1-D)\, W_p^0 \tag{23}$$

where D^* is the global damage indices of the considered structural mass, $W_p^0 = \int_V \rho_0 \Psi_0 dV$ is its fictitious ever-elastic potential energy due to the actual strains and $\mathbf{W}_p$ is the actual potential energy. Solving Equation (23) for D, yields the final expression:

$$D^* = 1 - \frac{W_p}{W_p^0} = \frac{\int_V \rho_0\Psi_0 dV - \int_V (1-d)\rho_0\Psi_0 dV}{\int_V \rho_0\Psi_0 dV} = \frac{\int_V d\rho_0\Psi_0 dV}{\int_V \rho_0\Psi_0 dV}. \tag{24}$$

In a finite element context, expression (24) takes the following operational form:

$$D^* = 1 - \frac{\sum_e a^T \int_{V^{(e)}} B^T \sigma dV}{\sum_e a^T \int_{V^{(e)}} B^T \sigma^0 dV} \tag{25}$$

where $\sum_e$ denotes the sum over a number of finite elements, **a** is the mesh nodal displacement vector, **B** is the strain displacement matrix, $V^{(e)}$ is the volume of each finite element (e), $\boldsymbol{\sigma}$ is the actual stress vector and $\boldsymbol{\sigma}_0$ is the stress vector

ISSN 1582-3024

should the material preserve its original characteristics and undergo the actual strains.

CONCLUSION

In this paper the author presented a damage model which can be applied successfully to asses the structural conditions and estimate the safety level and durability of historical masonry constructions under static and dynamic loading. The global damage indices provides accurate quantitative data on the state of any component subpart of a damaged structure and its importance to the overall structural behaviour, being of invaluable help to the task of assessing the reliability, safety and definition of adequate repair of retrofitting strategies.

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