

### Contribution to non-linear constitutive modelling of masonry structures in 2D

Jiří Brožovský<sup>1</sup>, Alois Materna<sup>2</sup> and Ivan Kološ<sup>1</sup>

<sup>1</sup>Dept. of structural mechanics, VSB – Technical University of Ostrava, CZ708 33, Czech Republic

<sup>2</sup>Dept. of building structures, VSB – Technical University of Ostrava, CZ708 33, Czech Republic

#### Summary

*This paper discusses a non-linear constitutive model for modelling of masonry in 2D. The model consists from two independent parts: the model for mortar and the model for bricks or stones.*

*The reason of using two different constitutive models is our need to be able to modell individual bricks (or stones in the case of a stone masonry) and the mortar between the bricks.*

*We have decided to use a smeared crack approach for the modelling fo a mortar. It is an approach that is widely and successfully used for a constitutive modelling of concrete and we assume that it can be also applied to a mortar that is very similar to a concrete.*

*The model of mortar is based on an equivalent one-dimensional stress-strain relation that depends also on 2D state of stress (through the Kupfer's failure criteria that is used to obtain the limit stresses for the one-dimensional stress-strain relation) and on some special material properties such is fracture energy of mortar.*

*Bricks are modelled in a different way, As a generally brittle, they often can't be effectively modelly by the smeared crack approach. The damage of the brick (a crack) usually goes through the whole brick and it can be assumed that its occurrence depends on a stress intenzity on an area and not on the stress size in an individual material point. Thus we have decided to use a simple approach that can be describes as a very basic non-local material model.*

*After the ckack on the brick is detected the material properties are changed in a moment. Unlike the model for a mortar, it is assumed that there is no unloading curve for this model,*

**KEYWORDS:** finite element method, constitutive modelling, masonry, mortar, bricks, crack band model, smeared cracks.



J. Brožovský, A. Materna, I. Kološ

### 1. INTRODUCTION

The static analysis of masonry can be done in several ways. The most common is approach is a linear elastic constitutive modelling. It allows to provide a relatively simple computational analysis and many structures can be analysed with very simple computational models (beams and frames) and often even without need of a computer. Also the finite element analysis can be relatively easy and non-complicated. It is often said that linear elastic modelling is enough for a design of masonry structures because they are not allowed to work in situations when their behaviour can be non linear (under tension loads, for example).

But there are situations when the assumption of the linear elastic behaviour is not sufficient. For example there are often needs of analysis of already existing buildings or of historic structures and monuments that can work in a non-optimal mode for masonry. For these cases a lot of different non-linear constitutive models have been developed by many authors (for example [1]). Many of these approaches are very complex and they can offer a high level of accuracy of results if they are properly used. But it isn't often easy (or even possible) to get all the necessary input data that are needed for such models. In these cases the results can be even less precise than the traditional computing approaches. There are also a lot of not so advanced models that cannot offer so high precision of results but they usually require lower number of input data and may they may be easier to use.

In this paper we present a constitutive model for masonry that we are developing. This model was designed as a compromise between needs and possibilities of computing and access to input data (however, the proposed model still can be too complicated in some cases).

### 2. CONSTITUTIVE MODEL DESCRIPTION

#### 2.1. Overview

The constitutive model includes two independent parts: the model for mortar and the model for bricks. These parts are based on different assumptions and they are implemented in different ways. The mortar is implemented using the smeared crack approach and the crack-band model. Bricks are modelled with a simple variation of a non-local model approach. This division of the model into two parts allows us to create a models that can include individual bricks (or stones in a case of stone masonry that is common in historic buildings) and also locate the mortar in positions that are correspond with the real structure.

The proposed constitutive model is developed for 2D cases (for the plane stress case).



*Contribution to non-linear constitutive modelling of masonry structures in 2D*

### 2.2. Model for mortar

The constitutive law for mortar is controlled by an equivalent uniaxial stress-strain relation. The simplification of the problem from 2D to 1D is not ideal but it is relatively easy to develop and understand. To make the uniaxial relation to be more corresponding with the real behaviour the limits of the relation (strengths in tension and in compression) are computed from a 2D failure criteria (the Kupfer's criteria is used here) and depends on the actual 2D stresses.

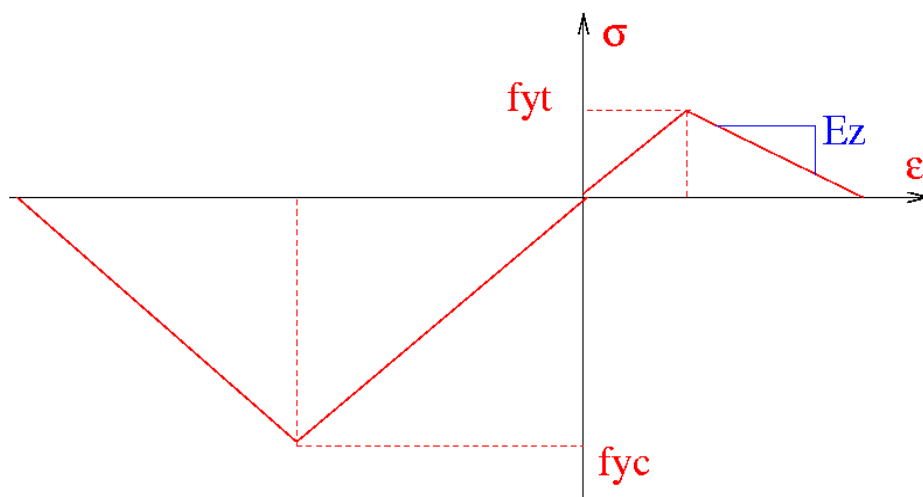


Figure 1. Uniaxial stress-strain relation for mortar

The used approach has several disadvantages. The one of the most important is that results depend of properties of a finite element mesh. It is obvious, because when a “damage” is detected the properties of the material became be reduced and they are reduced on whole area of the element (or – in our case – on a whole area that corresponds with an integration point of a finite element).

This issue can be minimized by usage of the Bazant's crack-band model when the properties of the stress-strain diagram (the “Ez” value in Figure 1) depends on the size of the cracked area (value L on Figure 2) and the material properties (the fracture energy  $G_F$ ). The fracture energy can be obtained from special experiments or it can be (with limited precision) computed (Karihaloo []).

The behaviour of the material in the compression is modelled in a similar way.



J. Brožovský, A. Materna, I. Kološ

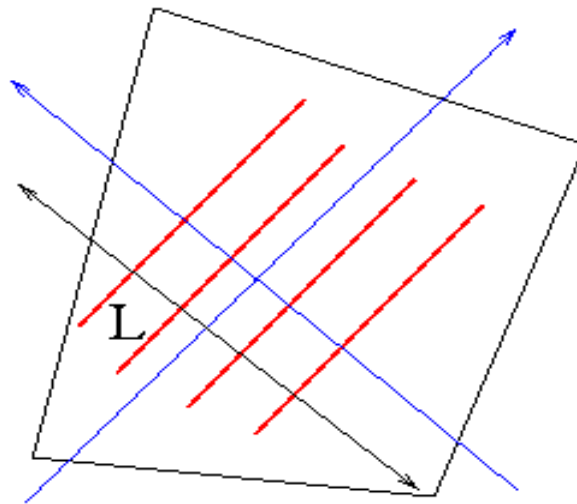


Figure 2. Length L of the cracked area

### 2.3. Model for bricks

The real bricks (or stones in stone masonry) have a different behaviour. They usually are much more brittle than the mortar. The material model for mortar can be also used for a modelling of bricks but it usually isn't ideal for this purpose.

We have prepared a different constitutive model. We assume that behaviour of bricks is linear elastic until the damage (crack) is detected. Usually, the crack goes through the full height (or width) of brick so it is obvious that the failure condition must respect this.

We have proposed a use an approach that is similar to non-local material models: the stresses in the brick are controlled on an area  $A$  with dimensions that are comparable with height of the brick. It should guarantee that crack is detected if the stresses in the brick are big enough to be able to create a crack. Thus a brittle damage of the brick should occur.

After the brick is damage the material properties should be changed. Now we have adopted the elasto-plastic behaviour of a material. After the crack is detected then the normal stiffness of the material (represented by Young's modulus) is reduced to zero. It is not an ideal approach because it means that the cracked material still carry the previous stresses but we selected it for the relatively ease of the implementation. After we will test the other parts of the algorithm (namely the computation of the failure condition) we will improve the behaviour of the cracked material to be more realistic.



*Contribution to non-linear constitutive modelling of masonry structures in 2D*

### 3. ILLUSTRATIVE EXAMPLE

#### Model description

To show the behaviour of the model we have prepared a simple (not very realistic) numerical example. The geometry is shown on the Figure 3. The sizes of bricks are 270x160 mm and the width of mortar is 10 mm in all cases.

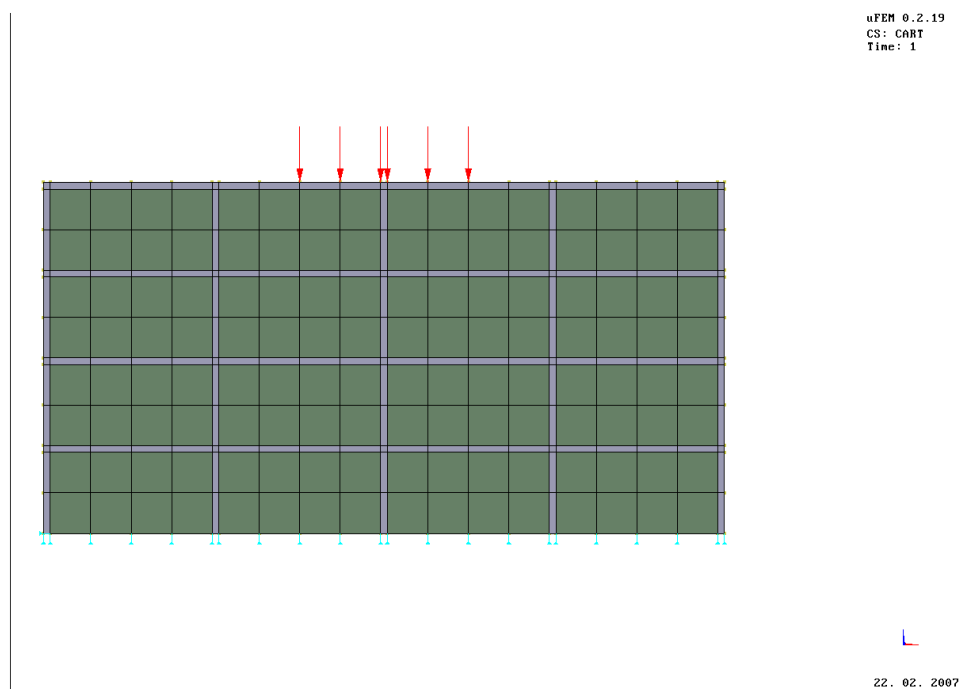


Figure 3. Illustrative example – finite element model

The Figure 4 shows the load-displacement relation that was obtained from the analysis of the model.

The relatively sharp change of the stress-strain curve is the result of a selected constitutive model of mortar – the bricks under the loading cracked in a moment and then the strength of the structure became reduced. From the Figure 4 it is obvious that the mortar has a relatively small influence of the results in this case.

The relatively high strength of a damaged material in this case was a result of a relatively large residual stiffness of a bricks (we used a elastoplastic model with hardening for the bricks here to help the convergence of solution).



J. Brožovský, A. Materna, I. Kološ

The results show that the model features a behaviour that we have expected to obtain.

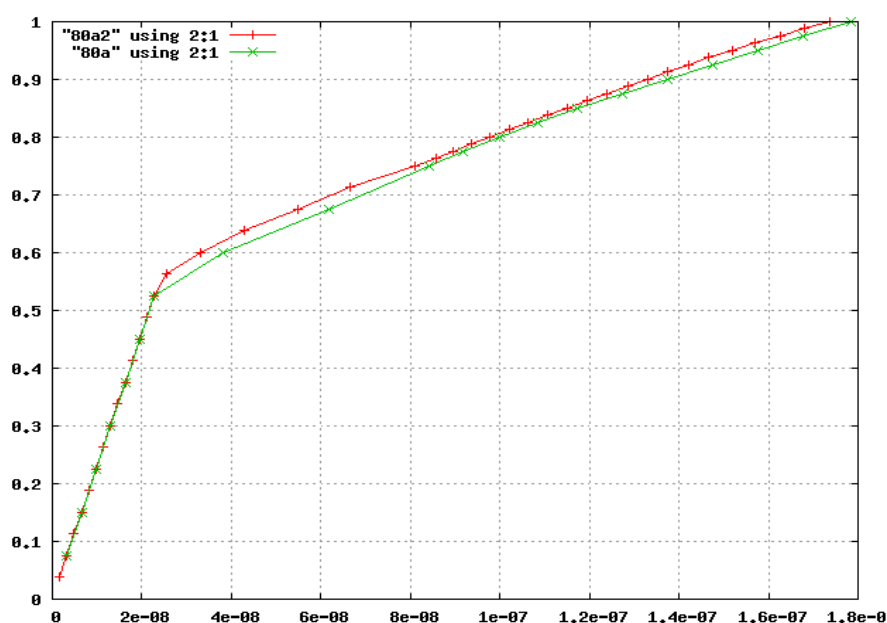


Figure 4. Computed stress-strain relation

## 4. CONCLUSIONS

The article shows a basic material model for masonry. It can be improved in several ways, namely in the area of the modelling of bricks.

### Acknowledgements

The works are supported from the Czech state budget through the Czech Science Foundation. The project registration number is GA CR 103/03/P389.

### References

1. Bazant Z. P., Planas, J., *Fracture and size effect in concrete and other quassibrittle materials*, CRC Press, Boca Raton, 1998.
2. Cervenka V., *Constitutive models for cracked reinforced concrete*, ACI Journal, vol. 82, 1985.
3. Cervenka V., *Inelastic finite element analysis of of reinforced concrete panels under in-plane load*, University of Colorado, Colorado, 1970

