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Influence of deformation at a heritage building support on stability of groined masonry arch

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Summary

Masonry arch stability criteria definition has been developed by numerical modeling of cracked arch shell structure of Riga cathedral. Settlement of supports induced strains in masonry arch shells which lead to stress redistribution and crack forming in masonry shell structures is serious problem for heritage buildings. Deformation criteria should be developed for safety exploitation of heritage masonry structures. The main task of the present research is - developing deformation criteria for evaluating of masonry shell construction stress-strain state. Three-dimensional scanning of arch structure by means of Leica 3D laser scanners were used to determine geometric model of masonry arch shell. Previously accomplished monitoring data was used and a new monitoring program was developed to define unequal settlements of supports. The settlements monitoring of Riga cathedral supports and numerical modeling to define settlement inequality were used as initial information to compare computer modeled deformed structure shape with existing cracked masonry structure. Geodesic monitoring by optical tools and mechanical tensometers have been used for determination of a crack expansion since 2005. Twenty one Optical Fiber Sensors of SOFO type are used now to measure deformations to establish safe exploitation limits. Multilevel analysis of masonry shell has been fulfilled by finite element software and applied for developing safe exploitation criteria.

KEYWORDS: structural masonry, masonry fracture, groined arch, heritage building safety, numerical modeling.

1. INTRODUCTION

Since past masonry structures were built by time-honored method of trial and error. The one of the first written rules and building code was “de Architectura” in ten volumes written by Marcus Vitruvius Pollio, roman architect approximately at 1st century. Medieval time masonry unions use time approved methods of masonry structure cross sections, shell geometry, proportions and forms. Masonry code in medieval Europe was distributed by Christianity influence as sacral building forms, proportions and architectural styles. An objective of investigation is deformed structure of Riga cathedral in Latvia capital city shown in figure 1. 13th century built masonry Gothic arch shells which are built from solid clay bricks are one of the first masonry shell covered structures in Latvia.

Figure 1. North front of Riga Cathedral

Cross arch and star arch shell where commonly used as type of heritage building shell. Long building process and changes in finale view of cathedral was usual by-effect in building process centuries ago. Riga cathedral had long erection process with stops and design changes. Few steps of erection and enlargement by changing city demands took place also. Fire took place in 1547 when tower and nearest roof parts were lost. A cross and star type masonry arches were partly rebuilt after Swedish / Russian war in 16th century. Medieval European Hansa union cities were usually located near river deltas. The supporting piles and subsoil situation therefore in Riga cathedral are survived today and found in not satisfied conditions. Weak subsoil under building foundations is common situation in Old Riga city. For lot of buildings wooden piled foundations are used to transfer loads to dense sandy layer through loose sand located under footings. Wooden pile life time practically is not limited under ground water minimum level but pile condition after eight centuries of building life time were found in unconditional situation. Seismic risk in Latvia region is low but support deformations proceeds in most of Old Riga piled supported buildings.

Structural weakness or overloading, dynamic vibrations, settlement, and in-plane and out-of-plane deformations can cause failure of masonry structures. To prevent the accidental situation in heritage buildings safety criteria must be specified determining deformation limit between masonry arch shell interacting parts. Surveillance des Ouvrages par Fibres (SOFO) optical fiber sensors will be mounted on arch shell cracks for deformation monitoring. Cathedral arch internal surface laser scanning were made by tools of three-dimensional 3D Leica laser scanner for geometrical description of structure. Whole building thee-dimensional model is available at modern software computing. The plastic analyze approach is developed to define safety exploitation of deformed structure. Proposed methodology on existing heritage buildings reduce necessary time for first approximation of deformed structure. Numerical modeling of masonry structures by modern computing tools demand additional material parameters such as elasticity, stiffness, compression, tensile and shear resistance of stone material and lime mortar, friction angle and cracking energy. Cracked masonry shell part interaction also must be specified by friction because of safe exploitation possibility after crack forming. Thrust line in non-cracked arks show center of compression force in cross section and for medieval buildings almost is located on center of cross section predicted by geometric proportions carefully improved in mason unions. Based on "middle third rule" later computing method also is in safe exploitation zone. Uncracked masonry arch shells can be easily computed using modern GEM software's and plastic material approach. After support deformations and crack forming in shell structures thrust line change position in cross section and internal forces relocates significantly. Therefore the main task of present research is safety exploitation criteria definition for deformed masonry arch shell of Riga cathedral.

2. MASONRY COMPUTING EVOLUTION

The main problems in the structural analysis of historic buildings are following:

1. Lack of data about geometric dimensions,
2. Material properties of the inner parts those are not visible exteriorly of the structural members that are huge in cross-sectional dimensions,
3. Difficulties in identifying the characteristics of construction materials,
4. Excessive cost of detailed laboratory analyses,
5. Variety of the data due to construction techniques and natural material utilization,
6. Altering material properties even along the same structural member due to long-lasting construction process,
7. Uncertainties in construction process and steps,
8. Indefiniteness of general stability and strength continuity due to the existing damage on the structure,
9. Inability in the application of modern construction materials, structural analysis and loading conditions.

However modern computing technique mainly based on FEM software gives possibility to detail analyze of heritage buildings. The historical review is necessary to understand the computing evolution and background of modern computing technologies.

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2.1. Geometrical approach

Time honored geometry of arch structures was approved by geometric methods till 17th century. Arch and cross arch is a statically indeterminate structure therefore the first calculation methods were developed to simplify design methodology. In 17th century Hook's ca. 1670 proposed hanging flexible line thrust line definition method was brilliant idea for very first arch geometry approach. Adequately weight distributed hanged line give ideal tension line which in inverse form gives ideal compression line or trench line as brilliant solution of statically indeterminate structure. That type of solution is useful for vertically loaded arch structure but doesn't include horizontal loading. In Riga cathedral three arch horizontal interacting forms the horizontal loading eliminate this methodology usage for that type of buildings. Developed by Moseley (1833) "new Theorem in Statics" where the Principle of Least Pressure was proposed describes structures widely. Later Moseley (1843) specialized this approach to arches. Methodology, proposed by Villarceau (1853) reduces the statically indeterminate level of arch by inserting the three hinges to statically determinate system, was mostly useful analytic method till development of elastic analysis method. Towards the end of the 17th century, the studies of Galileo on strength analysis brought the medieval structural theory to the end. In those times, the scientific deal was mainly in the analysis of string-resembling the arch form- under vertical forces. In 1826, Navier put forward that the buildings should stand by means of calculations of the stresses on its structural members rather than the application of some geometric rules.

2.2. Elastic analysis

In end of 19th century elastic analysis was developed. In fact, until ca. 1880, engineers divided arches into "elastic", made of wood or wrought iron, and "rigid", made of masonry. Poncelet (1852) was conscious of the problem and in his historical review of arch theory suggested to apply the elastic theory to masonry arches in order to obtain a unique solution. Already in the 1860=s some elastic analysis of masonry arches were made. Winlker (1879) make a discussion of elastic material approach to masonry structure. However, he added a discussion on the "Störungen" that can affect the position of the line of thrust. Their main origins were: the deformation of the centering during construction, the yield of the buttresses under the thrust and the effect of changes of temperature. All these perturbations would produce some cracking of the arch and Winkler was well aware this would affect notably the position of the line of thrust, which could be very different from the calculated. Winlker suggested that some means of controlling the position of the line of thrust by inserting internal hinges during construction. Castigliano (1879) applied his theory of elastic systems also for masonry bridges. Elasticity theory gives admissible results till elasticity limit of material. In heritage building supporting structures compressive stress level is not

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high and elastic material behavior is applicable. In early stage of elastic analysis develop for masonry structures engineers showed a certain resistance because of material anisotropic and irregularity. Elastic material approach in masonry shell systems uncertain results is giving therefore computing approach development seams necessary. The historical development review of first computational methods was done by Heyman [1,2].

2.3. Kinematic approach and arch collapse mechanism

Hinge forming by loading, thermal variations or geometric changes is most of arch collapse reasons. Material plasticity in hinges in self-weight loaded masonry shell system is main reason of safety limit exceeding initiated by support deformation. Support deformation forms the hinges in arch masonry structure as interaction contact by sliding surface. Safe exploitation criteria therefore must be defined. The kinematic or 'mechanism' method was first introduced by Heyman [3]. Heyman assumptions for arch safety analyze method were infinite compressive strength and friction resistance and zero tensile strength. He pointed out that plastic limit analysis can apply well to the case of masonry gravity structures, such as piers and arch bridges. In his book, for the single span arch, Heyman assumed the arch will collapse when four hinges form. Furthermore, he assumed that the arch has infinite compressive strength and sliding failures cannot occur. The hinges can alternately form at the intrados or extrados of the arch, and at failure, the thrust line must pass through the hinges. Therefore, it is possible to determine the magnitude of the applied load which will cause the arch to collapse. By Heyman, [4,5] the kinematic approach takes into account a collapse mechanism activated by an adequate number of plastic hinges. As main criteria of safe exploitation definition was Heyman proposed methodology. When the thrust line in a cross section is adjacent to the ring of the arch, a hinge is opened in that point. According to the upper bound theorem from the theory of plasticity, the maximum load corresponding to some collapse mechanism is greater or equal to the maximum load corresponding to the real collapse mechanism. This theorem implies that when the thrust line is adjacent to the ring of the arch in four points then the arch is not safe. Friction is high enough between stones and sliding failure cannot occur. The masonry has an infinite compressive strength. More complicate collapse mechanism has been considered for twin-span models by Hughes [6]. The pioneer in using modern computational methods to determine the collapse load of block structures was Livesley [7], who attempted to solve the problem as a linear programming problem using the lower bound formulation of limit analysis. In his paper Livesley showed that the adoption of associative friction leads to an incorrect collapse mechanism. More importantly it may also give an overestimate of the true collapse load. When using linear programming to solve limit analysis problems, flow will always occur normal to the specified surface (i.e. according to the so-called ‘normality rule’).

3. MASONRY MATERIAL APPROACH

In present research are proposed two step computational approaches. A complete and detailed review of micro and macro material approach may be found in Lourenco [8] for mechanical aspects at both scales. As first stage of presented computational methodology is deformed structure modeling by elastic macro modeling approach to understand whole building situation and decrease computing amount. In second stage of analysis deformed structure situation must be investigated defining cracked masonry part interaction. Therefore micro modeling material approach is most convenient.

3.1. Macro modeling approach

Homogenizations of material included in most of building codes give simplify rules for material property description and usage in calculations. In previously done research other heritage brick building was investigated to find masonry material elasticity properties and work out material definitions according Latvian building code. The homogenization of material means generalization of stone and mortar properties for elastic material property definition adopted for computational usage in elastic stage. Hooke law defined in 1676 long lasting linear elastic material approach was used for analytic calculations. As main parameter for elastic material approach elastic modulus was defined. Elasticity dependence from brick and masonry class defined from cube resistances of brick and mortar material shown in figure 2. Not cracked cross groined arch loading capacity computational analyze for 1912 year built building using defined lower elasticity modulus in previously done investigations earlier.

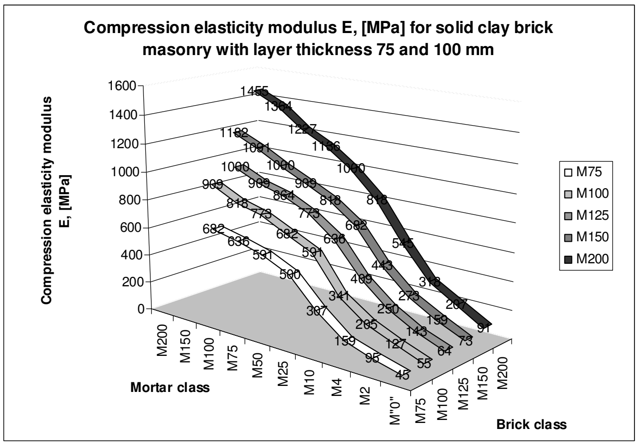

Figure 2. Latvian building code methodology defined mean elasticity modulus for elastic masonry approach

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To understand the heritage masonry there are 3 point brick bending tests done. Shown in figure 3 bending test results highlight main problem of heritage masonry material definition – load bearing resistance result dispersion induced by home manufacturing process variability.

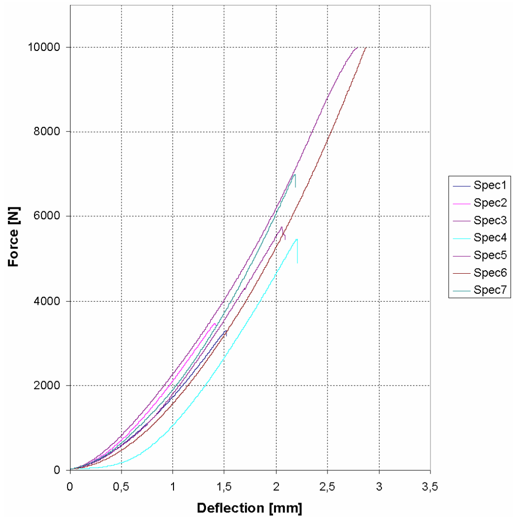

Figure 3. Three point brick bending test results

3 point bending test shows different tensile resistance by crash force. Test also shows equal elasticity for all specimens and brittle material approach of stone material. Structural analysis in elastic stage using FEM linear masonry approach is not acceptable for close to compression resistance calculations. In that cause plasticity theory must be used. Not only brick manufacturing significantly affect whole masonry material properties but also various stone material usage, block size and form variations, mortar component variations and joint thickness variations. Our aim is to give possibility to understand and analyze masonry arch, in fact, any combination of them, i.e., a masonry building. The 3D surface scanning with 20mm precision is so detailed that the internal structure can be described easily. Behind plaster layers forming the regular forms and levels of masonry wall surface a most irregular internal structure is found. Homogeneity, isotropy, uniform mechanical properties, etc., all the common assumptions of modern conventional structural analysis cannot be made in this case without violence to the most basic common sense. Taken from Riga Technical University building material laboratory test report Nr 65-2000, masonry compression resistance, 3 point bending test given tensile resistance was used for definition of masonry property. Used in computational software material behavior is approximate by material property variations but can give very close deformed stage structural understanding. Used in FEM software Staad Pro material properties are described as: Prime modulus of

elasticity (E_0 = 3600 MPa), average modulus of elasticity (E_{mean} = 818 MPa), Yong's modulus (G = 1200 MPa), poisson's ratio (0.2), thermal linear expansion coefficient (α_t = 0.000005 1/degree), density (ρ = 18 kN/m3), brick compression resistance (R_1 = 4.5 MPa), mortar compression resistance (R_2 = 2.5 MPa), masonry compression resistance (R = 1.1 MPa), masonry centric tensile resistance (R_t = 0.05 MPa), masonry shear resistance for head joint (R_{sq} = 0.11 MPa), masonry tensile resistance in bending for head joint (R_{tb}=0.08 MPa), resistance to main tensile stresses (R_{tw} = 0.08 MPa), coefficient of creep effect (v = 2.2).

$$\tau_{red} \leq R_{sq} + 0.8 * \mu * \sigma_0 \tag{1}$$

Defined in equation 1 by Latvian building code methodology limiting tangential stresses, where μ = 0.7 friction coefficient by joint mortar; σ_0 – compression stresses average value from lightest loading with reduction coefficient 0.9. Tangential limiting value in second stage of calculation is defined as crack forming value for shear forces in macro modeling material approach. Macro modeling approach is simplifying material description and may normally be used in analytic FEM structural computing by Rikards [9]. The macro modeling of masonry as a composite is latest developed material approach by Rots [10]. Lourenço and Rots [11] macro-modeling technique, specifically formulated for the analysis of masonry constructions, is based on lumping all inelastic phenomena to the joints by means of a composite interface model. This model, stemming from plasticity, comprehends three different failure mechanisms, namely, a straight tension cut-off for mode I failure, the Coulomb friction model for mode II failure as well as an elliptical cap for compression and combined shear-compression failure. As in the previous case, Lourenço and Rots model requires the values of the initial axial and shear stiffnesses K_N and K_T as input data.

3.2. Micro modeling approach

Micro modeling of masonry material is precise stone material interaction description possibility but also significantly increase computing time and amount. In early stage of micro modeling dry joint approach was used. In latest developments the block and the mortar in the joints are represented by continuum models, whilst the interface unit-mortar is represented by discontinuous elements. The Young model, the Poisson coefficient and the inelastic properties of the units and the mortar are taken into account. Micro modeling is widely investigated and described by Lourenco and Rots [12] and find mortar material and stone material plasticity, load bearing tensile and compression resistances depending from joint position, friction angle a.c. Micro modeling material approach must be used in second step of computational investigation to analyze support settlement deformed structure situation. For those purposes additional masonry material properties must by involved.

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4. MONITORING DATA COLLECTION

After World War Two Riga cathedral was closed as church and reorganized to concert hall. Soviet government put serious amount of funding to uplift and repair the structure, installed new ventilation, heating and fire safety systems. Latest funding in those systems renovation, crack monitoring and geological survey make a possibility to install long term monitoring system and make the present research.

4.1. Cracked arch photo fixation

Since 1959 cracks has been survived in Riga cathedral. Photo fixation made by heritage building protection institutions show lack of cracks in masonry shells. This is useful information to understand the new crack forming and prolongation of existing in long term monitoring.

4.2. Arch masonry building sequence and layer orientation fixation

From 1960 to 1961 each arch has been sketched and described by J. Stukmanis. In very careful way each brick and block sizes, form, orientation and position in masonry structure. Shown in figure 4 arch orientation, rib positions, brick sizes and forms were fixed in his work. Also description of damages and cracks for each arch was written totally on 68 pages and 30 sketches done. Collected in State Cultural Monument Archive materials about structure investigations and reconstruction process give we better understanding of existing situation.

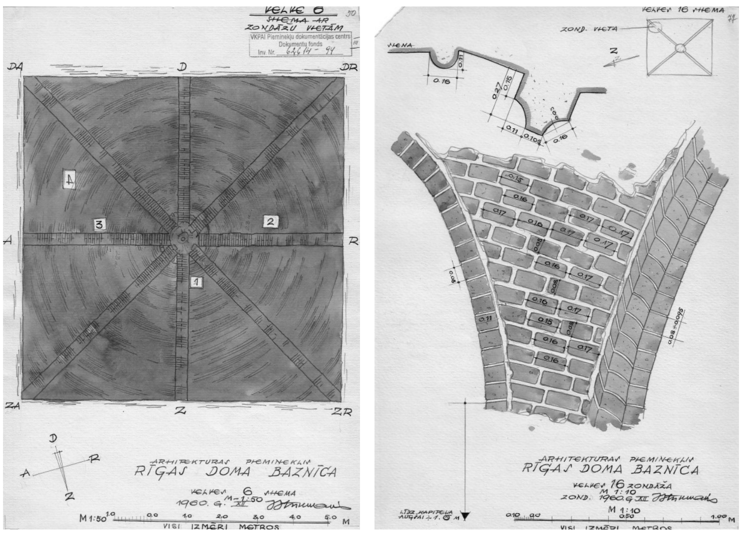

Figure 4. Careful fixation of all cross arch shells was done by J. Stukmanis

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Fixation of damages after World War Two is very useful information in nowadays to understand changes since that time. Crack prolongation, widening and new cracking fixation of masonry structure therefore can be done by present crack monitoring. After crack fixation plastering of arch surface was accomplished and cracks hided.

4.3. Cathedral scanning by 3D Leica laser scanners

Building scanning gives the possibility of virtual three dimensional presentations in tourist web sites. This very detailed scanning by three dimensional Leica laser scanner give precise geometric surface defining for computational analyze of structure. Made by Kalinka and Reiniks, specialists from geodesic company Merko laser scanning in 2006 used for FEM computational analyze. Precision of 20mm on internal surfaces and less detailed for outside is today used for structures. Increase out coming virtual model precision theoretically gives whole structure computing possibility. Future development of computational hardware will provide us with powerful method of masonry structure analyze.

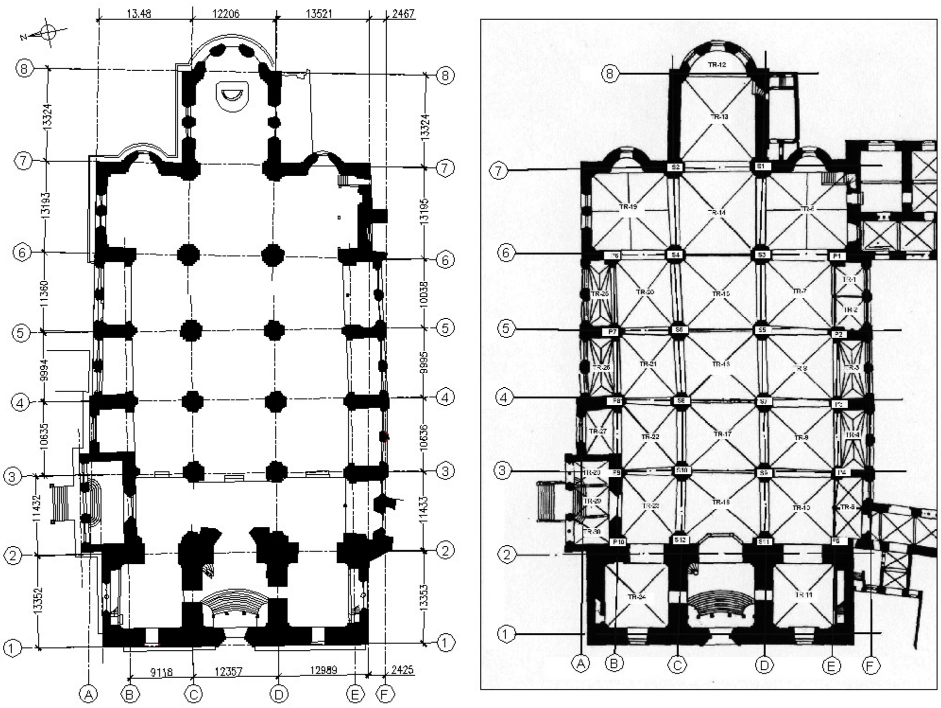

Figure 5. From full 3D model made plan and arch marking in plan made by Stukmanis

Figure 5 shows building plan section of model highlights geometrical uniformity of structure in comparing with Slavietis, Seglins and Drugis in 1959 made hand tool measurements. Scanned Gothic arch geometry also is preferable in comparing with Erdmanis 1963 proportionality findings shown in figure 6. Proportion findings are simplifying method of geometric approach to use the hand calculations of thrust line. Geometrical inequalities lead to serious difference between existing situation and real situation in cross section.

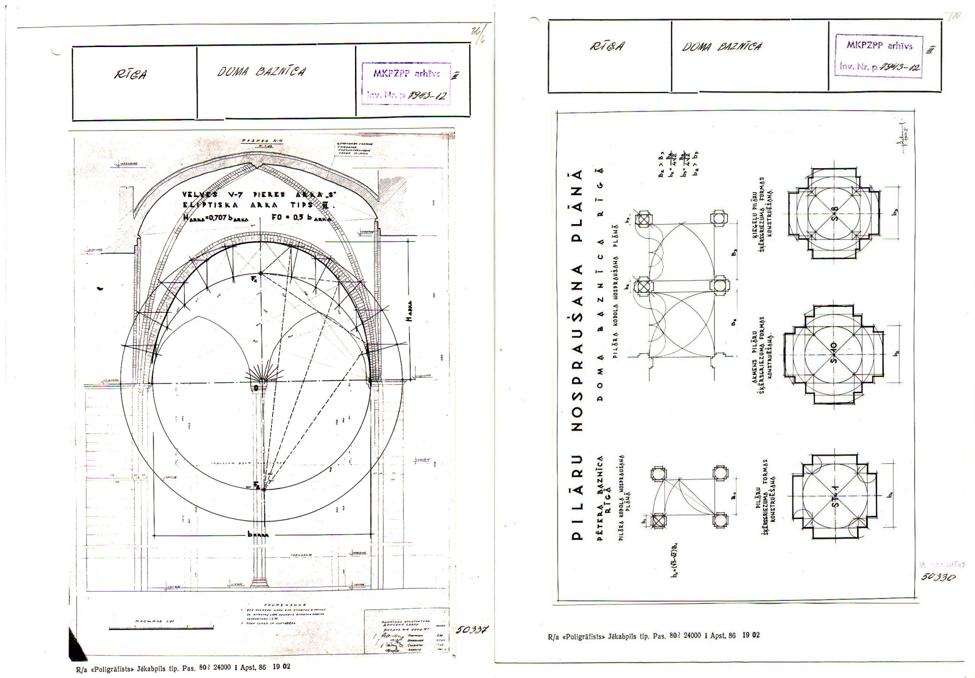

Figure 6. Proportionality findings by Erdmanis in 1963

By means of modern computational FEM software there is a possibility to analyze the whole building. We find the laser scanning as very fast geometric modeling for FEM software.

4.4. Microclimate monitoring

Staff managed microclimate monitoring for internal temperature and humidity was performed from January 2002 to June 2004. Temperature oscillation gives significant influence on masonry deformations and crack movement. In developed now SOFO type based monitoring temperature external and internal measure is included.

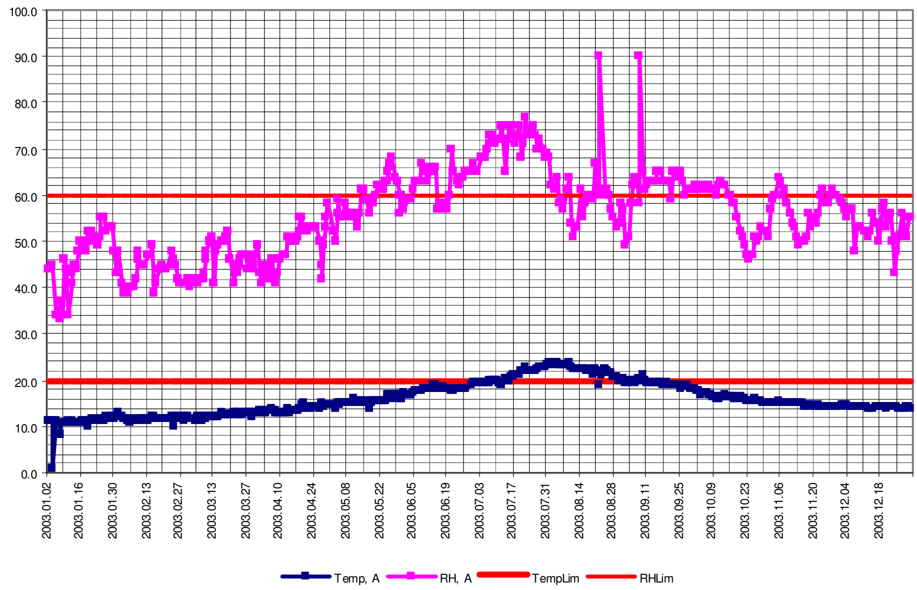

Figure 7. 2003 year internal temperature and humidity monitoring by staff

At figure 7 shown staff made monitoring for temperature and humidity for year 2003. Line Temp,A shows temperature variation during 2003 year period. RH,A line shows humidity variations along 2003 year period. Temp,Lim and RH,Lim lines show lower limiting values favorable to fungous forming. Temperature initiated deformations simulated by computer modeling show significant influence on crack forming, prolongation and movements. Cyclic temperature loading in eight hundred years of cathedral life time affect not only internal comfort but also stress situation in cross arch system seriously changing thrust line position. Solar radiation effect on cathedral external surface can be defined as load on FEM elements also.

4.5. Support condition survey

Support condition and subsoil situation survey in last two years made by CM GIB Geotechnical Company presented by Celmins and Markvarts show weak soil layer presence under footings. Started in 2005 geotechnical survey is more detailed than in former Soviet period ever done. Cross section of piled supported column footing is shown in figure 8. Marked as 7'''D low density sand layer around the wooden pile footing is main Riga cathedral part unequal deformation reason.

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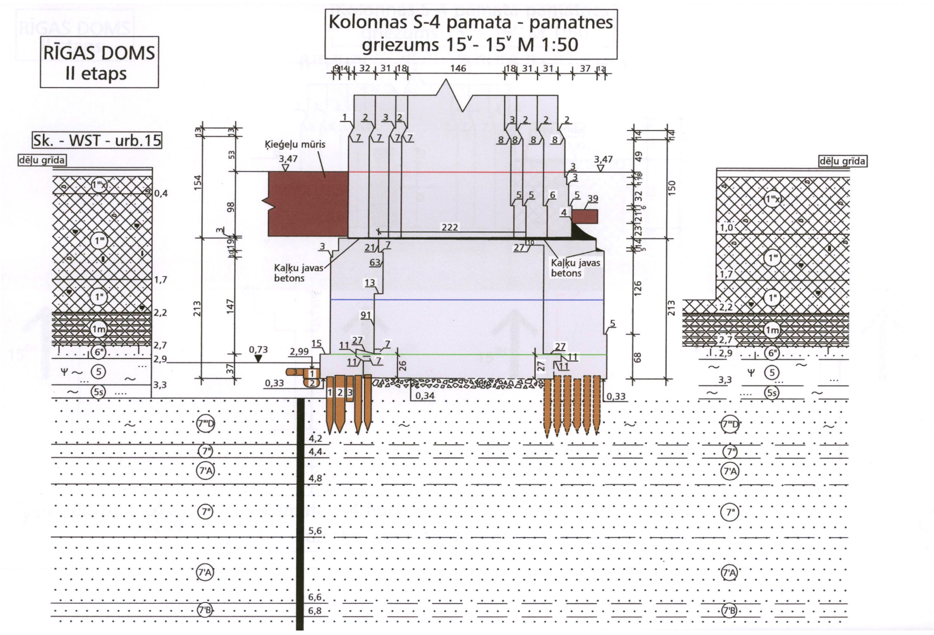

Figure 8 Column footing section in building middle span

Also an unsuited condition wooden piling was found. Support deformations along time counted as main reason of crack forming in masonry arch shell system. This is object of present research to fix the safe exploitation limits. Extreme support deformation of masonry shell system change force lines and part interacting stresses in shell section. Cracking changed thrust line position in cross section is safe situation definition criteria. Present deformed situation and LBS Konsultants make support deformation simulation by Plaxis in 2005 show approximate unequal support deformations in 16 cm by various parts of cathedral. Future support deformations predict crack prolongation, plastic hinge forming and unequal support deformation possibility.

To understand ground water flow and level changes Riga Department ordered ground water control monitoring from Balt-Ost-Geo Company. The ground water table monitoring during 2006 showed approximately 0.4m wooden pile coverage and less. In situation when wooden pile caps are not covered with ground water table wood structure degradation is possible. Solutions must be found to prevent air exposure possibility.

4.6. SOFO deformation monitoring system development

Surveillance des Ouvrages par Fibres (SOFO) optical fiber sensors monitoring system received from Smartec and would be mounted in few following months to

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change mechanical tensometer crack monitoring to a long term. Optical sensor mounting cracks to provide masonry part movement in five years is decided to control safe exploitation. According with mechanical tensometer monitoring reports presented from 2005 crack oscillation varies 1-2mm every year depending on season. The main reason of optical tensometer usage is computational control of measurement data, long lasting life period and minimized side factor's influence on measurement's quality.

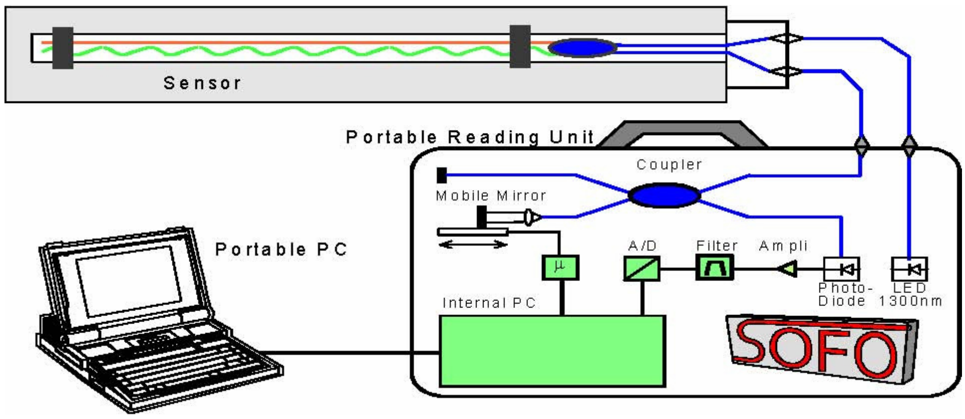

Figure 9 SOFO principles of optic tensometer

The SOFO measuring system gives precision of 0.02mm therefore defining high level of monitoring. Used as reading unit Smartec Bee allows control of 24 optical sensor units and communicates with registration PC trough telephone line. Additional thermo sensors for external and internal temperature control are included in measured program. Internal memory of reading unit gives possibility of data downloading by reasonable schedule. Battery support of reading unit gives possibility of non-stop monitoring. Support deformation caused crack widening is the way of safe criteria exploitation definition.

5. STABILITY CALCULATION AND SAFETY CRITERIA DEFINITION

As discussed before critical thrush line position must be found on deformed structure of masonry arch shell structure. Laser three dimensional scanning geometrical models were used to define geometric forms of Gothic arch system. The latest FEM software usage in heritage building masonry calculations in two stages is developed. In first stage shown in figure 10 linear elastic material approach is used. Support deformation loading is applied by Staad Pro software. In second stage of analyze deformed structure shape and plastic material approach is used to define stability by thrust line position in section.

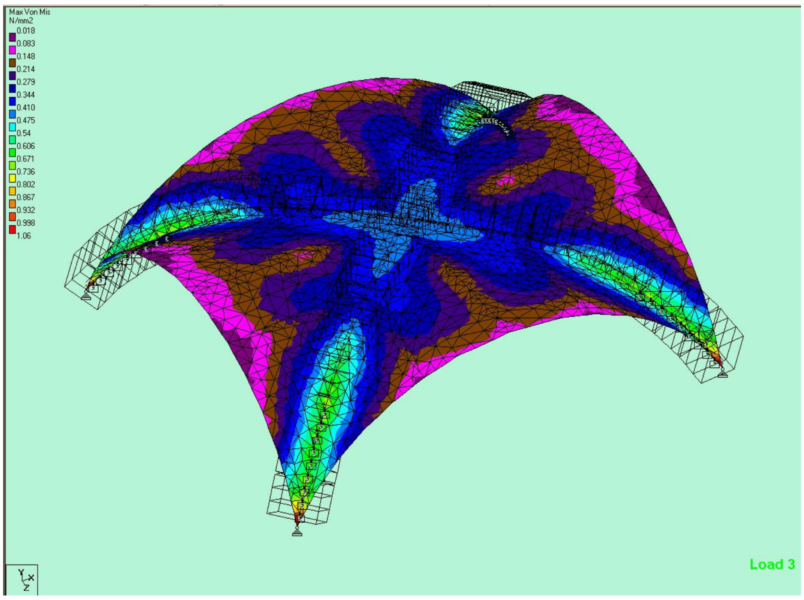

Figure 10 First stages analyzed with Staad Pro elastic material approach using

The second stage of deformed structure includes the fixed and measured by monitoring support settlements, cracked part interaction and subsoil elasticity. Plaxis computing tool is developed for taking into account masonry plasticity and stiffness. Each arch part is stabile and safe till thrust line locate in curved shell cross section. Defined by Heyman four hinge forming as safety criteria is also used to find the support deformation limits by numerical computing.

6. CONCLUSIONS

The present research is a small part of huge monitoring and investigation amount done in Riga cathedral. All previously collected investigations data and calculations increase the cathedral structure detailed understanding. Our view about masonry material computing technologies and possible material assumptions received from available papers are used to prepare FEM model and computational strategy choice. Laser scanning based surface geometric data transfer to FEM computational software significantly reduce modeling time. Improving laser scanned precision and computational devices will give the powerful tool for structural analyze of heritage buildings. Detailed investigations of material properties, exploitation conditions and deformed situation must be taken into account. None methodology describing masonry material is possible to include the side effects of building process. It is necessary to point out the side effect of building process: significant dispersion of material properties; geometric and material variations; shell structure deformed conditions and various material interactions. Wind, solar radiation and temperature loading give no big influence on computational results but can be important. Two step computational method highlight deformed shape influence on thrust line position in cross section of arch shell. The monitoring improvement by long term SOFO optical tensometer tool is the way for detailed structure investigation and safe exploitation system establish of Riga cathedral. Methodology developed with curved arch surfaces laser scanning in reason to use data as geometrical model for FEM computing software is our aim of future improvement.

Acknowledgements

Thanks to Riga cathedral staff for huge amount of former Soviet time investigation and monitoring data, which is included in this paper. Involved investigator's permit give us big acknowledge to understand the cathedral building. Thanks to the Riga cathedral representatives understanding in the latest technology usage on arch long term monitoring advantage. Thanks a lot for complex monitoring development and funding in heritage building to keep it in good health.

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