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About some computation particularities of multicell polygonal cross-section bars

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Summary

A set of particularization of the thin-wall bars with polygonal multicell section theory is developed. The hypothesis of the indeformability of the section outline is assumed. There are derived the relations expressing the geometric characteristic functions I_{B^*} *-the moment of inertia at free torsion and* I_{ω^*} *- the generalized sectorial moment of inertia.*

KEYWORDS: multicell polygonal section, nonuniform warping, the generalized sectorial moment of inertia.

1. INTRODUCTION

Thin-walled structural elements with multicell box cross-section are frequently used in the achievement of structures due to the reasonable way of disposing the material over the section. In most cases, the loading plane doesn`t contain the shear center of section. The supports of these elements restrain the free warping of the section in these regions. This leads to a nonuniform warping of the section along the elements. The nonuniformity of the section warping generates additional normal stress distribution σ_{ω} , and shear stress, τ_{ω}, which are estimated applying calculus theories of the thin-walled bars stressed to warping.

In the literature are known two development directions of these theories:

a) when the bar section outline is considered nondeformable;

b) when it is considered the section deformability through the warping effect.

In what follows a theoretical analysis concerning the computation particularities specific to polygonal multi-cell sections is affected.

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2. PARTICULAR FORMS OF EXPRESSING THE COMPUTATION THEORY

Additional stress functions generated by the nonuniform warping of the section are obtained from the known relationship expressing the stress-strain dependence. In this way, for the normal stress, σ_{ω^*}, it will be obtained the expression:

$$\sigma_{\omega^*} = E\varepsilon_x = -E\omega^* \overline{\beta}'' \tag{1}$$

where: ω^* is the generalized sectorial characteristic given by relation:

$$\omega^* = \int_0^s \left(r - \frac{\psi_i}{t} \right) ds \tag{2}$$

$\overline{\beta}$ - the warping angle of the section; ψ_i - the distribution function of the shear stress provided by pure torsion for "*i*" cell. In the particular case of a hollow polygonal section, this function represents the solution of the equations system:

$$\begin{cases} \psi_1 \overline{L}_1 - \psi_2 \overline{L}_{1,2} = 2\Omega_1 \\ -\psi_1 \overline{L}_{1,2} + \psi_2 \overline{L}_2 - \psi_3 \overline{L}_{2,3} = 2\Omega_2 \\ -\psi_2 \overline{L}_{2,3} + \psi_3 \overline{L}_3 = 2\Omega_3 \end{cases} \tag{3}$$

where: $\overline{L}_1, \overline{L}_2, \overline{L}_3$ are the reduced lengths of the cell outlines forming the section, given by the relations:

$$\overline{L}_1 = \oint_{C_1} \frac{ds}{t}, \qquad \overline{L}_2 = \oint_{C_2} \frac{ds}{t}, \qquad \overline{L}_3 = \oint_{C_3} \frac{ds}{t} \tag{4}$$

$\Omega_1, \Omega_2, \Omega_3$ - the areas defined by the average line of the component cells of that section:

$$\Omega_1 = \frac{1}{2} \oint_{C_1} r_1 ds, \qquad \Omega_2 = \frac{1}{2} \oint_{C_2} r_2 ds, \qquad \Omega_3 = \frac{1}{2} \oint_{C_3} r_3 ds \tag{5}$$

r_1, r_2, r_3 -the reduced areas of the three including outlines; $\overline{L}_{1,2}, \overline{L}_{2,3}$, the reduced lengths of the average lines limiting the cells, are given by:

$$\overline{L}_{1,2} = \int_a^b \frac{ds}{t}, \; \overline{L}_{2,3} = \int_c^d \frac{ds}{t},$$

a, *b*, respectively *c, d* being the extreme limits of these average lines. In the particular case of the polygonal area single cell, function ψ is:

$$\psi = \frac{2\Omega}{\overline{L}},$$

Ω being the area limited by the average line of the cell and $\overline{L}$ - the reduced length of its outline.

The function of the total shear stress at point situated on the multicell section area is obtained by summing of the shear function generated by the free torsion (τ_B) and those generated by the nonuniform warping (τ_ω):

$$\tau_{s,x} = \tau_B + \tau_\omega = G\frac{\psi_i}{t}\overline{\varphi}' - E\frac{S_{\omega^*}}{t}\overline{\beta}''' \tag{6}$$

where: $\overline{\varphi}$ is the relative shape angle of the section at free torsion; S_{ω^*}-the generalized sectorial static moment which is estimated by means of relation:

$$S_{\omega^*} = \int_0^s \omega^* ds + S_{\omega_0^*}$$

The resulted torsion moment over the polygonal multicell section is obtained by equivalence relation:

$$M_x = \int_A \tau_{s,x} r dA \tag{7}$$

Replacing relation (6) in (7) we obtain:

$$M_x = \int_A \left(G\frac{\psi_i}{t}\overline{\varphi}' - \frac{ES_{\omega^*}}{t}\overline{\beta}''' \right) \cdot rdA = G\int_A \left(\frac{\psi_i}{t} rdA \right) \cdot \overline{\varphi}' - E\left(\int_A \frac{S_{\omega^*}}{t} rdA \right) \overline{\beta}''' \tag{8}$$

Denoting:

$$I_{B^*} = \int_A \frac{\psi_i}{t} rdA \tag{9}$$

and

$$I_{\omega^*} = -\int_A \frac{S_{\omega^*}}{t} rdA \tag{10}$$

relation (8) becomes:

$$M_x = GI_B\overline{\varphi}' - EI_{\omega^*}\overline{\beta}''' \tag{11}$$

I_{B^*} being the inertia product at free torsion and I_{ω^*} - the generalized sectorial inertia moment.

The particular forms (9) and (10) are specific to polygonal multicell sections, the analytical demonstration of these relations being presented in what follows. Thus:

a) Relation (9) is derived when considering that in the case of the polygonal multicell section one can write:

$$\int_A \frac{\psi_k}{t} r dA = \sum_{k=1}^{n} \oint_k \frac{\psi_k}{t} r ds = \sum_{k=1}^{n} \psi_k \oint_k r ds \tag{12}$$

and having in mind that:

$$\oint_k r ds = 2\Omega_k$$

relation (12) becomes:

$$\int_A \frac{\psi}{t} r dA = 2\sum_{k=1}^{n} \psi_k \Omega_k = I_{B^*}$$

b) In order to derive relation (10) it is assumed that additional shear stresses, τ_ω, can be considered uniformly over the wall thickness, the element being with thin walls. Thus, the expression of the shear stress function may be expressed with relation:

$$f_\omega = \tau_\omega t$$

The stress flow function at any point of the average line of the section is:

$$f_{\omega^*} = E\bar{\beta}''\int_0^s \omega^* t ds + f_{\omega_1^*} \ ,$$

$f_{\omega_1^*}$ being the value of additional shear stresses in the initial point ($s = 0$).

In the case of a structural element with "l" length, the additional shear stress flow over the section at this end ($s = l$) is:

$$\sum_{j_1} f_{\omega_1} = \sum_{j_2} f_{\omega_2} \tag{13}$$

where $\sum_{j_1}$, $\sum_{j_2}$, show the summing over the initial and end sections which are connected in joint j.

From Umanski s theory it is well known that for any closed cell, the kinematic probability of the displacement is:

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$$\oint_k \frac{f_{\omega^*}}{t} ds = 0 \tag{14}$$

consequently the additional shear stresses, τ_ω, are self-balanced.

Taking into account that additional stress distribution, τ_ω, is proportional with the generalized sectorial static moment, S_{ω^*}, Eqs. (13) and (14) can be written under the form:

$$\sum_{j_1} S_{\omega_1^*} = \sum_{j_2} S_{\omega_2^*} \tag{15}$$

and

$$\oint_k \frac{S_{\omega^*}}{t} ds = 0 \tag{16}$$

Thus the integral of the right member of relation (10) becomes:

$$\int_A \frac{S_{\omega^*}}{t} r dA = \sum_{i=1}^{n} \left(\int_0^l \frac{S_{\omega^*}}{t} r dA \right)_i = \sum_{i=1}^{n} \left[\int_0^l S_{\omega^*} \left(r - \frac{\psi_i}{t} \right) ds + \int_0^l S_{\omega^*} \frac{\psi_i}{t} ds \right]_i \tag{17}$$

However, taking into that:

$$d\omega^* = \left(r - \frac{\psi}{t} \right) ds$$

and the hypothesis of the constant character of function ψ along the cell perimeter, the last of identity (17) becomes:

$$\sum_{i=1}^{n_c} \left(\int_0^l S_{\omega^*} d\omega^* + \psi_i \int_0^l \frac{S_{\omega^*}}{t} ds \right) \tag{18}$$

n_c being the total number of cells of the polygonal section.

Integrating by parts the first term in the brackets and taking into consideration that:

$$\frac{dS_{\omega^*}}{ds} = \omega^* t$$

it results:

$$\int_0^l S_{\omega^*} d\omega^* = \omega^* S_{\omega^*} \Big|_0^l - \int_0^l \omega^{*^2} t ds = \omega_2^* S_{\omega_2^*} - \omega_1^* S_{\omega_1^*} - \int_0^l \omega^{*^2} ds \tag{19}$$

After replacing (19) into (18), it results:

$$-\sum_{i=1}^{n_c}\left(\int_0^l \omega^{*2} tds\right)_i + \sum_{i=1}^{n_c}\left(\omega_2^* S_{\omega_2^*} + \omega_1^* S_{\omega_1^*}\right)_i + \sum_{i=1}^{n}\left(\psi_i \int_0^l \frac{S_{\omega^*}}{t} ds\right) \tag{20}$$

n_c being the side number of the polygonal section.

The first term of relation (20) becomes:

$$-\sum_{i=1}^{n_c}\left(\int_0^l \omega^{*2} tds\right)_i = -\int_A \omega^{*2} dA = -I_{\omega^*}$$

The second term of the equation (20) represents the summation of the products of sectorial characteristics from all the extremities of the cell sides wich are concurrent in the joints of the cross-section.

Taking into account that the values of the sectorial coordinates are the same and having in mind Eq. (15), we obtain:

$$\sum_{i=1}^{n_c}\left(\omega_2^* S_{\omega_2^*} + \omega_1^* S_{\omega_1^*}\right)_i = \sum_{j=1}^{n_n}\left(\sum_{j_2} \omega_2^* S_{\omega_2^*} - \sum_{j_1} \omega_1^* S_{\omega_1^*}\right) =$$

$$= \sum_{j_1}^{n_n} \omega^* \left(\sum_{j_2} S_{\omega_2^*} - \sum_{j_1} S_{\omega_1^*}\right) = 0$$

where n_n represents the joint number of the polygonal section.

The third term of equation (20) can be transformed in a summation over all cells:

$$\sum_{i=1}^{n_c}\left(\psi \int_0^l \frac{S_{\omega^*}}{t} ds\right) = \sum_{k=1}^{n_c} \psi_k \oint_k \frac{S_\omega}{t} ds = 0$$

based on equation (16), therefore:

$$I_{\omega^*} = -\int_A \frac{S_{\omega^*}}{t} rdA$$

3. CONCLUSIONS

In this paper are presented some particularities of the theory concerning the calculus of thin-wall structural element with polygonal multicell section, and are derived the additional normal and shear stress functions, σ_ω and τ_ω, also the resulting torsional moment, M_x. There are derived new analytical expressions of the geometric characteristics: moment of inertia at torsion, I_{B^*}, and the generalized sectorial moment, I_{ω^*}.

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