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# Identification Methods of the Nonlinear Systems Subjected to Seismic Actions

Doina Stefan, Violeta-Elena Chitan
[1]*Structural Mechanics, Technical University "Gh. Asachi" Iasi, 700050, România*

Summary

*The need to identify the nonlinear systems subjected to stochastic actions leaded to the development of the classical and/or deterministic methods toward some methods capable to take into account the improved models for structure and excitation.*

*The identification of the nonlinear systems can be carried out using specific approaches that are based on the reparability hypotheses or on the change of the nonlinear system with an equivalent linear system that has the features closely related to the initial one.*

*Current nonlinearities that occur in the structural response during a strong earthquake are geometrical and/or physical (nonlinearity of the stiffness and the damping). In order to underline the nonlinearity of the stiffness, in this paper we take advantage of the modeling method for the structural response using a non-stationary linear model and the parameter identification of the equivalent stationary linear model, both for the SDOF and the NDOF systems.*

*Explaining the nonlinearity of the stiffness can be done in two specific ways: the structure response modeling through the nonstationary linear model and the parameter identification of the equivalent stationary linear model; the real stiffness valuation of the structure during each vibration cycle using the internal force calculus of the structure.*

KEYWORDS: nonlinear system, stationary linear model, nonlinearity of the stiffness, structural response.

## 1. INTRODUCTION

In the context of continuous efforts to improve the knowledge in the field of dynamic-seismic behavior of a building, the direction is toward the study of structural behavior starting from the tests in situ.

This kind of approach gives the means for: structural quality and security level checking in the conditions of real exploitation; validation of the existent analytical

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models of behavior, also for the a-priori hypotheses of the structural response prediction; knowledge accumulation, necessary to improve the dimensioning methods and the well argued working out of the new behavior models.

Knowing precisely or often good enough the features of a system implies the discerning use of various theoretical and experimental approaches. The system identification doesn't mean anything else but the convenient processing of some information achieved from the tests, in order to obtain a physical or mathematical model, as sample and easy-to-use as possible.

An identification process consists of: the choice of the model structure; the choice of the criterion to compare the model with the real structure; estimation of the model parameters.

The need to identify the nonlinear systems subjected to stochastic actions leaded to the development of the classical and/or deterministic methods toward some methods capable to take into account the improved models for structure and excitation. For this reason, the non-linear system identification can be carried out using specific approaches that are based on the reparability hypotheses or on the change of the nonlinear system with an equivalent linear system that has the features closely related to the initial one.

Current nonlinearities that occur in the structural response during a strong earthquake are physical and/or geometrical. In the case of physical non-linearity the equations of motion settled in the hypothesis of visco-elstic linear behavior do not remain valid already, and generally, the forces that occur are expressed using non-linear functions, with respect to displacement and velocity, this leading to the non-linear stiffness and non-linear damping.

To underline the non-linear stiffness, two methods are usually carried out: the structural response modeling using a non-stationary linear model and the parameter identification of the equivalent stationary linear model; the estimation of the effective structural stiffness during each cycle of vibration using the computation of internal structural forces.

## 2. THE EQUIVALENT LINEARIZATION METHOD

Using the fact that linear systems are solved easily, the study of the nonlinear systems can be performed with sufficient accuracy by changing the nonlinear system with an equivalent linear system having the features closely related to the initial one. This technique of linearization for the systems subjected to stochastic actions was proposed for the first time by Booton in 1953.

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### 2.1. SDOF Systems

The simplest mechanical system is the SDOF oscillator, which has the equation of motion as follows:

$$m\ddot{y}(t)+h(y,\dot{y})=f(t) \tag{1}$$

where:

$y$, $\dot{y}$, $\ddot{y}$ are denoting the displacement, velocity and acceleration;

$h(y,\dot{y})$ denotes a function that describes the nonlinearity of the oscillator;

f(t) denotes the stochastic action.

The linearization method consists of the change of nonlinear oscillator with an equivalent linear one. Equation (6.6) becomes:

$$m\ddot{\tilde{y}}(t)+c_{eq}\dot{\tilde{y}}(t)+k_{eq}\tilde{y}(t)=f(t) \tag{2}$$

where:

$y$, $\dot{y}$, $\ddot{y}$ are denoting the displacement, velocity and acceleration of the equivalent linear system;

$c_{eq}$ denotes the viscous damping of the equivalent system;

$k_{eq}$ denotes the stiffness of the equivalent linear system;

($c_{eq}$ and $k_{eq}$ must be found out).

It is normaly to say that the nonlinear system and the equivalent linear system do not have identically motions, so the solutions $y, \dot{y}, \ddot{y}$ do not verify exactly the equation (2), this becoming:

$$m\ddot{\tilde{y}}(t)+h(\tilde{y},\dot{\tilde{y}})=f(t)+\varepsilon(t) \tag{3}$$

where: ε(t) is the error that allows the measurement of the difference between the equations (1) and (2).

$$\varepsilon(t)=h(\tilde{y},\dot{\tilde{y}})-c_{eq}\dot{\tilde{y}}(t)-k_{eq}\tilde{y}(t) \tag{4}$$

In order to find out $c_{eq}$ and $k_{eq}$, the error ε(t) must have the minimum value. This method suggests a minimization process for ε(t), starting from the identity:

$$\frac{\partial E[\varepsilon(t)^2]}{\partial C_{eq}}=\frac{\partial E[\varepsilon(t)^2]}{\partial K_{eq}}=0 \quad (E[\varepsilon(t)^2]-math.operator) \tag{5}$$

Developing the equation (5) we obtain the system:

$$K_{eq}=\frac{E[h\cdot\tilde{y}]E[\dot{\tilde{y}}^2]-E[h\cdot\dot{\tilde{y}}][\tilde{y}\cdot\dot{\tilde{y}}]}{E[\tilde{y}^2]E[\dot{\tilde{y}}^2]-E[\tilde{y}\cdot\dot{\tilde{y}}]^2}$$
$$C_{eq}=\frac{E[h\cdot\dot{\tilde{y}}]E[\tilde{y}^2]-E[h\cdot\tilde{y}][\tilde{y}\cdot\dot{\tilde{y}}]}{E[\tilde{y}^2]E[\dot{\tilde{y}}^2]-E[\tilde{y}\cdot\dot{\tilde{y}}]^2} \qquad (6)$$

where:

h is as a matter of fact $h(y,\dot{y})$;

$E[\tilde{y},h(\tilde{y},\dot{\tilde{y}})]$ and $E[\dot{\tilde{y}},h(\tilde{y},\dot{\tilde{y}})]$ can be expressed as functions of y and $\dot{y}$ and $c_{eq}$ and $k_{eq}$ are functions depending on the terms of the covariance matrix, denoted cov[Y] and having the following format:

$$\mathrm{cov}[Y]=\begin{bmatrix}E[\tilde{y},\tilde{y}] & E[\tilde{y},\dot{\tilde{y}}]\\ E[\dot{\tilde{y}},\tilde{y}] & E[\dot{\tilde{y}},\tilde{y}]\end{bmatrix} \qquad (7)$$

## 2.2. NDOF Systems

The nonlinear NDOF system has the following equation of motion:

$$M\ddot{y}(t)+h(y,\dot{y})=f(t) \qquad (8)$$

where:

f(t) is a representative gaussian process of the excitation;
W(t) is a stationary gaussian process;
g(t) is a vector with terms as functions that are measurable in time;
y is the displacement vector.

$$f(t)=W(t)\cdot g(t) \qquad (9)$$

The equivalent linear system has the form:

$$M\ddot{\tilde{y}}+C_{eq}\dot{\tilde{y}}+K_{eq}\tilde{y}=f(t) \qquad (10)$$

where:

$C_{eq}$ is the equivalent damping matrix that depends on the time (because g(t) is not constant);
$K_{eq}$ is the equivalent stiffness matrix.

The vector of errors is defined by the relation:

$$\varepsilon(t)=h(\tilde{y},\dot{\tilde{y}})-C_{eq}(t)\dot{\tilde{y}}-K_{eq}(t)\tilde{y} \qquad (11)$$

This vector is computed as a function of variables of the linear system and it is minimized using the least square criterion. The $C_{ij}(t)$ and $K_{ij}(t)$ coefficients from the $C_{eq}(t)$ and $K_{eq}(t)$ matrices are computed from:

$$\begin{aligned} &\frac{\partial E[\varepsilon^T(t)\varepsilon(t)]}{\partial C_{ij}(t)} = 0 \qquad i = 1, N \quad ; \quad j \le i \\ &\frac{\partial E[\varepsilon^T(t)\varepsilon(t)]}{\partial K_{ij}(t)} = 0 \end{aligned} \tag{12}$$

Using equation (11) we can write:

$$\begin{aligned} &E[h(\tilde{y},\dot{\tilde{y}})\tilde{y}^T] - C_{eq}(t)E[\dot{\tilde{y}},\dot{\tilde{y}}^T] - K_{eq}(t)E[\tilde{y},\dot{\tilde{y}}^T] = 0 \\ &E[h(\tilde{y},\dot{\tilde{y}})\dot{\tilde{y}}^T] - C_{eq}(t)E[\tilde{y},\tilde{y}^T] - K_{eq}(t)E[\tilde{y},\tilde{y}^T] = 0 \end{aligned} \tag{13}$$

From above, the $C_{ij}(t)$ and $K_{ij}(t)$ terms can be obtained.

The excitation is a gaussian one, this leading to a gaussian response and allowing to write the $E\left[h(\tilde{y},\dot{\tilde{y}}),\tilde{y}^T\right]$ and $E\left[h(\tilde{y},\dot{\tilde{y}}),\dot{\tilde{y}}^T\right]$ terms as functions of the Y(2n x 2n) covariance matrix terms. The computation of the $E\left[h(\tilde{y},\dot{\tilde{y}}),\tilde{y}^T\right]$ and $E\left[h(\tilde{y},\dot{\tilde{y}}),\dot{\tilde{y}}^T\right]$ terms is easily carried out if we assume the fact that the displacements and velocities are two by two independent.

$$C_{ij}(t) = E\left[\frac{\partial h_i(\tilde{y},\dot{\tilde{y}})}{\partial \tilde{y}}\right] \qquad K_{ij}(t) = E\left[\frac{\partial h_i(\tilde{y},\dot{\tilde{y}})}{\partial \dot{\tilde{y}}}\right] \tag{14}$$

$$Y = \begin{bmatrix} E[\tilde{y}\tilde{y}^T] & E[\tilde{y}\dot{\tilde{y}}^T] \\ E[\dot{\tilde{y}}\tilde{y}^T] & E[\dot{\tilde{y}}\tilde{y}^T] \end{bmatrix} \tag{15}$$

The linear system has n(n+1)/2 unknowns, which are the $C_{ij}(t)$ and $K_{ij}(t)$ i=1,n terms and they are computed with respect to the terms of the Y(t) matrix.

## 3. NONLINEAR SYSTEM IDENTIFICATION. THE MODEL OF THE NONLINEAR OSCILLATOR

It is considered the fact that the measured response at the roof level can be modeled using the nonlinear oscillator, which has the equation of equilibrium as follows:

$$\ddot{y} + f(y,\dot{y}) = -p\ddot{y}_{ter} \tag{16}$$

Assuming for the mass and the damping to remain constant, and for the stiffness to be variable, the force f has the expression:

$$f = c_0\dot{y} + k_0(t)y \tag{17}$$

where:

$c_o$ is the damping constant over the unit of mass;

$k_o(t)$ is the structural stiffness over the unit of mass.

The equation (16) can also be written:

$$\ddot{y} + c_0\dot{y} + k_0(t)y = -p\ddot{y}_{ter} \tag{18}$$

In order to compute the response of the assumed model, the non-linear differential equation (17) must be solved for each iteration step of the numerical algorithm for minimization.

The equation of equilibrium in incremental variables is:

$$\Delta\ddot{y} + c_0\Delta\dot{y} + k_0(t)\Delta y = -p\Delta\ddot{y}_{ter} \tag{19}$$

where:

$$\begin{aligned} \Delta\ddot{y} &= \ddot{y}(t+\Delta t) - \ddot{y}(t) \\ \Delta\dot{y} &= \dot{y}(t+\Delta t) - \dot{y}(t) \\ \Delta y &= y(t+\Delta t) - y(t) \\ \Delta\ddot{y}_{ter} &= \ddot{y}_{ter}(t+\Delta t) - \ddot{y}_{ter}(t) \end{aligned} \tag{20}$$

There are many ways to solve the equation (19). The best one is the step-by-step approach. The dynamic equilibrium is stable at the beginning and at the end of each time step Δt, inside in which it is assumed a linear variation of acceleration and the structural features remaining constant during all the range (fig.1).

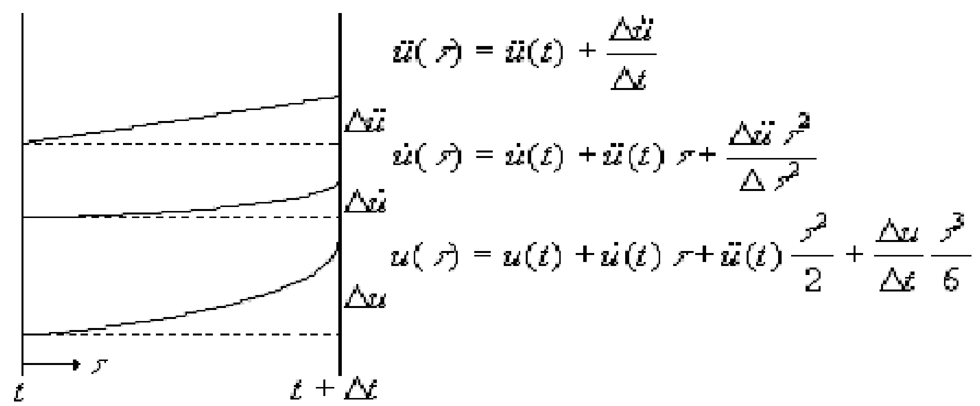


Figure 1. The dynamic equilibrium

The incremental values for velocity and displacement are:

$$\Delta\dot{y}(t) = \ddot{y}(t)\Delta t + \Delta\ddot{y}(t)\frac{\Delta t}{2} \tag{21}$$

$$\Delta y(t) = \dot{y}(t)\Delta t + \ddot{y}(t)\frac{\Delta t^2}{2} + \Delta\ddot{y}(t)\frac{\Delta t}{2} \tag{22}$$

Using the incremental displacement as a basic variable, one can obtain the incremental acceleration from equation (22), which is after substituted in equation (21), this leading to:

$$\Delta\ddot{y}(t) = \frac{6}{\Delta t^2}\Delta y(t) - \frac{6}{\Delta t^2}\Delta\dot{y}(t) - 3\ddot{y}(t) \tag{23}$$

$$\Delta\dot{y}(t) = \frac{3}{\Delta t^2}\Delta y(t) - 3\dot{y}(t) - \frac{\Delta t}{2}\ddot{y}(t) \tag{24}$$

Substituting equations (23) and (24) in equation (19) we obtain:

$$\left[\frac{6}{\Delta t^2}\Delta y(t) - \frac{6}{\Delta t^2}\Delta\dot{y}(t) - 3\ddot{y}(t)\right] + K_0\left[\frac{3}{\Delta t}\Delta y(t) - 3\dot{y}(t) - \frac{\Delta t}{2}\ddot{y}(t)\right] + K_0(t)\cdot\Delta y(t) = -p\ddot{y}_{ter} \tag{25}$$

or:

$$\tilde{K}(t)\cdot\Delta y(t) = \Delta\tilde{\ddot{y}}_{ter} \tag{26}$$

where:

$$\tilde{K}(t) = \frac{6}{\Delta t^2} + \frac{3}{\Delta t}C_0 + K_0 \tag{27}$$

$$\Delta\tilde{\ddot{y}}_{ter} = -p\Delta\ddot{y}_{ter} + \frac{6}{\Delta t}\dot{y}(t) + 3\ddot{y}(t) + C_0\left[3\dot{y}(t) + \frac{\Delta t}{2}\ddot{y}(t)\right] \tag{28}$$

Therefore, the incremental displacement is computed starting from equation (26) and dividing the effective incremental load by the effective stiffness.

## 4. CONCLUSIONS

A nonlinear system can be defined as a system that doesn't obey to the superposition principles. This "non-property" occurs to the extremely various systems for which it doesn't exist general solving methods, but only special methods, adapted to each class of problems.

The identification of the nonlinear systems can be carried out using specific approaches that are based on the reparability hypotheses or on the change of the nonlinear system with an equivalent linear system that has the features closely related to the initial one.

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