Structural Mechanics

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The state of stresses in RC beams with installation holes located in compressed zone

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Summary

This paper presents a question of influence of installation holes located in the compressed zone of the reinforced concrete beam on functioning. In order to research the actual functioning of the element a numerical model based on Finite Element Method has been created.

Two independent analyses were performed in which linear and nonlinear constitutive relations for concrete and for steel have been used. The results acquired have been compared and discussed.

KEYWORDS: RC beam, nonlinear FEM analysis, nonlinear material, numerical model, numerical analysis.

1. INTRODUCTION

Designing process of reinforced concrete structures (RCS) is based on current state of knowledge. Codes of Practice (CP) in many countries are based on this knowledge. It can be considered that if an element is designed according to requirements of the CP and it fulfills requirements of border states (in both phases: service and mount) as well as construction requirements, there is certain probability that it will be safe and will properly carry adequate loads.

The problem occurs when there is need to specify the state of safety of reinforced concrete element not designed in accordance with requirements of the CP and especially when it doesn't fulfill construction requirements. Then it is necessary to perform a complete analysis of stresses and strains according to rules of mechanics. And here a problem arises: according to rules of a linear analysis it is not possible to specify correctly the state of stresses in reinforced concrete elements in the II (2nd) phase. So there is a need to make the analysis based on physical and geometrical nonlinearities in reinforced concrete.

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2. NONLINEAR MODEL

Fundamental physical stress-strains relations based on assumption of linear elastic relation come from the Hook's law. The structure deformations are not proportional to the loads and this irregularity in material of a structure. This kind of material is called plastic.

From the point of view of theory of elasticity, material turns to undetermined state after exceeding limit value of stresses (criterion of plasticity) defined by constitutive relationships between stresses and strains. There are many criteria of plasticity in complex state of stresses [1,2,3]. The most popular are: Trasca and Von Mises (M-H-V) yield criterion for describing materials such as steel and aluminum; and Coulomb and Druckner-Prager yield criterion for describing such materials as concrete, rocks, soils and sands. In literature, for describing plasticity in reinforced concrete the most popular are Von Mises criterion (M–H–V) for steel and Druckner-Prager criterion for concrete under compression. The most divergences among researchers of the problem have been caused by concrete under tension. The solutions used [4.5.6.7] are mainly connected with adopted calculation techniques. In this paper the Finite Element Method as a basic analytical tool has been used. Within a framework of this method the smeared crack approach for modeling concrete under tension has been used.

Basic assumption of theory of plasticity in FEM formulation is decomposition of strains to elastic and plastic parts:

$$\varepsilon = \varepsilon^{e} + \varepsilon^{p} \tag{1}$$

Such a model is called elasto-plastic. The most important consequence of adopting this material model relation between total stresses for certain time t and total strains for time t, is additional parameter – a function of stresses and strain history. This function is taken into account by internal parameter of state □, governed by a specific evolution law. Properties of elasto-plastic material are determined by assumptions:

- the elastic relation between total stresses and strains is given by:

$$\sigma = \mathbf{D}\varepsilon^{e} \tag{2}$$

- the yield condition determines the state of stresses when the plastic flow is initiated. This condition is determined by vector function of stresses and by the parameter of state &ʒ, according to relationship:

$$f(\sigma, \kappa) = 0 \tag{3}$$

- the flow rule specifies the plastic strain rate vector as a function of the state of stresses in relationship:

$$\dot{\varepsilon}^{p} = \sum_{j=1}^{n} \dot{\lambda}_j \frac{\partial g_j}{\partial \sigma} \tag{4}$$

for n functions of plastic potential g_j which can also be considered as a function of the stress vector and state parameter g_j(♦,&).

The plastic multiplier $\dot{\lambda}_j$ is determined from Kuhn-Tucker conditions:

$$\begin{cases} f \leq 0 \\ \dot{\lambda}_j \geq 0 \\ \dot{\lambda}_j f = 0 \end{cases} \tag{5}$$

- the hardening hypothesis describes evolution of the internal state parameter &. Generally, changes of & parameter are given by a function of the stresses vector and the plastic strain rate vector, i.e.:

$$\dot{\kappa} = h(\sigma, \dot{\varepsilon}^{p}) \tag{6}$$

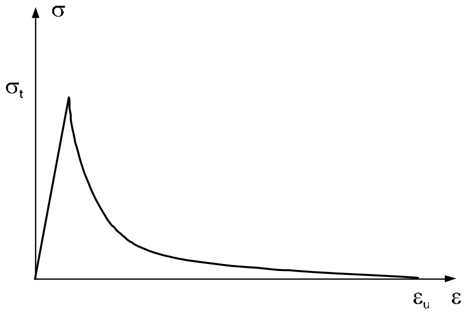

Fig. 1 – The stress-strain relation in crack.

The smeared crack approach puts forward two concepts: smeared cracks with fixed direction which is determined when the crack is initiated (constant direction of the crack during the whole iteration) and the rotated cracks (the crack direction changes during iteration according to redistribution of main stresses). Transient concept is multi-directional crack model which makes an assumption that a new crack arises when angle between direction of vector of actual main stresses and direction normal to the old crack exceeds certain value but the old one is still active.

The fundamental feature of smeared crack approach is taken from decomposition of the total strain into elastic strain and a crack strain as:

$$\varepsilon = \varepsilon^{e} + \varepsilon^{cr} \tag{7}$$

During increasing of loading, the new cracks simultaneously occur and everyone of them makes new local co-ordinate system n-t. For the crack strain for crack i which is given by:

$$e_i^{cr} = \left[\varepsilon^{cr}{}_{nn,i}, \gamma^{cr}{}_{nt,i} \right]^T \tag{8}$$

total strain vector in crack is given by:

$$e^{cr} = \left[e_1^{cr}, e_2^{cr}, \ldots, e_n^{cr} \right]^T \tag{9}$$

Basic cracking parameters of the smeared crack concepts are: tensile strength of concrete which determines when crack appears, fracture energy which determine how much of energy is dissipated when a unit of crack appears (the area under ♦📫♏ curve in crack in fig. 1, shear retention relation in crack, the tension softening relations as a relation between crack stress and crack strain (shape of descending part of relations in fig. 1).

3. DESCRIPTION OF PROBLEM.

During the analysis of large precast concrete structure [8] it has been noticed that almost every beam has from 4 to 8 holes located in compressed zone. Through these holes go installation pipes.

Fig. 2 – Shown Precast concrete beams with installations holes.

Detailed analysis of the project has shown that no analysis of influence of these holes on load-carrying capacity and operating conditions had ever been carried out.

The analysis has been made for 11,54 m long precast concrete beam (0,40x1,025). It had 8 holes (see fig. 2) 3,60 m from the end of the beam 4 installation holes (15cm in diameter), have been made. They have been placed 20cm from the upper edge (i.e. the compressed zone).

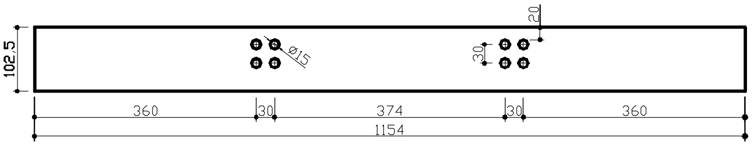

Fig. 3 – Draft of the beam. Installation holes displayed.

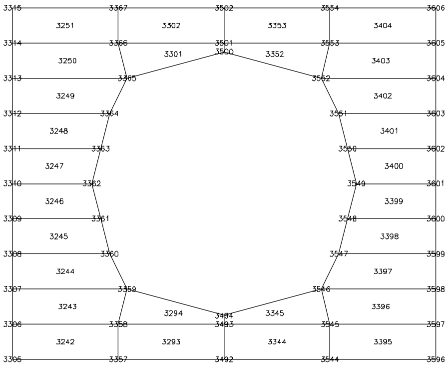

Fig. 4 – Scheme of nodes arrangement around the hole.

4. DESCRIPTION OF NUMERICAL MODEL.

The Finite Element Model has been designed with 4-node plain stress elements (fig. 4 and 5). Reinforcement has been modeled as embedded bars in structural elements – reinforcement strains have been computed on the basis of the displacement field of the mother element. This approach guarantees full connection of finite element and reinforcement (without any bond-slip). In numerical model both linear and nonlinear constitutive principles for concrete and steel have been used. The computer model has been performed only for a support zone 4 m long. It has been assumed that material was homogeneous – cross-sections only along neutral axis have been examined.

5. CALCULATIONS AND RESULTS.

For numerical calculation the actual load from the design project (216,78kN/m) has been taken into account. This load yields the bending moment at 3568,89kN/m.

In the figures 6 and 7 the linear static analysis results have been presented.

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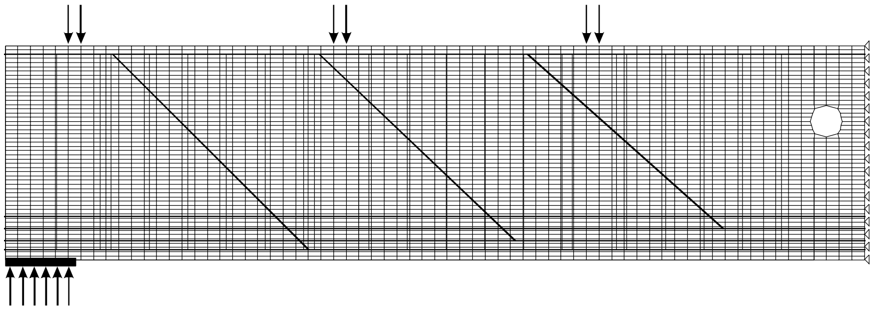


Fig. 5 – Scheme of finite elements arrangement.

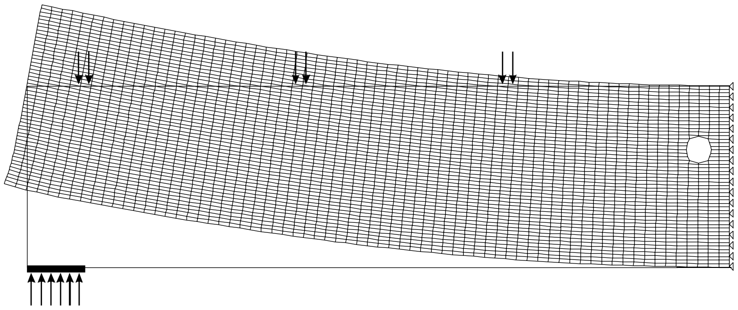


Fig. 6 – Beam deformations draft – linear static analysis.

It can be noticed that elastic relations in concrete don’t allow to analyze properly the stress distribution in concrete. According to this analysis the tensile stresses reach 11,71 MPa, which is not true. Therefore the correctness of stress distribution and its values around the hole could be questioned.

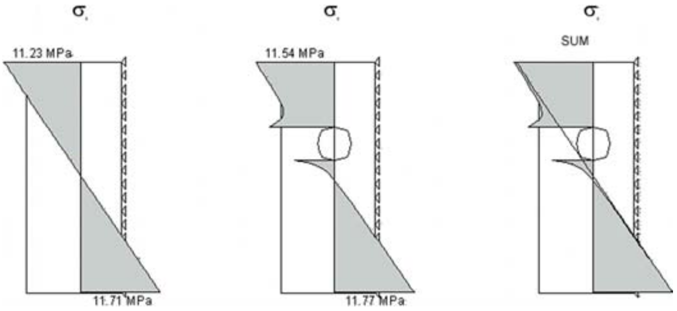


Fig. 7 – Stress distribution around the hole – linear static analysis.

In order to eliminate this problem the nonlinear analysis has been performed.

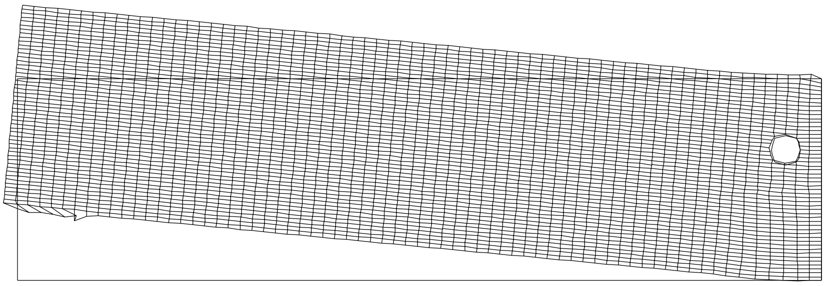


Fig. 8 – Element deformation for Lf= 2,496 (damage of the beam) – nonlinear analysis.

The numerical analysis has been carried out with use of Newton-Rawston iterative algorithm for solving a nonlinear system of equations. For iteration process the arc-length spherical method for prediction of load increment and energy norm criterion for convergence criterion of equilibrium iteration have been used. During numerical analysis load has been progressively increased in order to fulfill equation:

$$L_f \cdot \overline{\mathbf{K}} \times \overline{\mathbf{v}} = \overline{\mathbf{F}} \tag{10}$$

Where:

- L_f – scalar nonlinear load coefficient,
- K – global stiffness matrix,
- v – general displacements vector,
- F – general loads vector.

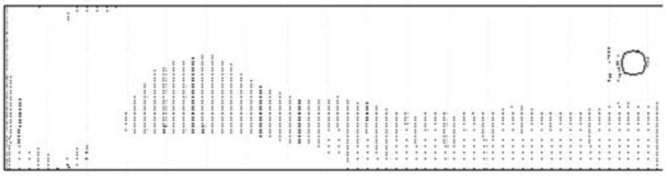
Response of the structure has been presented as a relation between selected degree of freedom in function of nonlinear load coefficient (Lf) which allows to determine critical values of loads at which structure fails (see fig. 10).

Structure damage mechanism can be observed through the research (albo investigation) of model deformation during the nonlinear analysis (fig. 8). The breaking of the beam close to the hole can be noticed very clearly. This has been caused by (the lack of) elastic recovery of other parts of researched element (no flexion).

Crack development has presented the state of stresses in the structure. In fig.9 areas of active cracks have been displayed.

An important fact is appearing of skew cracks (in the support zone) for coefficient of deformation Lf equal to 1,0 which has been observed in real structure as well. During damage phase almost the whole tensile zone of the beam has been cracked.

Cracking zone for L=0,785



Cracking zone for L= 1,242

Cracking zone for L= 2,496

Fig. 9 – Development of cracking zone for different values of nonlinear loads coefficient Lf.

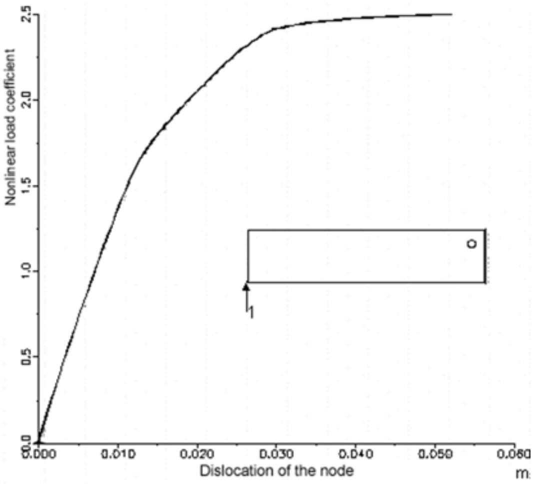


Fig. 10 – Relation between displacement of node 1 and nonlinear load coefficient L_f.

Through nonlinear analysis and stress distribution it is possible to determine crack development and neutral axis position of cross-section of RC beam (fig. 12). As contrasted with the nonlinear analysis, the linear analysis of RC structure has proved almost totally useless (fig. 11).

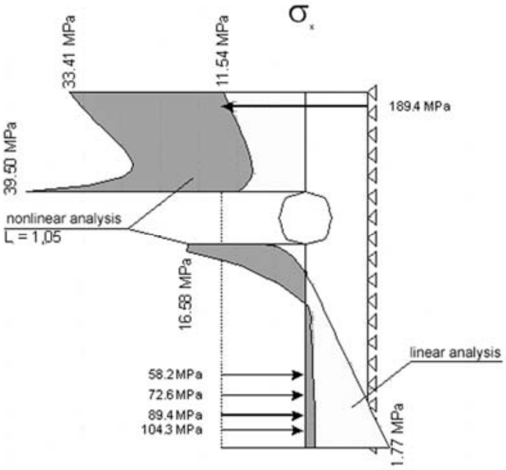


Fig. 11 – Stress distribution around the hole received from nonlinear analysis.

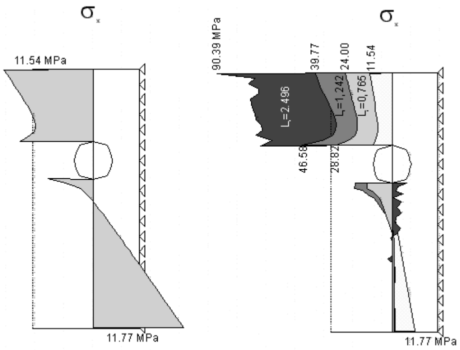


Fig. 12 – Stress distribution from linear and nonlinear analysis for various loads coefficients. Left – linear analysis, Right – nonlinear analysis.

6. CONCLUSIONS

In result of the analysis of the computer model an image of stresses around the hole has been received (fig. 7 and fig.11, 12). It can be seen very clearly there occurred a disorder in stress distribution around the hole. It is significant that stress increment at the edge of the hole reaches up to 70% of material strength (compression strength of concrete). However this disorder decays and about 10 cm from the hole edge stresses reach regular values (similar to those of elements without holes).

The analysis of a nonlinear loads coefficient has shown that the beam fails at large values of this coefficient (about 2,5 times of service load) which leaves large safety margin for the structure. Therefore it can be stated that the influence of the hole is not significant for decreasing of the beam safety.

Additionally the nonlinear computer model has been proved highly reliable for the analyses of cracked RC elements.

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