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A new form of the Active Moments Method

Constantin Ionescu
Civil Engineering, University, Iaşi, Zip code: 700482, Romania

Summary

The paper desires to pay a tribute to Professors A. Şesan and N. Orlovschi, which professed last century at the Faculty of Civil Engineering and Installations from Iasi.

These professors imagined a calculation procedure for frames, named active moment method, as a response to the displacement (deformation) method.

The structure calculation by displacement methods is conducted on a base system, obtained by introducing fictive connections which deters the possible displacement of the nodes – revolutions and translations.

The elements in question in the displacement method are the nodes real displacements (written Z_i*). Under the exterior forces action and displacements, on the elements in question direction, in the complementary connections appear reactions. Total reactions from complementary connections must be equal to zero. This way it can be obtained the condition equations.*

Despite the active moments method, the basic system is similar to that in the displacements method, but the element in question are "active moments" of nodes $\boldsymbol{M_i}$ *and displacement (kinematic chains),* $\boldsymbol{M_A}$*.*

The active moment is defined as the moment which by its action on the base system, on the node **"i"** *or in the degree of freedom* **"a",** *creates the real displacement of the node* **"i"***, and the real displacement of the nodes which are part of the kinematic chain* **"a".**

The condition equations from the active moments method expresses the structure static equilibrium and has two types: nodes equilibrium equations and kinematic chains equilibrium equations. Must be mentioned the fact that none of the papers written by the authors didn't demonstrated the way in which the equilibrium equations have been obtained.

In the present paper have been obtained the equilibrium equations by active moments method by using the girder, node and nodes and beams chain equilibrium conditions with the help of the methodology "Gh. Em. Filipescu". This way has been obtained calculation relations for the extremity moments more general than the ones used in the displacements method, relations (4.6.) and (4.19.).

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Also, it has been distinguished an alternative of the active moments method, where the elements in question are node active moments and active displacement forces.

KEYWORDS: structure calculus, condition equations, active moments, active forces, displacements, rigidities.

1. INTRODUCTION

The engineering technique evolution, in different time periods, has been determined by the necessity of refuge and attendance of the human activities which imposed a diversified volume of constructions. Due to this fact, has increased the number of problems connected to the search of different constructive forms which could better undertake the actions with a minimal construction materials consume.

The resistance structures composition and calculation has been made, long ago, based on the experience and only slightly on the theoretical elements.

Latter, based on the phenomena observation have been established constructive rules and calculation models, repetitively verified on the constructions. Generalizing the experience and development of the experimental researches have been developed the theoretical calculation basis for the constructions.

The current stage of theory development and structure calculus is due to engineers and science people who have discovered the solutions for the problems of the resistance structures design and execution for different constructions by experimental methods but also by theoretical ways.

Considering the investigations, have resulted general calculation methods, main principles of construction organization and future horizons of the researches in the area of one of the most interesting branches of the Construction Mechanics and Construction Dynamics.

In the science people gallery, which have contributed to the development of Construction Mechanics can be mentioned J.C.Maxwell, O. Mohr, K. Culman, L. Cremona, pe S. Timoşenko, I.P. Prokofiev and others.

In Romania, great engineers and professors as Gh. Em. Filipescu, A. Beleş, M. Hanganu, C. Avram, Alex. Gheorghiu, A. Şesan, M. Ifrim, N. Orlovschi and others have obtained remarkable results in the area of Constructions Static and Dynamics.

2. STATEMENT OF EQUILIBRIUM CONDITION

In the paper "The frame calculation by circumvolution method" published by A. Şesan and N. Popescu, in 1954, in the Journal of University "Al. I. Cuza" and Polytechnic Institute in Iasi, are presented several definitions regarding the static structures calculus undetermined by displacements method.

It is mentioned that in that moment were known two methods of expressing the equilibrium equations. The first one had been published in Wien, in 1943, by R, Guldan. The equilibrium condition was expressed by two types of equations:

- Equilibrium equations on node ($\Sigma M_i = 0$):

$$d_i\varphi_i + \sum_k K_{ik}\varphi_i + K_{is}\Delta_{is} + K_{ij}\Delta_{ij} + S_i = 0 \qquad (2.1.)$$

- Equilibrium equations of floor ($\Sigma T_k=0$):

$$\sum_u \overline{K}\varphi_s + \sum_a \overline{K_j}\varphi_j + D_u + S_u = 0 \qquad (2.2.)$$

In these equilibrium equations have been made the notations:

- φ_i, φ_k, φ_s – nodes unknown angular displacements;
- Δ_k – unknown liniar displacements of the nodes;
- K_{ik} – rigidity at circumvolution of a bar extremity;
- $\overline{K}_s$ - rigiditatea la deplasare transversală a unei extremităţi de bară;
- S_i, D_u *şi* S_u – terms resulted from the actions effect.

Starting from the same base system with stucked displacements, I.P.Procofiev has published, in 1948, another form of static equilibrium equations:

- Node equilibrium equations:

$$\sum_{j=1}^{N} r_{ij}Z_j + \sum_{s=a}^{G} r_{is}Z_s + R_{ip} = 0 \quad i = 1,2,...,N \qquad (2.3.)$$

- Kinematic chain equilibrium equations:

$$\sum_{i=1}^{N} r_{si}Z_i + \sum_{e=a}^{G} r_{se}Z_e + R_{sp} = 0; \quad s = a, b, \ldots, G \qquad (2.4.)$$

Where: $Z_1, \ldots, Z_j$ are nodes angular unknown displacements ,
$Z_k, \ldots, Z_n$ – nodes linear displacements.

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Analyzing the two forms of the equilibrium equations statement, it can be observed that the first way in which it is used the forces equilibrium (T = 0), expressed by forces projections on perpendicular directions on beams, diminishes the application area only to frames with floors parallel beams. The second way, presented by Prokofiev, is based on effects overlapping (of the diagrams) and it is more general than the first one, because it increases the methods area of application also to the frames with inclined pillars.

It must be mentioned also the paper of P. Mazilu, in Bucharest, in 1946, entitled: “The Frames Calculation – Displacements and Virtual Mechanical Work in Cross Method”. The paper has been elaborated, as the author says, between 1942 –1943 and “introduces in Cross Method the principle of virtual displacements and virtual mechanical work”.

In the same direction is written also the paper “Another Form of the Condition Equations for the Deformation Method” with the authors: A. Şesan and N. Popescu. The authors demonstrate that “the use of mechanical work for the condition equations establishment – node and displacement – can be generalized for any frames type”.

The cited article and the paper named “Variants and Simplification of the Moments Distribution Method “, published by A. Sesan and N. Orlovschi, puts the basis of active moments method by introducing the notions of node and chain active moment.

In this paper, it is mentioned the use of “revolutions and displacements” as unknown elements of the condition equations which is a “disadvantage”, because it operates with “insignificant” values, “modified” by multipliers and because of that the authors propose to express the equilibrium condition by elements in question - “active moments”.

3. ABOUT THE “ACTIVE MOMENTS” METHOD

The “active moments” method was created in 1954 by professor A. Şesan and his collaborators: N. Orlovschi and N. Popescu. This is an alternative of displacements method which operates with unbalanced moments: “active moments“ of nodes, $\mathbf{M_i}$ and displacement (kinematic chains), $\mathbf{M_A}$.

The active moment is defined as the moment which by activating, on the base system, on **“i”** node or in the liberty degree **“a”,** determines the node **“i”** real displacement, also the real displacement of the node which are part from the kinematic chain **“a”.**

The condition equations in the active moments method express the structure of static equilibrium and are of two types:

Nodes equilibrium equations:

$$\sum_i M = 0; \qquad -M_i + \sum_{j=1}^{N} \mu_{ij} M_j + \sum_{a}^{G} \mu_{ia} M_a + \mathfrak{m} = 0; \quad (i = 1, 2, ..., N); \tag{3.1.}$$

- Kinematic chains equilibrium equations:

$$\sum_a M = 0; \qquad -M_a + \sum_{i=1}^{N} \mu_{ai} M_i + \sum_{b}^{G} \mu_{ab} M_b + \mathfrak{m}_a = 0; \quad (a = a,b,...,G); \tag{3.2.}$$

Where have been done the notations:

$$\mu_{ij} = -d_{ji} t_{ji}; \qquad \mu_{ji} = -d_{ij} t_{ij}; \qquad \mu_{ij} \neq \mu_{ji}; \tag{3.3.}$$

$$\mu_{ia} = \sum_i \pm \nu'_{ij,a}; \qquad \nu'_{ij,a} = \frac{\overline{K}_{ij} \beta_{ij,a}}{\sum_A (\overline{K}_{ij} + \overline{K}_{ji}) \beta_{ij,a}}; \tag{3.4.}$$

$$\mu_{ai} = \frac{1}{\beta_a} \sum_a d_{ij} (1 + t_{ij}) \beta_{ij,a}; \qquad \mu_{ia} \neq \mu_{ai}; \tag{3.5.}$$

$$\mu_{ab} = -\frac{1}{\beta_a} \sum_{a,b} (\nu'_{ij,a} + \nu'_{ji,b}) \beta_{ij,a}; \qquad \mu_{ab} \neq \mu_{ba}; \tag{3.6.}$$

$$\mathfrak{m}_i = \sum_i \mathfrak{m}_{ij}; \qquad \mathfrak{m}_a = \frac{1}{\beta_a} \left(\sum_a (\mathfrak{m}_{ij} + \mathfrak{m}_{ji}) \beta_{i,a} + \sum_k P_k \delta_{k,a} \right); \tag{3.7.}$$

For these coefficients calculation are used the rigidities for the rotation of the bars extremities and for the transversal displacement of the bars extremities, $\mathbf{K_{ij}}$ şi $\mathbf{K_{ij}^{\Psi}}$, the distribution and transmission: $\mathbf{d_{ij}}$, and $\mathbf{t_{ij}}$ and two proportionality coefficients: $\boldsymbol{\beta_{ij}}$ and $\boldsymbol{\beta_A}$.

The proportionality coefficient of the bar rotation **ij** noted $\boldsymbol{\beta_{ij}}$ is calculated by considering a bar rotation which is equal to the unit, by which can be determined the other bars rotations.

If we consider that on a fictive bar acts an active moment, $\mathbf{M_A}$, than the proportionality coefficient of this bar is $\boldsymbol{\beta_A}$. The calculus is done with the relation:

$$\beta_a = \frac{\sum_a (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}})\beta_{ij,a}^2}{\sum_a (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}})\beta_{ij,a}} \quad (3.8.)$$

4. DIRECT DEVICE FOR CONDITION EQUATION DEDUCTION IN THE METHOD OF "ACTIVE MOMENTS"

The deformed position of a certain frame can be defined by the mean of nodes displacements: rotations and translations.

The nodes angular displacements (**z_i, i = 1, 2, 3, ..., N**) are independent variables, while the linear displacements (**z_s, s = a, b, c, ..., G**) are interdependent by the mean of a number of parameters equal to the number of liberty degrees of the kinematic system, obtained by introducing articulations in nodes and in bearing constraint. The result is that the number of independent parameters, which geometrically define the deformed position of a structure, equals the nodes number to which is added the number of degrees of the frame elastic liberty. The nodes translations are part of a kinematic chain which relies on a single parameter, **z_s.**

For a bar **ij,** the rotation noted ψ_{ij} is expressed considering the parameter z_s with the relation:

$$\psi_{ij,s} = \beta_{ij,s} z_s \quad (4.1.)$$

Where $\beta_{ij,s}$ is the bar rotation ij when the parameter $z_s = 1$.

4.1. The girder equilibrium conditions

We extract a bar **ij** from a base system, of a certain structure, corresponding to the displacements method, activated by burdens and displacements (nodes rotation and bar rotation), fig.4.1. The general expressions of the end moments can be expressed as:

$$\begin{aligned} M_{ij} &= M_{ij}(z_i) + M_{ij}(z_j) + M_{ij}(\psi_{ij}) + M_{ij}(p) \\ M_{ji} &= M_{ji}(z_i) + M_{ji}(z_{jj}) + M_{ji}(\psi_{ij}) + M_{ji}(p) \end{aligned} \quad (4.2.)$$

Or as notations in fig.4.1:

$$\begin{aligned} M_{ij} &= K_{ij} z_i + t_{ji} K_{ji} z_j - \overline{K_{ij}^{\psi}} \psi_{ij} - \mathfrak{m}_{ij} \\ M_{ji} &= t_{ij} K_{ij} z_i + K_{ji} z_j - \overline{K_{ji}^{\psi}} \psi_{ij} + \mathfrak{m}_{ji} \end{aligned} \quad (4.3.)$$

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In the case in which the bar **ij,** considered in the research, is component of different chains and nodes corresponding to the liberty degrees **a, b, c, ..., G,** than the expression of bar rotation can be written as a sum of rotations corresponding to each liberty degree:

$$\psi_{ij} = \psi_{ij,a} + \psi_{ij,b} + \psi_{ij,c} + ... = \sum_{s=a}^{G} \psi_{ij,s} \tag{4.4.}$$

or taking into consideration the relation (4.1):

$$\psi_{ij} = \sum_{s=a}^{G} \beta_{ij,s} z_s \tag{4.5.}$$

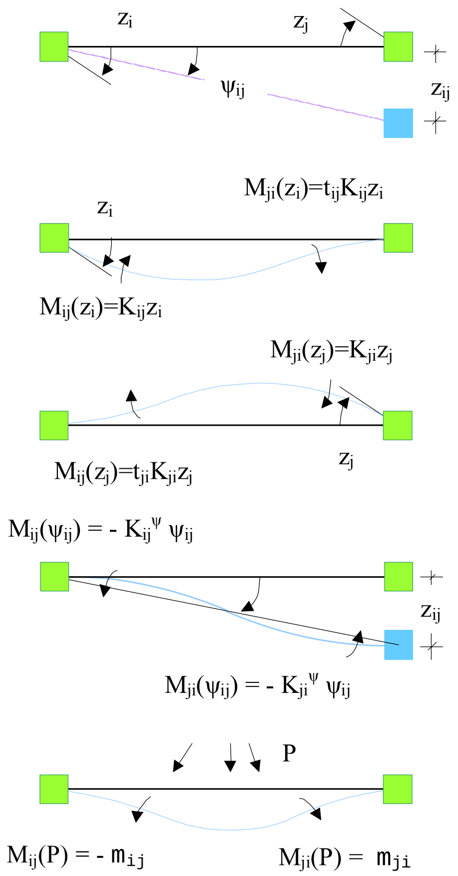

Fig. 4.1 Situations of girder and node loads considering the statement of equilibrium conditions – bar end moments *ij* for different load situations

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The end moments, the relations (4.2.), as per relations (4.4.) and (4.5.) and notations from fig.4.1, become:

$$M_{ij} = K_{ij} z_i + t_{ji} K_{ji} z_j - \sum_{s=a}^{G} \overline{K_{ij}^{\psi}} \beta_{ij,s} z_s - \mathrm{m}_{ij}$$
$$M_{ji} = t_{ij} K_{ij} z_i + K_{ji} z_j - \sum_{s=a}^{G} \overline{K_{ji}^{\psi}} \beta_{ij,s} z_s + \mathrm{m}_{ji} \qquad (4.6.)$$

4.2. Node equilibrium conditions

Considering a node **i** taken from a certain structure, fig, 4.2., in which compete several bars, activated by a couple $\mathbf{M_i}$. The node will rotate. The node rotation measurement, $\mathbf{z_i}$, will equal each bar extremity rotation measure which competes in the node. In each bar extremity the end moments, which appear are proportional to the rotation measurement $\mathbf{z_i}$ ($M_{ij} = K_{ij}z_i$).

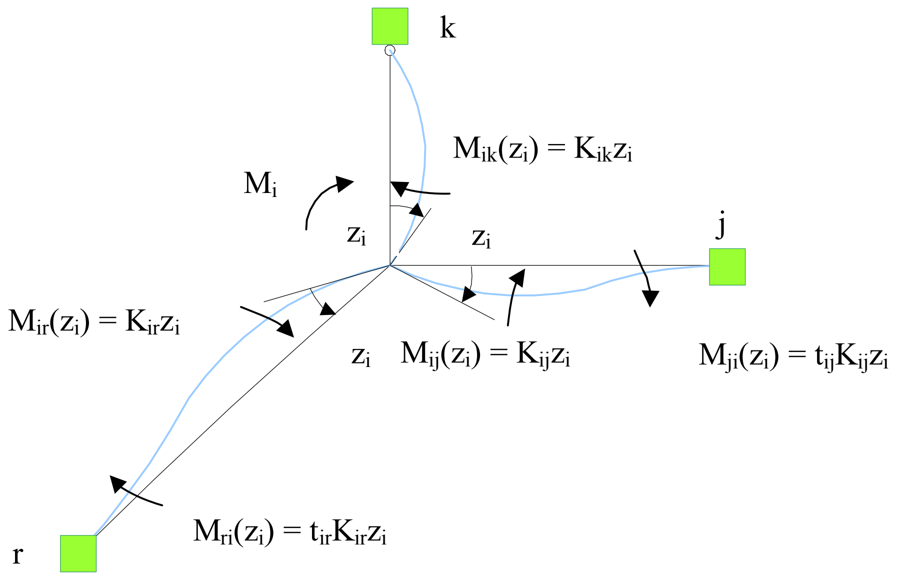

Fig. 4.2 The end moments of the bars which compete in node i when the node is loaded with a rotation of node, z_i, or with an active moment of node, M_i

Expressing the node **i** equilibrium, by a moment's equation:

$$\sum_i M_i = 0; \qquad \Rightarrow M_i = \sum_{j=1}^{N_i} K_{ij} z_i \qquad (4.7.)$$

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Where:

$$z_i = \frac{1}{\sum_{j=1}^{N_i} K_{ij}} M_i \tag{4.8.}$$

Where - $\mathbf{N_i}$ represents the number of nodes connected by bars to node **i**.

4.3. Chain equilibrium condition

It is detached, from a certain frame, fig. 4.3.a., a chain of bars and nodes corresponding to a liberty degree, depending on the parameter $\mathbf{z_s}$. We act on this chain with an active moment, $\mathbf{M_s}$, which determines the real displacement, $\mathbf{z_s}$, or, the rotations, $\boldsymbol{\psi_{ij,s}}$, without the node to have the possibility to rotate.

We express the equilibrium of these bars and nodes chain by a moment's equation:

$$\sum_s M = 0 \qquad \Rightarrow \mathrm{M_s} = \sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \psi_{ij,s} \tag{4.9.}$$

And considering the relation (4.1.) it is obtained:

$$M_s = z_s \sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s} \tag{4.10.}$$

Where:

$$z_s = \frac{1}{\sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s}} M_s \tag{4.11.}$$

Where - $\mathbf{B_s}$ number of bars for the chain s.

In order to determine the reaction from the connection of liberty degree it turns to the faulty in fig. 4.3.c. and the displaced in fig.4.3.d. The articulated scheme of the nodes and bars chain loaded with the moments on the bars and the reaction $\mathbf{R_s}$ is in equilibrium. The equilibrium is expressed by an equation of virtual mechanical work by producing a kinematic displacement compatible with the connections. It results:

$$R_s \Delta^{ar} - \sum_{ij}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \psi_{ij,s} \psi_{ij,s}^{ar} = 0 \tag{4.12.}$$

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$$R_s \Delta^{ar} - \sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s} z_s \beta_{ij,s} \Delta^{ar} = 0 \qquad (4.13.)$$

Where

$$R_s = \sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s}^2 z_s \qquad (4.14.)$$

And

$$z_s = \frac{1}{\sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta^2_{ij,s}} R_s \qquad (4.15.)$$

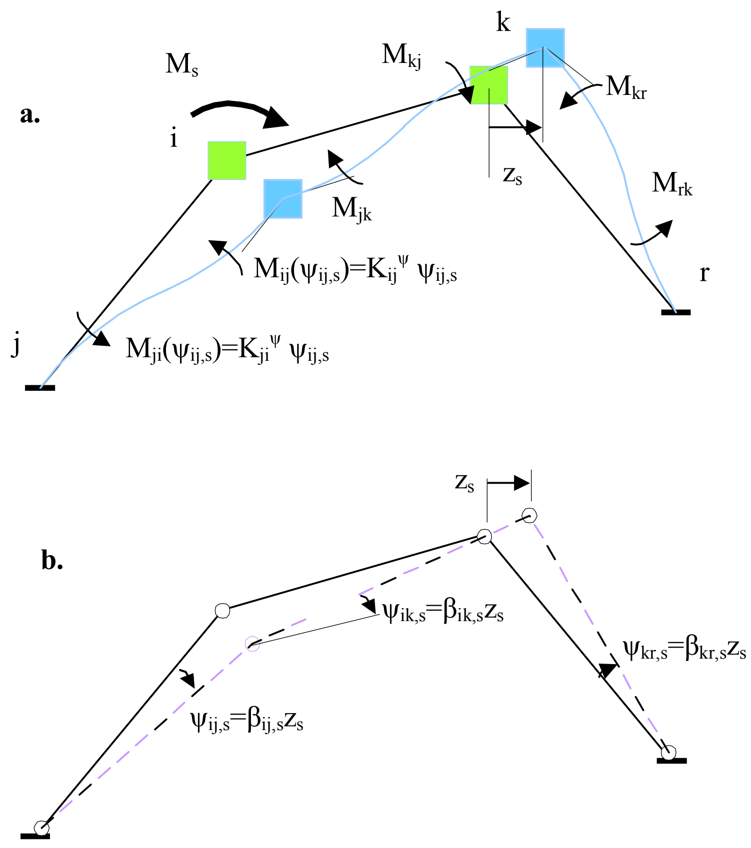

Fig.4.3. Loading situations for a nodes and bars chain in order to express the equilibrium condition: a. Bars and nodes chain loaded with the active moment of displacement, M_a; b. the connection between the bar rotation, $\psi_{ij,s}$ and displacement z_s;

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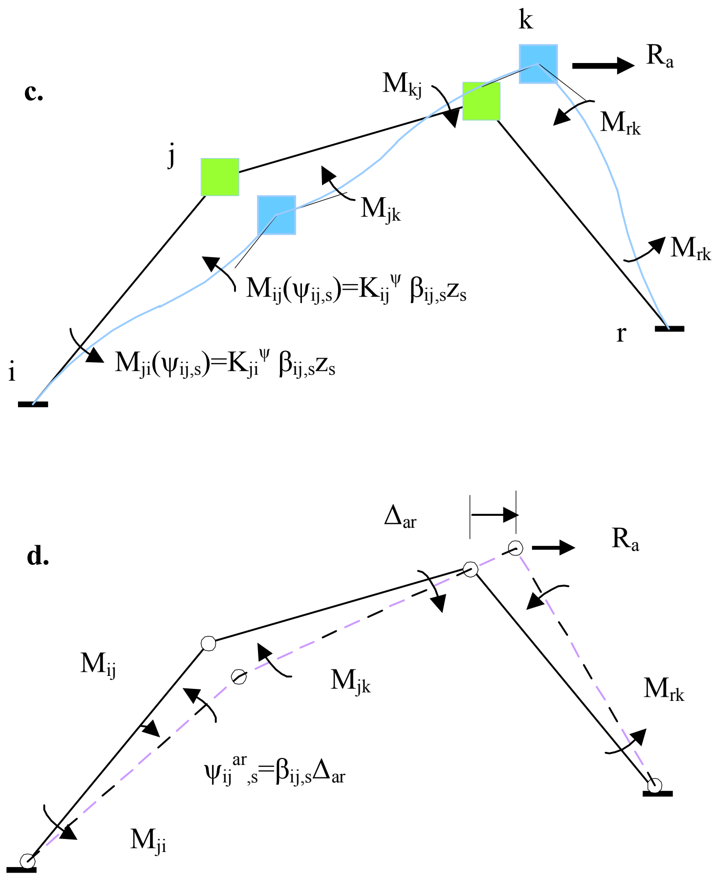

Fig .4.3 (continuation) Loading situations for a nodes and bars chain in order to express the equilibrium condition: d. The articulated scheme for a bars chain having in the bars extremities applied end moments, M_{ij} and the reaction from the liberty degree hardcore; c. Nodes and bars chain loaded with the real displacement z_s, on the liberty degree direction has been introduced the corresponding reaction R_a

Equalizing the relations (4.11.) and (4.15) it results:

$$\beta_s = \frac{R_s}{M_s} = \frac{\sum_{ij=1}^{B_s}(\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}})\beta_{ij,s}^2}{\sum_{ij=1}^{B_s}(\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}})\beta_{ij,s}} \tag{4.16.}$$

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$$R_s = \beta_s M_s = \frac{\sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s}^2}{\sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s}} M_s \tag{4.17.}$$

β_s is, from the active moments method point of view, the rotation of a fictive bar on which would act the active displacement moment.

Based on the relations, (4.8.), (4.11.) and (4.16.) can be written the general expressions of bar moments loaded with burdens, with node active moments, **M_i,** and active displacement moments, **M_s,** or with node active moments and active displacement force, **R_s**:

$$M_{ij} = \frac{K_{ij}}{\sum_{j=1}^{N_i} K_{ij}} M_i + t_{ji} \frac{K_{ji}}{\sum_{i=1}^{N_j} K_{ji}} M_j - \sum_{s=a}^{G} \frac{\overline{K_{ij}^{\psi}} \beta_{ij,s}}{\sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s}} M_s - m_{ij} \tag{4.18.}$$

$$M_{ji} = t_{ij} \frac{K_{ij}}{\sum_{j=1}^{N_i} K_{ij}} M_i + \frac{K_{ji}}{\sum_{i=1}^{N_j} K_{ji}} - \sum_{s=a}^{G} \frac{\overline{K_{ji}^{\psi}} \beta_{ij,s}}{\sum_{ij=1}^{B_s} (\overline{K_{ij}^{\psi}} + \overline{K_{ji}^{\psi}}) \beta_{ij,s}} M_s - m_{ji} \tag{4.19.}$$

Also,

$$M_{ij} = d_{ij} M_i + t_{ji} d_{ji} M_j - \sum_{s=a}^{G} v_{ij,s} M_s - m_{ij}$$

$$M_{ji} = t_{ij} d_{ij} M_i + d_{ji} M_j - \sum_{s=a}^{G} v_{ji,s} M_s + m_{ji} \tag{4.20.}$$

And

$$M_{ij} = d_{ij} M_i + t_{ji} d_{ji} M_j - \sum_{s=a}^{G} v'_{ij,s} R_s - m_{ij}$$

$$M_{ji} = t_{ij} d_{ij} M_i + d_{ji} M_j - \sum_{s=a}^{G} v'_{ij,s} R_s + m_{ji} \tag{4.21.}$$

Where:

$$d_{ij}=\frac{K_{ij}}{\sum_{j=1}^{N_i}K_{ij}};\qquad \nu_{ij,s}=\frac{\overline{K_{ij}^{\psi}}\beta_{ij,s}}{\sum_{ij=1}^{B_s}(\overline{K_{ij}^{\psi}}+\overline{K_{ji}^{\psi}})\beta_{ij,s}};\qquad \nu_{ij}^{'}=\frac{\overline{K_{ij}^{\psi}}\beta_{ij,s}}{\sum_{ij=1}^{B_s}(\overline{K_{ij}^{\psi}}+\overline{K_{ji}^{\psi}})\beta_{ij,s}^{2}};\qquad (4.22.)$$

Observation:

1. The expressions 4.8 and 4.11 indicate the fact that the real displacements of the nodes can be determined based on the node and displacement active moments.

2. Analyzing the relations (4.3.), (4.6.) and (4.20.) we can say that the last two relations are more general, because the end moments are applicable to any straight bar from a static undetermined structure, with rigid nodes, loaded with burdens, node rotations and parameters of liberty degrees, node active moments and active displacement moments.

It is considered, a certain structure driven by a force system. The structure will deform and in the bars extremities appear bending moments, determined by the relations (4.5), (4.18) and (4.19). In order to determine the elements in question: nodes different displacements (**z_i** şi **z_s**) or, node and displacement active moments (**M_i** şi **M_s**) we will use the methodology of Gh. Em. Filipescu. In this method are being used the labile base systems activated by the given forces and the end moments (**M_{ij}**, **M_{ji}**). Are being used two types of equations:

- Continuity equations. In the case studied, here, in the paper, these equations reduced to the level of each bar, brought us to the relations (4.5), (4.18) and (4.19);
- Static equilibrium equations:

$$\sum_{j=1}^{N_i}M_{ij}=0;\qquad \mathrm{i}=1,2,3,..\,.,\mathrm{N}\qquad (4.23.)$$

$$\mathrm{LMV}_{(s)}=0;\qquad \mathrm{s}=\mathrm{a,b,c,...,G}\qquad (4.24.)$$

Explaining the equations (4.23) and (4.24) by end moments, the relations (4.5), (4.19), and (4.21), can be obtained these equation systems:

$$\sum_{j=1}^{N_i}K_{ij}z_i+\sum_{j=1}^{N_i}t_{ij}K_{ij}z_j-\sum_{s=a}^{G}\overline{K_{ij}^{\psi}}\beta_{ij,s}z_s-\sum_{j=1}^{N_i}\mathrm{m}_{\mathrm{j}}=0\qquad (4.25.)$$

$$\sum_{ij=1}^{B_s} K_{ij}\left(1+t_{ij}\right)\beta_{ij,e}z_i + \sum_{ij=1}^{B_s} K_{ij}\left(1+t_{ij}\right)\beta_{ij,e}z_j - \sum_{s=a}^{G}\sum_{ij=1}^{B_s}\left(\overline{K_{ij}^{\psi}}+\overline{K_{ji}^{\psi}}\right)\beta_{ij,s}\beta_{ij,e}z_s +$$

$$+\sum_{ij}^{B_s}\left(-\mathrm{m}_{ij}+\mathrm{m}_{ji}\right)\beta_{ij,e} + \quad + \sum_{k=1}^{N_p} P_k\delta_{k,e} = 0; \quad i=1,2,...,N: \quad i\neq j; \quad e=a,b,...,G; \tag{4.26.}$$

Or

$$r_{ii}z_i + \sum_j r_{ij}z_{jj} + \sum_s r_{is}z_s + R_{ip} = 0; \qquad \mathrm{i}\neq \mathrm{j}; \qquad \mathrm{i}=1,2,..., \mathrm{N}; \tag{4.27.}$$

$$\sum_i r_{si}z_i + \sum_j r_{ij}z_j + \sum_s r_{se}z_s + R_{sp} = 0; \qquad \mathrm{i}\neq \mathrm{j}; \qquad \mathrm{e}=1,2,...,\mathrm{G}; \tag{4.28.}$$

And

$$\sum_{j=1}^{N_i} d_{ij}M_i + \sum_{j=1}^{N_i} t_{ji}d_{ji}M_j - \sum_{s=a}^{G}\sum_{j=1}^{N_i} \nu_{ij,s}M_s - \sum_{j=1}^{N_i}\mathrm{m}_{ij} = 0; \quad \mathrm{i}\neq \mathrm{j}; \quad \mathrm{i}=1,2,...,\mathrm{N}; \tag{4.29.}$$

$$\sum_{ij=1}^{B_s} d_{ij}\left(1+t_{ij}\right)\beta_{ij,e}M_i + \sum_{ij=1}^{B_s} d_{ji}\left(1+t_{ij}\right)\beta_{ij,e}M_j - \sum_{s=a}^{G}\sum_{ij=1}^{B_s}(\nu_{ij,s}+\nu_{ji,s})\beta_{ij,s}\beta_{ij,e}M_s +$$

$$+\sum_{ij}^{B_s}\left(-\mathrm{m}_{ij}+\mathrm{m}_{ji}\right)\beta_{ij,e} + \quad + \sum_{k=1}^{N_p} P_k\delta_{k,e} = 0; \quad i=1,2,...,N; \quad i\neq j; \quad e=a,b,...,G; \tag{4.30.}$$

Or

$$M_i + \sum_j \mu_{ij}z_j + \sum_s \mu_{is}M_s + \mathrm{m}_{ip} = 0; \tag{4.31.}$$

$$\sum_i \mu_{si}M_i + \sum_j \mu_{sj}M_j + \sum_e \mu_{es}M_e + \mathrm{m}_{sp} = 0; \tag{4.32.}$$

Also

$$\sum_{j=1}^{N_i} d_{ij}M_i + \sum_{j=1}^{N_i} t_{ji}d_{ji}M_j - \sum_{s=a}^{G}\sum_{j=1}^{N_i} \nu'_{ij,s}R_s - \sum_{j=1}^{N_i}\mathrm{m}_{ij} = 0; \quad \mathrm{i}\neq \mathrm{j}; \quad \mathrm{i}=1,2,...,\mathrm{N}; \tag{4.33.}$$

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$$\sum_{ij=1}^{B_s} d_{ij}\left(1+t_{ij}\right)\beta_{ij,e}M_i+\sum_{ij=1}^{B_s} d_{ji}\left(1+t_{ij}\right)\beta_{ij,e}M_j-\sum_{s=a}^{G}\sum_{ij=1}^{B_s}(\nu'_{ij,s}+\nu'_{ji,s})\beta_{ij,s}\beta_{ij,e}R_s+$$

$$+\sum_{ij}^{B_s}\left(-m_j+m_i\right)\beta_{ij,e}+\sum_{k=1}^{N_p}P_k\delta_{k,e}=0;\quad i=1,2,...,N;\quad i\neq j;\quad e=a,b,...,G;$$

(4.34.)

Regarding the previous relations certain comments must be done, such as:

- The relations (4.27) and (4.28) represent the condition equations system for a structure static undetermined, in the displacement method. The elements in question are the displacements. The free terms are reactions, and the coefficients are unitary reactions and are determined with the above relations:

$$r_{ii}=\sum_{j=1}^{N_i}K_{ij};\qquad r_{ij}=t_{ij}K_{ij};$$ (4.35.)

$$r_{is}=-\sum_{j=1}^{N_i}\overline{K_{ij}^{\psi}}\beta_{ij,s};\qquad r_{si}=\sum_{ij=1}^{B_s}K_{ij}(1+t_{ji})\beta_{ij,s};$$ (4.36.)

$$r_{es}=\sum_{ij=1}^{B_s}(\overline{K_{ij}^{\psi}}+\overline{K_{ji}^{\psi}})\beta_{ij,s}\beta_{ij,e};\qquad R_{ip}=-\sum_{k=1}^{N_i}m_j;$$ (4.37.)

$$R_{ep}=\sum_{ij=1}^{B_e}(-m_j+m_i)\beta_{ij,e}+\sum_{k=1}^{N_p}P_k\delta_{k,p};$$ (4.38.)

- The relations (4.21) and (4.32) represent the condition equation system for a structure static undetermined, in the method of active moments. The elements in question are node and bars active moments. The free terms and coefficients are determined with the relations:

$$\mu_{ii}=\sum_{j=1}^{N_i}d_{ij};\qquad \mu_{ij}=t_{ji}d_{ji};\qquad i\neq j;$$ (4.39.)

$$\mu_{is}=-\sum_{j=1}^{N_i}\nu_{ij,s};\qquad \mu_{si}=\sum_{ij=1}^{B_s}d_{ij}(1+t_{ij})\beta_{ij,s};\qquad i\neq j$$ (4.40.)

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$$\mu_{ej} = \sum_{ij=1}^{B_e} d_{ji}(1+t_{ji})\beta_{ij,e}; \qquad \mu_{es} = -\sum_{ij=1}^{B_s}(\nu_{ij,s}+\nu_{ji,s})\beta_{ij,e}; \tag{4.41.}$$

$$\mathrm{m}_{ip} = -\sum_{j=1}^{N_i}\mathrm{m}_{ij}; \qquad \mathrm{m}_{ep} = \sum_{ij=1}^{B_e}(-\mathrm{m}_{ij}+\mathrm{m}_{ji})\beta_{ij,e} + \sum_{k=1}^{N_p} P_k\delta_{k,e} \tag{4.42.}$$

- The relations (4.33) and (4.34) represent the condition equations system for a structure static undetermined, in an alternative of active moments method. The elements in question are node active moments and active displacement forces.

4.5. The indirect device for condition equation deduction from active moment methods starting from the equation system of the displacement method.

Corroborating the relations (4.8) with (4.35) and (4.11) with (4.16) and (4.37) for e = **s**, can be obtained the expressions:

$$z_i = \frac{1}{r_{ii}} M_i; \tag{4.43.}$$

$$z_s = \frac{\beta_s}{r_{ss}} M_s; \tag{4.44.}$$

Are introduced the relations (4.43) and (4.44) in the condition equation system of the displacement method (2.3) and (2.4), results:

$$\frac{r_{ii}}{r_{ii}} M_i + \sum_j \frac{r_{ij}}{r_{jj}} M_j + \sum_s \frac{r_{is}}{r_{ss}} M_s + R_{ip} = 0; \tag{4.45.}$$

$$\sum_i \frac{r_{si}}{r_{ii}} M_i + \sum_j \frac{r_{sj}}{r_{jj}} M_j + \sum_s \frac{r_{se}}{r_{ss}} \beta_s M_s + R_{sp} = 0; \tag{4.46.}$$

After dividing the equation (4.46.) by β_s it is obtained:

$$\sum_i \mu_{si} M_i + \sum_j \mu_{sj} M_j + \sum_e \mu_{es} M_e + \mathrm{m}_{sp} = 0; \tag{4.47.}$$

$$M_i + \sum_j \mu_{ij} z_j + \sum_s \mu_{is} M_s + \mathrm{m}_{ip} = 0; \tag{4.48.}$$

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The equation system (4.45) and (4.46) is identical with (4.31) and (4.32), and the system (4.47) and (4.48) is identical with the equation system formed from (2.3) and (2.4). Consequently, between the coefficients and free terms of displacement method and active moment method are the relations:

$$\mu_{ii} = \frac{r_{ii}}{r_{ii}} = 1; \qquad \mu_{ij} = \frac{r_{ij}}{r_{jj}}; \qquad \mu_{is} = \frac{r_{is}}{r_{ss}}; \tag{4.49.}$$

$$\mu_{si} = \frac{r_{si}}{r_{ii}} \frac{1}{\beta_s}; \qquad \mu_{sj} = \frac{r_{sj}}{r_{jj}} \frac{1}{\beta_s}; \qquad \mu_{se} = \frac{r_{se}}{r_{ss}}; \tag{4.50.}$$

$$m_{ip} = R_{ip}; \qquad m_{sp} = \frac{R_{sp}}{\beta_s}; \tag{4.51.}$$

3. CONCLUSIONS

The equilibrium equations from active moment method by using the equilibrium conditions for girder, node and nodes and bars chain, by the mean of "Gh. Em. Filipescu" methodology, have been obtained. The equations system obtained in subchapter 4.2., and the relation (4.27.), has interchangeable lateral coefficients.

Has been identified a variant of active moments method, where the elements in question are node active moments and displacement active forces.

Have been obtained calculus relations for end moments more general than the ones used in displacement method, the relations (4.6) and (4.19).

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