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The use of the bispectrum for analysis dynamic parameters of rail fastening

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Summary

"Higher Order Statistics" (HOS) is extension of second-order characteristics such as the auto-correlation function and power spectrum. HOS analysis is emerging as a new powerful technique in signal analysis, offering insight into non-linear coupling between frequencies and potential applications in many areas where traditional linear analysis provides insufficient information. This contribution describes the HOS theory and possibilities application to experimental data acquisition from measurements of rail fastening parameters.

KEYWORDS: Higher Order Statistics, Cumulants, Bispectrum, Railway Superstructure

1. INTRODUCTION

The basic claim put in particular components of the railway track is their functional reliability and the interrelated minimum costs of their manufacturing, repair and maintenance. Each part of the rail structure is exposed to an extensive static and dynamic stresses. This fact is of great importance especially in the design and construction of high-speed railway lines.

Different methods and various criteria have been applied to test the railway superstructure, and different mathematical methods are used to evaluate the signals measured. The method exciting the structure by a mechanical impact is often used in dynamic tests. The excitation by impact is advantageous for the determination of proper frequencies of a given system. Another possibility is the loading of the structures specimens by a continuous excitation by means of vibrators. This makes it possible to focus attention on certain frequencies or to the interval of the frequencies. For the evaluation, frequency spectra transfer functions, frequency response functions, etc. have been composed. From the viewpoint of the railway operation security, it is expedient to focus our attention also on new methods that will offer the information about the quality of particular components (or of the

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whole structure) both from the viewpoint of different failures and from the assembly viewpoint.

It should be said that until the present time practical applications of the evaluation of the results of measuring by "elementary" methods such as typical statistic and frequency analyses have prevailed. At the present time, there exist a number of mathematical methods through which discrete data may be processed and which are especially suitable for processing the signal measured. However, in the technological practice these methods appear only rarely although these may offer more information about the signals measured and also about the structure tested. These methods could not be used because of low efficiency of the hardware and the technology of measuring. At the present time, these methods are not fully utilized owing to the insufficient awareness of technologists.

2. THEORETICAL BACKGROUND

"Higher Order Statistics" (HOS) is extension [1, 2] of second-order characteristics such as the auto-correlation function and power spectrum to higher orders. HOS analysis is emerging as a new powerful technique in signal analysis, offering insight into non-linear coupling between frequencies and potential applications in many areas where traditional linear analysis provides insufficient information.

The second-order analyses work fine if the signal has a Gaussian (Normal) probability density function, but many real-life signals are non-Gaussian. The easiest way to introduce the HOS measures is just to show some definitions so that the reader can see how they are related to the familiar second-order measures. In the text to follow, are definitions for the time-domain and frequency-domain third-order HOS measures, moments, cumulants, high-order spectra or polyspectra, etc. [3, 4].

2.1 Moments

The time process $x(n)$ can be characterized in many ways, for example by its amplitude, its energy or its waveform. The probability density function (pdf) of the process provides detailed information about the distribution of the amplitudes of the process which can be used to characterize the process. A set of quantities which describes the shape of this "pdf" are the moments. The first-order moment m_1 of process $x(n)$ is just its mean, and it provides a measure of location of the „pdf". The second-order moment is the variance, a measure of the spread of the "pdf". Higher-order moments exist too, such as the skewness and kurtosis. Now, moments are statistical measures which characterize signal properties. The first four moments of the process are defined by the following equations:

$$m_1 = E\{x_n\} \tag{1}$$

$$m_2(\tau) = E\{x(n)\cdot x(n+\tau)\} \tag{2}$$

$$m_3(\tau_1,\tau_2) = E\{x(n)\cdot x(n+\tau_1)\cdot x(n+\tau_2)\}. \tag{3}$$

$$m_4(\tau_1,\tau_2,\tau_3) = E\{x(n)\cdot x(n+\tau_1)\cdot x(n+\tau_2)\cdot x(n+\tau_3)\} \tag{4}$$

2.2 Cumulants

Cumulants are specific non-linear combinations of these moments. The first-order cumulants of process is the mean. The second, third and fourth-order cumulants of process are defined by the following equations [5]:

$$c_2 = m_2 - m_1^2 \tag{5}$$

$$c_3 = m_3 - 3\cdot m_2\cdot m_1 + 2\cdot m_1^3. \tag{6}$$

$$c_4 = m_4 - 4\cdot m_3\cdot m_1 + 3\cdot m_2^2 + 12\cdot m_2\cdot m_1^2 - 6\cdot m_1^4 \tag{7}$$

2.3 Polyspectra

This term is used to describe the family of all frequency-domain spectra, including the those of the 2nd order. Most HOS work based on polyspectra and focus their attention on the bispectrum (third-order polyspectrum) and the trispectrum (fourth-order polyspectrum). Polyspectra consist of higher order moment spectra and cumulant spectra and can be defined for both the deterministic signal and random processes. Moment spectra can be very useful in the analysis of deterministic signals (transient and periodic), whereas cumulant spectra can play a very important role in the analysis of stochastic signals. The kth-order polyspectrum [3, 5] is defined as the Fourier transform of the corresponding cumulant sequence:

$$C_2(f) = \sum_{k=-\infty}^{\infty} c_2(k)\cdot e^{-i2\pi fk} \tag{8}$$

$$C_3(f_1, f_2) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} c_3(k,l)\cdot e^{-i2\pi f_1 k}\cdot e^{-i2\pi f_2 l}. \tag{9}$$

$$C_4(f_1, f_2, f_3) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_4(k,l,m)\cdot e^{-i2\pi(f_1 k + f_2 l + f_3 m)} \tag{10}$$

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which are the power spectrum, respectively the bispectrum and trispectrum. These can be estimated in a way similar to the power spectrum, but more data is usually needed to get reliable estimates. Note that the bispectrum is a function of two frequencies, whereas the trispectrum is a function of tree frequencies. In contrast to the power spectrum which is real-valued and non-negative, bispectrum and trispectrum are complex valued and contain a phase information.

Bispectrum a technique that is often used, can be estimated in a way similar to Welch periodogram method for the power spectrum estimation but the lengths of data required to obtain consistent estimates are longer than those required for the power spectrum estimation.

The variance of the estimate can still be reduced by averaging over multiple segments of data. To extract an information from the non-normalized bispectrum is often difficult because the variance of the bispectrum estimate at a particular frequency pair *(k,l)* depends not only on the data length but also on the power of the signal at frequencies *k, l* and *k+l.* The bicoherence is a useful normalized bispectrum. The bicoherence at any frequency pair *k, l* can be interpreted as a fraction of power at frequency *k+l* which is a phase coupled to the component at *k+l*. Bicoherence is estimated as [4]

$$b(f_1,f_2) == \frac{\left|C_3(f_1,f_2)\right|^2}{P(f_1)\cdot P(f_2)\cdot P(f_1+f_2)} \tag{11}$$

where $C_3(f_1, f_2)$ is the estimate of the bispectrum, and $P(f)$ is the estimate of the power spectrum.

3. EXPERIMENTAL RESULTS

The text to follow presents an example of the laboratory measurements and the analyses of the dynamic parameters of the rail fastening specimen. The measurements were carried out mainly with respect to the transfer properties of this fastening of rails under different assembly and operational states (the effects of tightening and releasing).

The test specimen was assembled of a part of the concrete sleeper B 91, on which a rail of structural shape UIC 60 using the elastic fastening of VOSSLOH SKL14 type was fastened.

To test the specimen of the rail grid, the method of measuring the response to the random excitation was used. The excitation was implemented in the horizontal direction towards the rail head by means of an efficient vibration exciter B&K

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4818. The frequency range of the observed area was determined ranging from 5 Hz to 1500 Hz.

The measuring system PULSE 3560C by the firm Brüel& Kjaer was used for the experiment control. The acceleration responses were measured by an accelerometric sensing unit fixed to the sleeper 10 cm from the fastening. Let us remark that the acceleration sensing unit was fixed to the structure measured by means of the bee-wax.

The measurement resulted in the scanned and digitally recorded electrical signal proportional to the instantaneous value of the acceleration in place of the sensing unit fixation. Frequency spectra and bicoherence were developed from the time responses.

The graphs in Figures 1 and 3 represent a dynamic response of the rail fastening with the standard tightened bolts.

Figure 1 shows a graph of the amplitude spectrum calculated by means of a direct application of Fourier´s transformation to the signal measured. Significant components on frequencies 15 Hz, 20 Hz, 75 Hz, 95 Hz, 150 Hz, 220 Hz and 550 Hz may be read from this graph. It is evident from this graph that particular frequency peaks are relatively uniformly distributed in the frequency region. Figure 3 shows the graph of the bicoherence. There are no distinctive maximum values noticeable here.

The graphs in Figures 2 and 4 represent dynamic responses of the rail fastening with loosened bolts. Figure 2 represents a frequency spectrum, and Figure 4 the bicoherence graph. From graph on Figure 2, significant components in frequencies 15 Hz, 75 Hz, 100 Hz, 160 Hz, 210 Hz and 550 Hz may be read. So, if the graphs of frequency spectra (Figures 1 and 2) are compared, it may be stated that the spectra are almost identical and the differences in significant components are not excessively important.

On the contrary, great differences are evident if the graphs of bicoherence in Figures 3 and 4 are compared. In the graph in Figure 4, significant components in frequencies and frequency pairs of 15 Hz, 75 Hz, 95 Hz, 120 Hz, 150 Hz, and 190 Hz are evident. Compared with the graph in Figure 2, there are significant maximum values in the graph in Figure 4.

These results from the fact that the nonlinearity of the transmission of vibration waves from the rail to the sleeper increases in the specimen with the rail improperly fixed. Thus, this phenomenon is very well detected by the calculated bicoherence.

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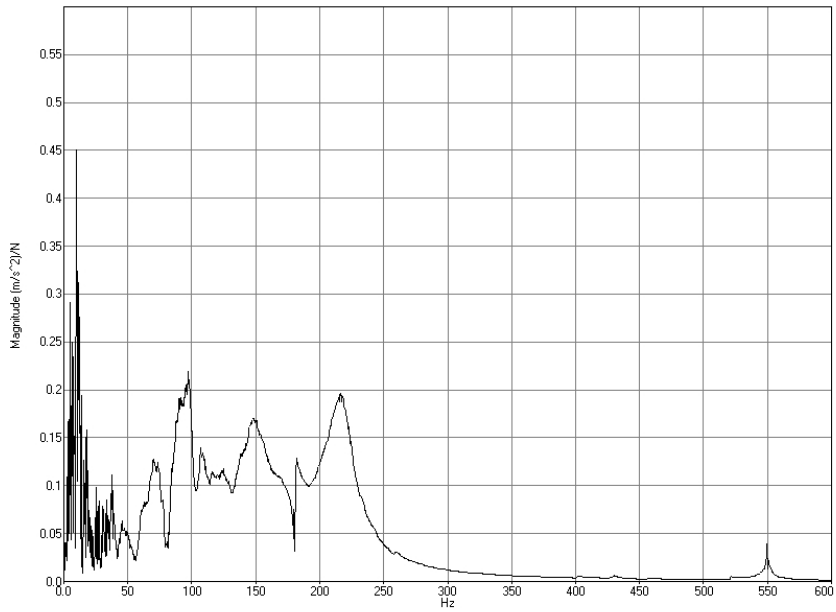

Figure 1 Frequency response function, rail fastening with the standard tightened bolts

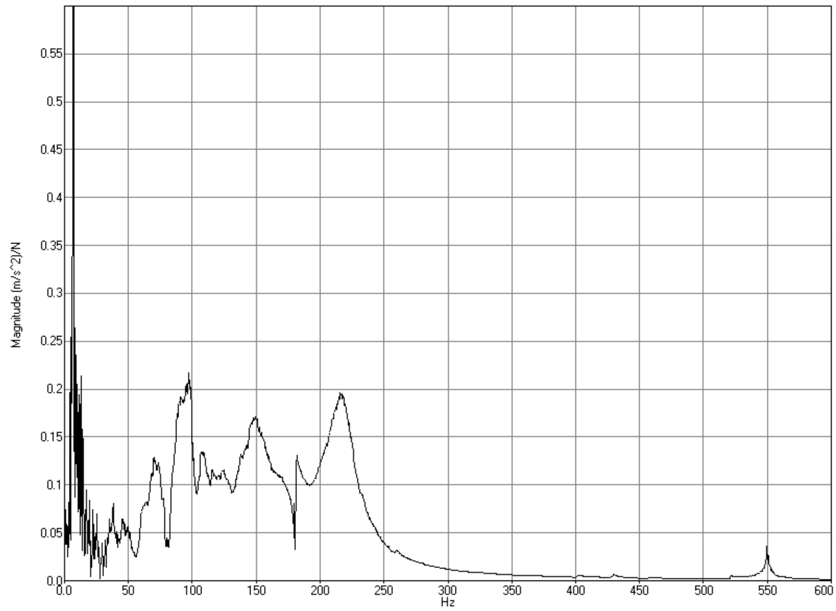

Figure 2 Frequency response function, rail fastening with loosened bolts

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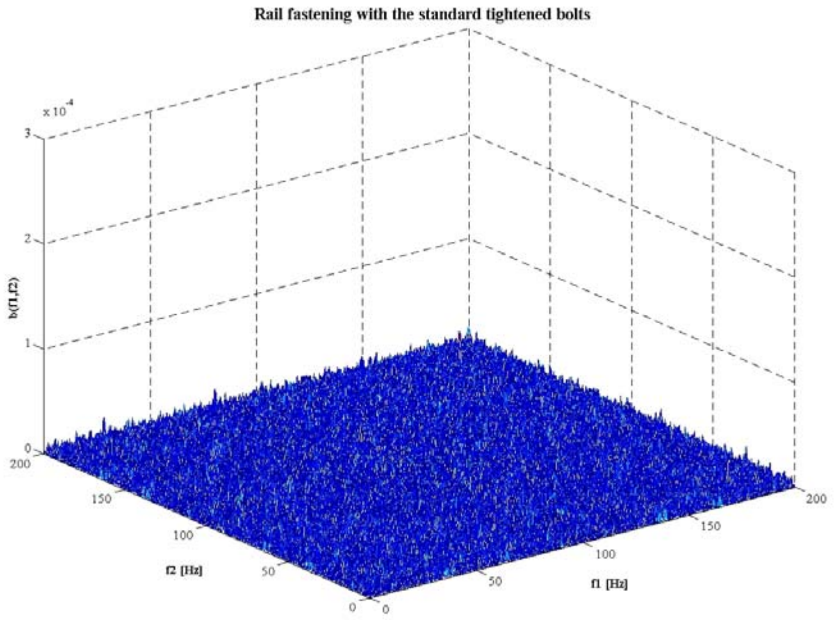

Figure 3 Bicoherence function, rail fastening with the standard tightened bolts

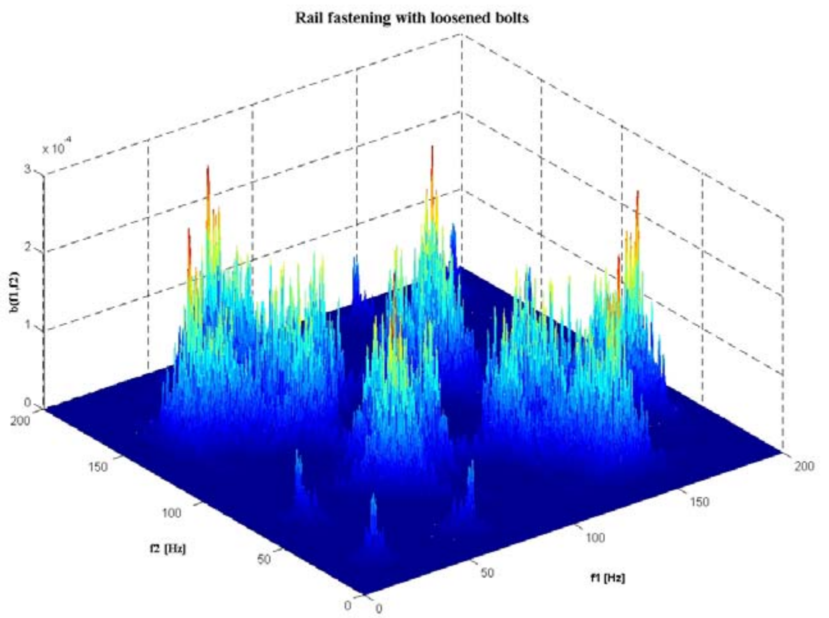

Figure 4 Bicoherence function, rail fastening with loosened bolts

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4. CONCLUSION

In conclusion, it may be stated, based upon the measurements and analyses carried out that the experiment carried out successfully verified the possibilities of utilizing the given method for assessing the quality of particular components of the rail structure. It follows from the text mentioned above that the method of higher order spectra may play an important role in future when analyzing railway structures.

The results obtained show that this method proves to be prospective for the analysis of the reaction of the rail fastening under different technological (constructional) state to the dynamic load, then in detecting different types of defects in fastening rails, sleepers and also the defects of rails proper.

Another important fact is that this method can be used to detect structural defects and to assess the quality of the superstructure, which is not essentially made possible when both the calculation models and the typical methods used until now are applied. Laboratory tests should also be verified by measurements in the field.

Based upon the results obtained, it may be assumed that the method presented is feasible in the region of the railway traffic. For example this method may be used in continuous diagnostics of various components of the rail structure or in the creation of a diagnostic system directly applied to commercial railway carriages etc.

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