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The Generalised Mohr-Coulomb (GMC) Yield Criterion and some implications on characterisation of pavement materials

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Summary

In this study a generalization of Mohr-Coulomb Yield Criterion has been developed, implemented, and tested in practice. The practical usefulness of the proposed model is demonstrated with a case study

In the ***G****eneralised* ***M****ohr-***C****oulomb (***GMC***) Yield Criterion one introduces in the Plasticity Mohr-Coulomb Theory the complete three-dimensional stress state* $(\sigma_I, \sigma_{II}, \sigma_{III})$*, thus generalising the well-known Mohr-Coulomb Theory* (σ_I, σ_{III})*. Indirectly, GMC also takes into account the influence of spherical tensor* $\tilde{\sigma}''$*, therefore* $\tilde{\sigma} = \tilde{\sigma}' + \tilde{\sigma}''$ *(*$\tilde{\sigma}'$ *= the deviatory tensor).*

According to Soil Plasticity Theory, and the constitutive characterisation, the material model can be described by the cohesion c and internal friction angle Φ*, or, alternatively, by the uni-axial strengths* R_c *and* R_t*. In the GMC model, the material is described by the generalised parameters* c^* *and* Φ^**.*

KEYWORDS: yield criterion; plasticity; cohesion and internal friction angle; pavement materials

1. INTRODUCTION

The object of the mathematical theory of plasticity is to provide a theoretical description of the relationship between stress and strain for a material which exhibits an elasto-plastic response. In essence, plastic behavior is characterized by an irreversible straining which is not time dependent and which can only be sustained once a certain level of stress has been reached.

In order to formulate a theory which models elasto-plastic material deformation three requirements have to be meet:

- An explicit relationship between stress and strain must be formulated to describe material behavior under elastic conditions, i.e. before the onset of plastic deformation;
- A yield criterion indicating the stress level at which plastic flow commences must be formulated;
- An incremental relationship between stress and strain must be developed for post-yield behavior, i.e. when the deformation is made-up of both elastic and plastic components.

The Mohr-Coulomb yield criterion is a generalization of the Coulomb (1773) friction failure low defined by:

$$\tau = c - \sigma_n tg\Phi \qquad (1)$$

where τ is the magnitude of the shearing stress, σ_n is the normal stress (tensile stress is positive), c is the cohesion and Φ is the angle of internal friction.

Graphically Eq. (1) represents a straight line tangent to the largest principal stress circle as shown in Fig. 1 and was first demonstrated by Mohr (1882).

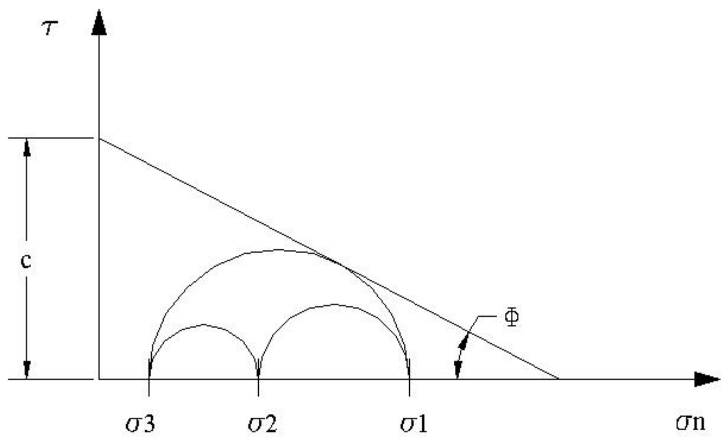

Figure 1. Mohr criterion

From Fig. 2, and for $\sigma_1 \geq \sigma_2 \geq \sigma_3$ Eq. (1) can be written as

$$(\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3)\sin\Phi = 2 \cdot c \cdot \cos\Phi \qquad (2)$$

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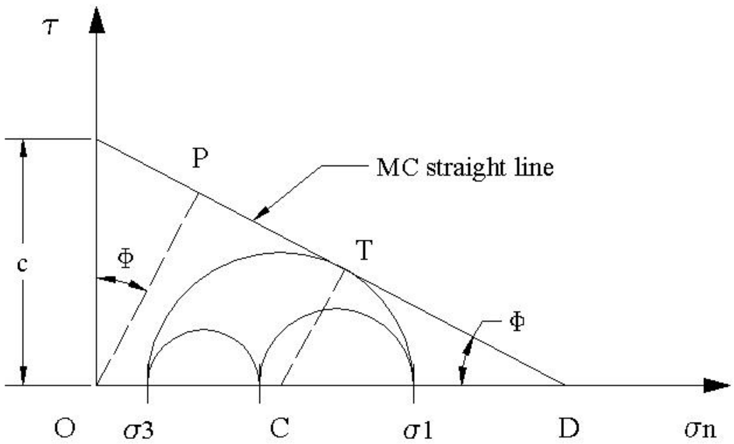

Figure 2. Mohr-Coulomb Yield Criterion

A complete derivation of the Mohr-Coulomb (MC) yield criterion is given in an forcoming paper [2].

2. NOVOZHILOV'S APPROACH OF MC YIELD CRITERION

The equivalent value of MC – yield criterion will now be expressed as follows:

$$\sigma_{ech}^{MC} = c \cdot \cos\Phi \tag{3}$$

or, in a Plasticity Theory format,

$$f(\sigma_1, \sigma_2, \sigma_3, c, \Phi) = \sigma_{ech}^{MC} - c \cdot \cos\Phi = 0 \tag{4}$$

The use of spheric/deviatoric decomposition of the stress tensor, $\tilde{\sigma} = \tilde{\sigma}' + \tilde{\sigma}''$, yields the following expressions for principal normal stresses used in Eq. (2) (see [2] for details):

$$\sigma_1 = \sigma'' + \frac{1}{3}\bar{\sigma}\left(-\sin\Theta + \sqrt{3}\cos\Theta\right)$$

$$\sigma_3 = \sigma'' + \frac{1}{3}\bar{\sigma}\left(-\sin\Theta - \sqrt{3}\cos\Theta\right)$$

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Therefore,

$$\sigma_1 - \sigma_3 = \frac{2\sqrt{3}}{3}\overline{\sigma}\cos\Theta$$

$$\sigma_1 + \sigma_3 = 2\sigma'' - \frac{2}{3}\overline{\sigma}\sin\Theta$$

and σ_{ech}^{MC} writes as:

$$\sigma_{ech}^{MC} = \sigma''\sin\Phi + \overline{\sigma}\left(\frac{1}{\sqrt{3}}\cos\Theta - \frac{1}{3}\sin\Theta\sin\Phi\right) = c\cdot\cos\Phi \qquad (5)$$

Comment: The MC - yield criterion is not a pure “shear” theory, because it contains both spherical part σ'', and the “deviatory” part, $\overline{\sigma}$.

3. THE GENERALISED MOHR-COULOMB (GMC) YIELD CRITERION

In Fig. 3 the effective stress $\overline{\sigma}$ is given an interesting geometric interpretation, due to V.M. Rosenberg [3].

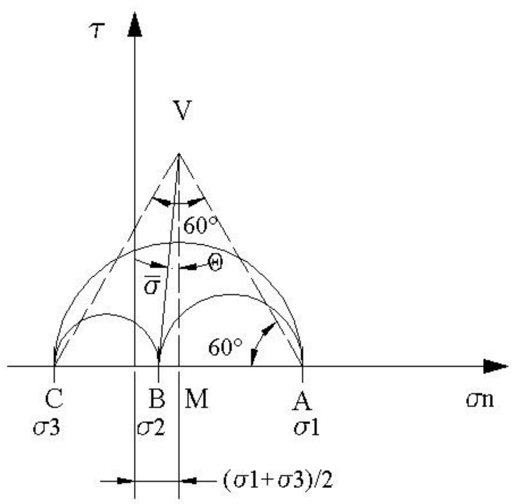

Figure 3. Rosenberg's method: $\overline{\sigma} = \|VB\|$; $\Theta = M\hat{V}B$; $\|VM\| = \overline{\sigma}\cos\Theta$

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A generalized Mohr-Coulomb yield criterion follows the line of classical MC, with the vortex point V following the straight line equation:

$$\frac{X}{d^*}+\frac{Y}{c^*}=1 \qquad (6)$$

where (see Fig. 4): c^* and d^* denote the generalized cohesion and generalized abscissas, respectively.

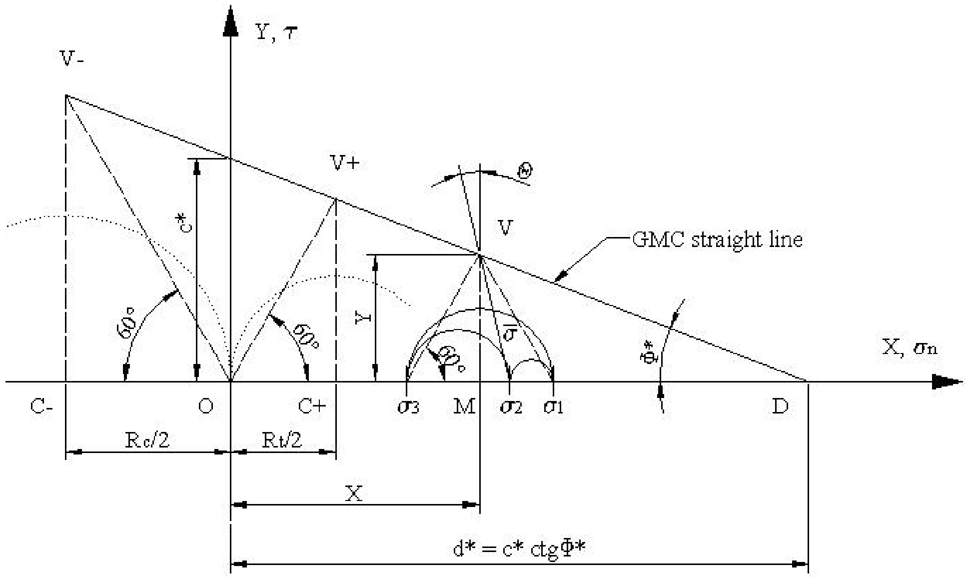

Figure 4. The Generalised Mohr-Coulomb (GMC) Yield Criterion

The Eq. (6) writes, successively, as follows:

$$\frac{\frac{\sigma_1+\sigma_3}{2}}{c^*\,ctg\Phi^*}+\frac{\bar{\sigma}\cos\Theta}{c^*}=1$$

$$\frac{\sigma_1+\sigma_3}{2}tg\Phi^*+\bar{\sigma}\cos\Theta=c^*$$

$$\frac{\sigma_1+\sigma_3}{2}\sin\Phi^*+\bar{\sigma}\cos\Theta\cos\Phi^*=c^*\cos\Phi^* \qquad (7)$$

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Now, following the trigonometric procedure of Novozhilov, one writes successively:

$$\sigma_1 + \sigma_3 = 2\sigma'' - \frac{2}{3}\bar{\sigma}\sin\Theta$$

and, therefore,

$$\frac{\sigma_1 + \sigma_3}{2}\sin\Phi^* = \left(\sigma'' - \frac{1}{3}\bar{\sigma}\sin\Theta\right)\sin\Phi^* \quad (8)$$

Eq. (7) finally writes as follows:

$$\sigma_{ech}^{GMC} = \sigma''\sin\Phi^* + \bar{\sigma}\left(\cos\Phi^*\cos\Theta - \frac{1}{3}\sin\Theta\sin\Phi^*\right) = c^*\cdot\cos\Phi^* \quad (9)$$

To compare, one observes that σ_{ech}^{GMC} (Eq. (9)) is a generalization of σ_{ech}^{MC} (Eq. (5)).

4. APPLICATION – CASE STUDY: ASPHALT MIXTURES

Laboratory tri-axial tests enable one to find out the intrinsic material characteristics of an asphalt mixture MASF 16 in the ambient temperature $T = 23$ ºC, lateral pressure $\sigma_3 = 2$ to 4 daN/cm^2, and a vertical loading in a regime of $v = 0{,}46$ mm/min.

The following MC – parameters were found:

$$\Phi = 36{,}90^{\circ}$$

$$c = 2{,}02 \text{ daN/cm}^2$$

Following the Generalized Mohr-Coulomb theory (GMC) the corresponding values are found as follows:

$$\Phi^* = arctg\left(\sqrt{3}\sin\Phi\right) = arctg\left(\sqrt{3}\sin 36{,}90\right) = 46{,}12^{\circ}$$

$$c^* = \sqrt{3}\cdot c\cdot\cos\Phi = \sqrt{3}\cdot 2{,}02\cdot\cos 36{,}90 = 2{,}80 \text{ daN/cm}^2$$

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5. CONCLUSIONS

For yield criteria with non-smoothly intersecting multiple yield surfaces e.g. Mohr-Coulomb (MC) and Generalized Mohr-Coulomb (GMC), a good return scheme has to be supplemented by proper care to account for the non-regular regions in the yield surface. The problem of determining if multiple yield surfaces are active, has recently received some attention. If these predictor-corrector algorithms have to incorporate the Koiter's generalization [4], some singularities should be taken into account [2].

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