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Non-linear constitutive model for mortar

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Summary

The non-linear constitutive model of a mortar is presented in the paper. The model is based on the equivalent one-dimensional stress-strain relation and on the smeared crack model. The limits of the one-dimensional relation are computed from the 2D state of stress with the help of the Kupfer failure condition. The smeared crack model is used for the modeling of tension crack propagation and the elasto-plastic behavior is assumed for the compression. It is assumed that concrete ca include up to two perpendicular crack direction. This model is similar to the ones that are used for a non-linear modeling of concrete.

KEYWORDS: constitutive models, finite element method, masonry, plasticity condition, smeared crack model, equivalent one-dimensional stress-strain relation.

1. INTRODUCTION

The masonry structures have been used for a very long time. In the present time it is often necessary to do a repair or reconstruction of many historical masonry buildings and engineering structures (bridges, for example). For this task, it is usually necessary to conduct a numerical analysis of the structure to understand its behavior and its carrying capabilities. The problem is that current technical standards (e.g. Eurocodes) don't provide instruction for assessment of existing structures (they have been created for a design and an assessment of new structures).

Additionally, current technical standards don't allow to use masonry for complicated states of stress (and especially for masonry loaded by tension loads) but it is often necessary to respect that historic structures may (in some cases) work also in tension or bending. Also, numerical analysis according to the Eurocodes should be done with the linear elastic model of material but masonry has highly non-linear behavior.

The masonry consists of two main components with very different properties - a mortar (which is quasi-brittle) and bricks or stones (they are brittle). The resulting

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material has complicated non-linear behavior with radically different properties in a tension and in a compression.

The paper presents only the non-linear model for the mortar. Its properties are very similar to a concrete. For this reason we use a constitutive model that is very similar to some constitutive models of concrete [9].

2. CONSTITUTIVE MODEL

2.1. Model overview

The model is based on the one-dimensional equivalent stress-strain relation (Figure 1). The relation uses so-called equivalent stress and strains that are defined (in this case) as a stresses and strains in the principal stress direction. We assume the initial linear behavior with softening (linear or non-linear) after the peak stress in tension. In a compression we assume the curvilinear behavior before the peak stress and a linear softening after the peak.

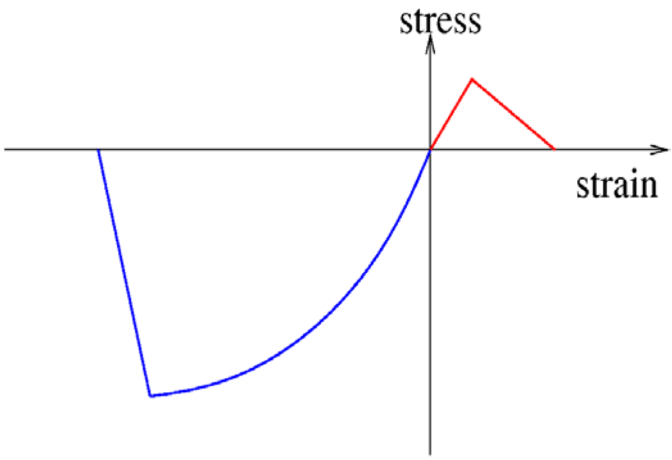

Figure 1. Equivalent one-dimensional relation (stress-strain diagram)

The limits of the equivalent one-dimensional relation (the tension and compression peak stresses and their related strains and also a strain limits) are obtained from two-dimensional (or three-dimensional) state of stress. The Kupfer or, alternatively, Chen-Chen failure condition is used for the computation of the limits.

The Kupfer [2] condition is more proven for this type of the analysis [1] but the Chen-Chen [3] relation is more universal and is designed to be used both for the 2D and the 3D. The Kupfer condition is defined only for 2D. This type of the

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model is often used for the concrete and is known to be very useful. For example, similar model is used in the successful commercial finite element code ATENA[4].

The behavior in unloading is assumed to be linear, with the initial Young modulus. It means that there is elastic and an inelastic part of deformation. Numerous authors use elastic unloading without inelastic part of deformations here [1] but the inelastic deformation is known to occur in the real material and this fact have to be respected in the model.

2.2. Modeling of behavior in tension

There is a problem related to this type of constitutive model and to its combination with finite element method: if peak stress occurs and behavior of model has to simulate the crack propagation, it is obvious that Young modulus have to be changed. But the size of the area of changing modulus is related to the size of the finite element on which the peak stress was detected. It means the behavior in a tension depends on the size of finite elements.

It can be tested that very fine mesh (with very small finite elements) can lead to absolutely useless results (the model is very soft and fails very fast). There is also second problem - the peak criteria depends on the stress but with finite element analysis the extreme stresses can be lower for meshes with large finite elements due to averaging of results. So a model with a fine mesh can also reach the peak stress faster than a model with less fine mesh. But the later problem is minor a less difficult than the first one.

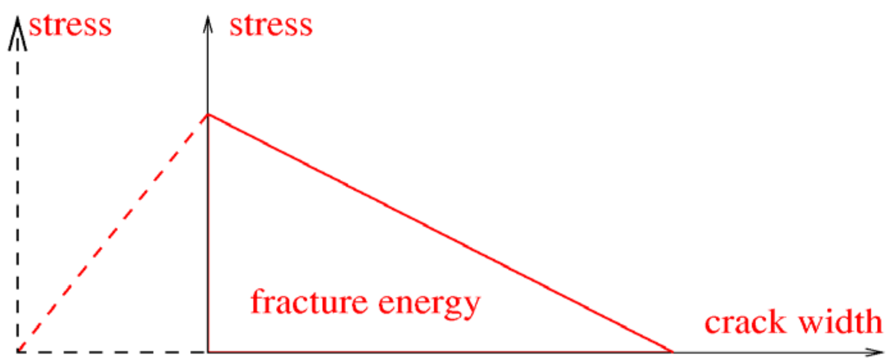

Figure 2. Relation between crack width and stress

Bazant [5] calls the problem a size-effect and offers a solution. The energy that is dissipated during the crack propagation can be measured and is can be used as a material property (but it also may vary and there is several definition of it that results to different sizes of this property - see [6] for details).

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It is usually referred as fracture energy. Because the fracture energy is related to the crack width and the width of a cracking area (in our case it is a finite element width - see Figure 2), it is possible to use it to modify the parameters of the tension part of the equivalent one-dimensional stress-strain relation (Figure 1).

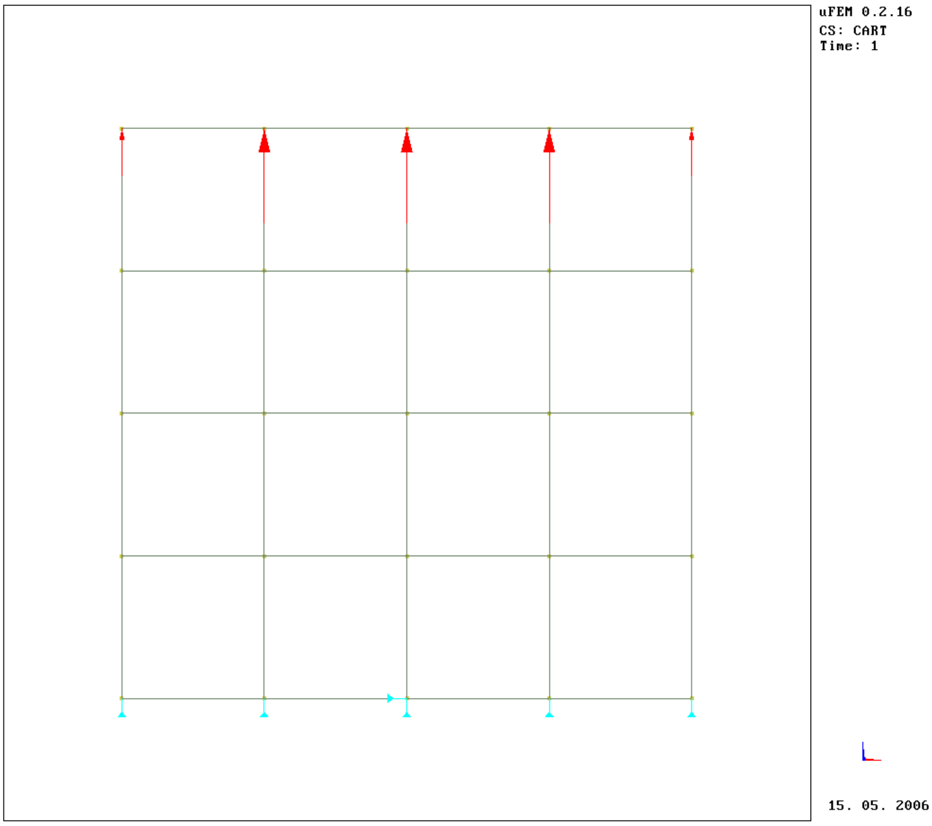

Figure 3. Finite element model of the illustrative example

3. NUMERICAL EXAMPLE

To illustrate the described constitutive model we provide a very simple numerical example. The computations were done in the uFEM finite element method software [7]. The example was conducted to show the behavior of the constitutive model in the tension.

The structure size is 0,4x0,4m and the width is 0,2m. The Young's modulus is 20 GPa, compression strength is 10,0 MPa and tension strength is 1,0 MPa. The forces represents the continuous load of a size of 5,2 kN/m.

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The structure was analyzed in 2D as a plane stress problem with help of the Finite Element Method with the four-node isoparametric finite elements (well-known serendipity family elements).

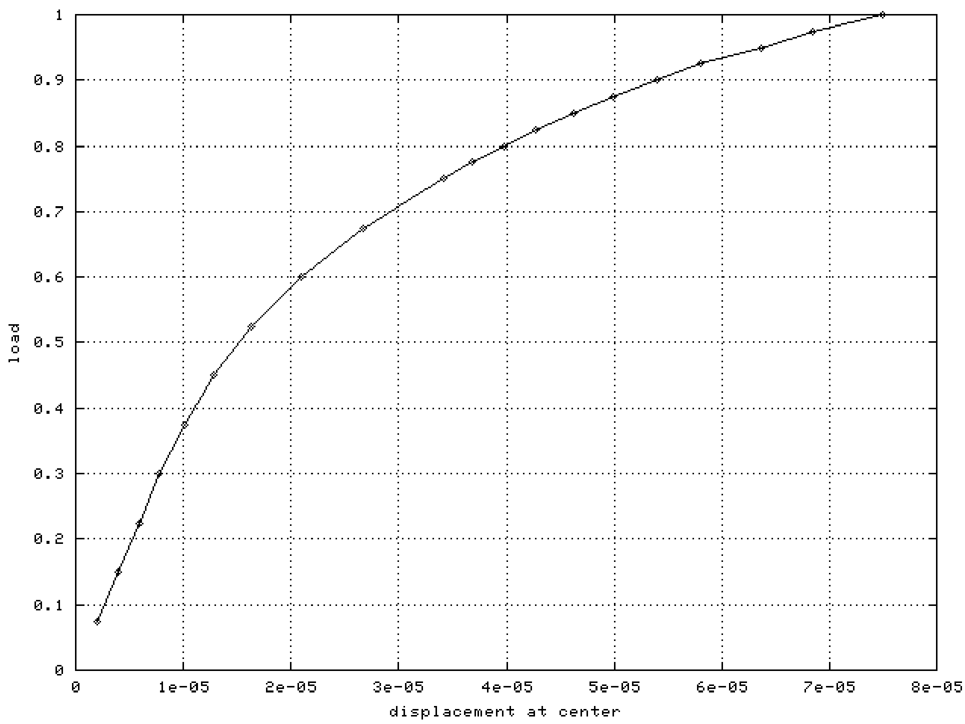

Figure 4. Load-displacement diagram for the illustrative example

The Newton-Raphson method was used for the control of the non-linear computational process. The solution consisted from 20 substeps. A total about 150 iterations were done.

The computational time for this solution was only few minutes on a 250 MHz workstation and under one minute on a modern computer (Sun Ultra 20 with Opteron 1,8 GHz processor).

The resulting load-displacement curve is shown in the Figure 4. The displacements were measured at the center of the structure. The load size shown in the Figure 4 is a relative (e.g. load size 1 is equal to 5,2 kN/m as mentioned above).

It is visible that behavior of the presented constitutive model is highly non-linear. The results presented here are only illustrative; however it is visible that the given approach can be used for modeling of mortar.

4. CONCLUSIONS

The article presents a non-linear constitutive model for mortar. This model is similar to ones that have been successfully used for modeling of a concrete because of high level of similarity of these material. The example solution that uses the model is also presented.

During further works we will concentrate on the verifying of the model and comparison with results of experimental laboratory tests.

We also work on the integration of this model with constitutive model for brittle materials that will simulate behavior of bricks and stones.

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