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On the dynamic behavior of some suspended roofs

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Summary

This article presents a simple suspended cable system, in which there are presented calculating formulas for tensions, mainstay loading, the arrow and its increase after taking into consideration the cable's load. In the second part, a roof system with a double curvature is presented, its behavior and how it deals with the resonance phenomenon .The possible elimination of the flutter effect that might appear at this type of roof is being observed, as well.

KEY – WORDS: cable, tensed membranes, arrow, double layer cable, suspended simple roof, vibration.

1. INTRODUCTION

Cabled roofs allow big openings, which are indicated when it comes to public constructions: show rooms, stadiums, trading centers, gymnastics halls, etc. The suspended roofs structures present a relatively high sensibility concerning the winds dynamic action. Being light, the roofs encourage the flutter effect under the action of the lifting aerodynamic forces. This effect is also facilitated by high amplitudes and by cables reduced damping.

The winds action manifests itself through fluctuations, which are nearly periodic and can lead to resonance phenomena.

Both the flutter, which is generated by vertical oscillations, coupled with waving oscillations, and the resonance phenomenon can appear especially on certain structural systems, but on principle, both of them must be analyzed, since they can lead to the structure's collapse.

Cabled suspended roofs are economical especially in case of big openings. Delicate problems also appear at the cables anchorage or their pretension, if pretension exists.

The conception of the cable system and of the covering elements is responsible for the structure's flexibility, which is not significant. And so, the roof surface can be well defined in space and its deforming to different actions, especially the static

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ones must not lead to important modifications of the deformed system, compared to the initial uncharged one.

Cabled roofs are made just as the thin curved board roofs are made, with certain specific differences. There can be roofs that have the form of simple curved roofs, for example cylinder surfaces or double curved ones – hyperbolic, parable shaped, etc. A type of roof which is frequently used is the bicycle wheel type, where the cables are anchored in an exterior ring and in a central device.

Suspended systems can be made with a simple surface, with a simple net of cables or with a double surface made of two layer cable, distanced on struts. Thus, a cabled roof has the shape of an imaginary membrane extended above cables put on a surface or on two surfaces which are connected to each other and complementary.

A significant size of these roofs is the ratio between the maximum arrow and the span which is normally between 1:15 – 1:20.

Suspended roofs significant loads are: its own weight and the weight of the covering membrane, the snow action, the wind action, a possible seismic activity. The winds' action always has a static component and a dynamic component which generate vibration phenomena, accompanied by the already mentioned effects: flutter and resonance.

Irrespective of the analysis made for the suspended system projection, its behavior is of great interest both in terms of its individual suspended cable static action and its answer to the dynamic action.

2. SUSPENDED CABLE ACTIONED BY STATIC LOADS

Depending on the types of structure, the mainstays and the cables' anchoring points are situated on the same level, a case frequently seen, or on different levels. Generally speaking, on suspended roofs the cables are stretched and - as mentioned earlier – the arrow / span ratio is low.

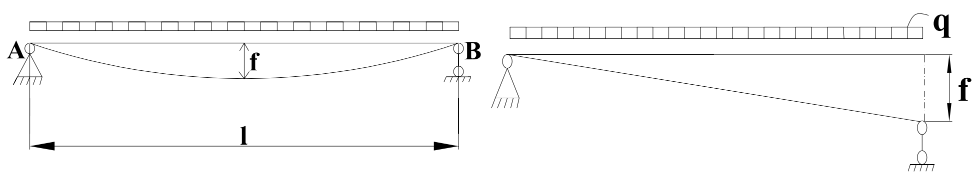

Figure 1. Uniform load of a cable

With stretched cables, the deformed shape is parable-shaped, if the charging is uniform between the mainstay points, Figure 1. It can have as source the cable's weight, the covering elements, and the snow.

Due to its flexibility in a transversal manner, the cable is subjected to tension only. The tension in cable T in the section area has the direction line tangent to the line that represents the deformed configuration of the cable.

When it comes to vertically distributed loads, the horizontal component of the T effort in the cable is constant.

From the immediate relation:

$$H = T\cos\alpha \tag{1}$$

α being the angle made by the horizontal and the tangent to the deformed curve.

The conclusion:

$$T = H/\cos\alpha \tag{2}$$

And obviously the minimum T_0 value is for $\alpha = 0$, that is, when the arrow is maximal f; so T increases towards the mainstays, having maximum values in the suspension points.

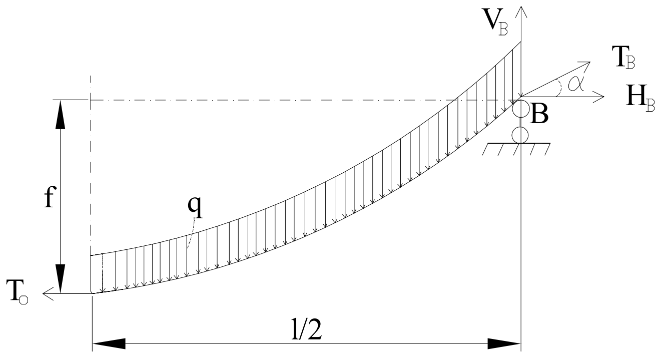

Figure 2. Equilibrium projection

From the equilibrium conditions, projection on the vertical and moment in relation to point B, represented in Figure 2, are obtained:

$$V_B - q\frac{l}{2} = 0 \tag{3.a}$$

$$T_o f - q\frac{l^2}{8} = 0 \tag{3.b}$$

ISSN 1582-3024

It yields:

$$V_B = q\frac{l}{2} \tag{4}$$

$$T_o = \frac{ql^2}{8f} = H = H_B \tag{5}$$

The T_{max} value is T_B and it immediately leads to:

$$T_{\max} = \sqrt{V_B^2 + H_b^2} = \sqrt{\left(\frac{q}{2}^2\right)^2 + \left(\frac{ql^2}{8f}\right)^2} = \frac{ql^2}{8f}\sqrt{1+16\left(\frac{f}{l}\right)^2} \tag{6}$$

The α angle made by the cable tangent in the support point results from:

$$tg\alpha = \frac{V_B}{H_B} = 4\frac{f}{l} \tag{7}$$

The initial length of the cable before it acts with q intensity is given by the relation:

$$L = l\left(1 + \frac{8}{3}\left(\frac{f}{l}\right)^2\right) \tag{8}$$

As a consequence of the force of intensity q, the cable length modifies becoming $L^{'} = L + \Delta L$ and:

$$\Delta L = \frac{TL}{EA} = \frac{T}{EA}\left(1 + \frac{8}{3}\left(\frac{f}{l}\right)^2\right) \tag{9}$$

Where:

E = the cable equivalent elasticity module

A = cable section equivalent area

The increase of the arrow Δf due to the stretch of the cable Δl is expressed by the relation:

$$\Delta f = \frac{\Delta L}{\frac{16}{15}\frac{f}{l}\left(5 - 24\frac{f^2}{l^2}\right)} \tag{10}$$

Through the increase of the arrow, the values of the efforts $T_0 = H$ and T_{max} are diminished according to the relations (5) and (6).

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3. CABLE VIBRATIONS FREQUENCIES

A tensed cable which vibrates is assimilated to a string and the frequencies of the free vibrations are calculated accordingly.

$$\omega_n = \frac{n\pi}{l} c \quad n = 1, 2, ... \tag{11}$$

Where c, meaning speed, has the expression:

$$c = \sqrt{\frac{H}{\rho A}} \tag{12}$$

H = the effort in the string, and $\rho A = m$ is the mass of unit length.

The n modules own vibrating frequency of a stretched cable, with no amortization, with an l length, perfectly flexible and, with a uniformly distributed load on a horizontal projection of m, intensity is given by the expression:

$$\varpi_n = \frac{n\pi}{l} \sqrt{\frac{T}{m}} \quad n = 1, 2, ... \tag{13}$$

Obviously, in the case of the uniform q loading $m=q/g$, where g = the gravitational acceleration.

Because T is proportional to load q, the natural frequency does not depend on the value of the load, with the mentioning of the fact that in calculating T the initial arrow f is used, and in the cable there are no other tensions.

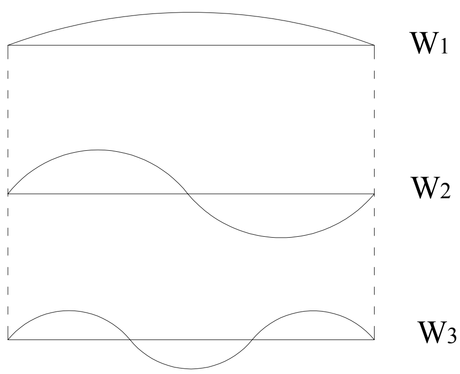

Figure 3. The first 3 vibration modes

In the stretched cable, for $T = \frac{ql^2}{8f} = H$, the ϖ_n frequencies become:

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$$\varpi_n = n\pi\sqrt{\frac{g}{8f}} \tag{14}$$

The frequencies increase with *n*, the smallest corresponding to *n* = 1, *n* is the number of the vibration waves. In Figure 3, the first 3 vibration modes are presented.

Frequencies are independent in relation to *m* if there are no other actions and / or tensions in the cable, which modify the answer in frequencies. In this way, the ballast of the cables can interfere, by increasing the *f* arrow, as well as the efforts in the cable. Efforts due to the cables pre-tensioning can also occur.

An exterior disturbing force with pulsating character, for example one that is given by the wind action, whose frequency is ω_e, modifies the answer in the cable frequencies, i.e. the cable vibration amplitudes are determined by the ratio ω_e / ω_n .

Near the values $\omega_e / \omega_n = 1, 2, 3$……may be seen an excessive increase of the amplitudes and there occurs the resonance phenomenon.

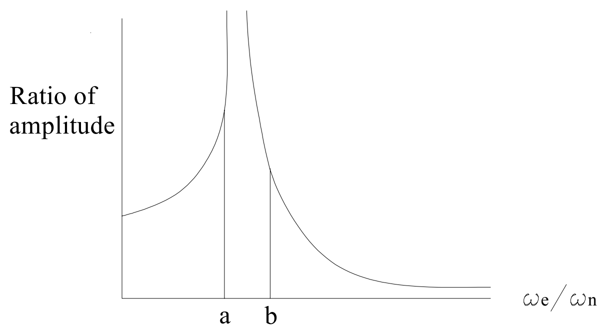

Figure 4. Interval of amplification

Therefore, in a graphic representation (Figure 4) in which appear the ratios ω_e / ω_n on the abscissa, there may be seen the intervals of amplification and resonance for which the suspended roofs have a destructive character.

The increase of the effort in the cable by the roof ballast or through the cable's pre-tensioning increases the roof rigidity and also, introduces amortizations through constructive means.

Erecting roofs with the shape of two shells by introducing double cables distanced through vertical stanchions leads to interconnected surfaces with a high rigidity which diminishes and even removes both the flutter and the resonance phenomenon.

ISSN 1582-3024

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4. EXAMPLE OF CALCULATION FOR A ROOF MADE OF ONE CABLE LAYER

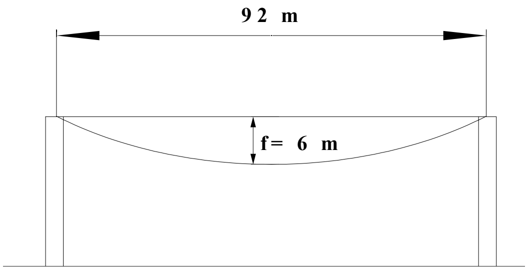

Figure 5. Section

The span is 92 m, the maximum arrow 6 m.

The dead load, the cable weight, included is:

$$q_p = 1000 N/m^2 .$$

The live load on the cables is:

$$q_a = 1556 N/m^2$$

The distance between 2 successive cables is:

$$l_c = 1.2m$$

Assuming

$$f = \frac{1}{15} l$$

It yields f = 6.13m.

The total load is:

$$q_t = \left(q_p + q_a\right)l_c = (1000 + 1556)1.2 = 2130\, N/m = 2130\, kN/m$$

The cable tension is present in relation (6):

$$T = q\frac{l^2}{8f}\sqrt{1 + 16\left(\frac{f}{l}\right)^2} = \frac{2.13092^2}{8 \cdot 6.13}\sqrt{1 + 16\left(\frac{6,13}{26}\right)^2} = 378kN$$

There can be used a galvanized steel cable with Ø = 5 cm and the section area A=15.6 cm^2, which has an elasticity module E=165.4742 KN/mm^2.

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In order to determine the cable elongation, expression (9) is used in which for *T*, has been assumed the value from the extremities and for *L* the value determined in relation (8), so:

$$L = l\left(1+\frac{8}{3}\left(\frac{f}{q}\right)^2\right) = 92\left(1+\frac{8}{3}\left(\frac{6.13}{92}\right)^2\right) = 93.089\,m$$

$$\Delta L = \frac{TL}{EA} = \frac{378x93{,}089}{165.47x42.1560} = 0.136\,m$$

It is possible to calculate the growth of the arrow Δf due to the elongations ΔL:

$$\Delta f = \frac{\Delta L}{\frac{16}{15}\frac{f}{q}\left(5-24\left(\frac{f}{l}\right)^2\right)} = \frac{0.136}{\frac{16}{15}\frac{6.13}{92}\left(5-24\left(\frac{6.13}{92}\right)^2\right)} = 0.391\,m$$

Next the cable's own frequencies may be calculated by means of relation (13):

$$\varpi_n = n\frac{\pi}{l}\sqrt{\frac{T}{q/g}} = n\frac{\pi}{92}\sqrt{\frac{378}{2.13/981}} = n\cdot 1.425\,Hz$$

For n = 1,2,3… the first pulsations are obtained, the fundamental one being 1.425 Hz.

The values T and ω_n being corrected, $f + \Delta f$ will be used for the arrow and $L' = L + \Delta L$ for the cable length.

$$T' = q\frac{l^2}{8(f+\Delta f)}\sqrt{1+16\left(\frac{f+\Delta f}{l^2}\right)^2} = \frac{2.130\cdot 92^2}{8\cdot(6.521)} = \sqrt{1+16\left(\frac{6{,}521}{92}\right)^2} = 359.2\,kN$$

$$\varpi'_n = n\frac{\pi}{L+\Delta L}\sqrt{\frac{t}{q/g}} = n\frac{\pi}{93.225}\sqrt{\frac{378}{2.15/9.51}} = n\cdot 1.406\,Hz$$

It can be concluded that the pulsation determined with the initial values of the cable length is not essentially different from the pulsation determined using the real length $L + \Delta L$.

Concerning the cable tension, a diminishing can be noticed once the arrow increases through the cable deformation (its elongations), a favorable effect for the suspended roofs, since through the cables' deformation there is produced a tendency of structural stabilization.

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5. ROOFS WITH DOUBLE SURFACES

The cables which form double surfaces coupled through vertical rigid bars suggest a behavior to applied loads which is similar to a beam, the inferior cables being further stretched while the tensions in the superior cables are diminished. Mention should be made that regularly the ratios $f : L$ remain, more or less, the same for the convex and concave cables and the charging capacity of the roof surface depends on the way in which the curve of the roof surface is made and less on the individual cables' curves.

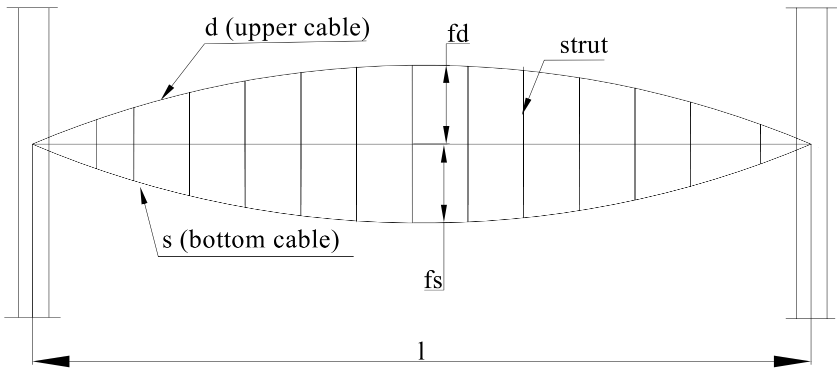

Figure 6. Double layer cable

The cables *d* and *s* are used with initial tensions *Td* and *Ts* and the value of these tensions is function of the number and the positions of the vertical struts, to their dimensions and weights of the two types of cable, Figure 6. When used, the charging is given by its own weight and by the weight of the struts.

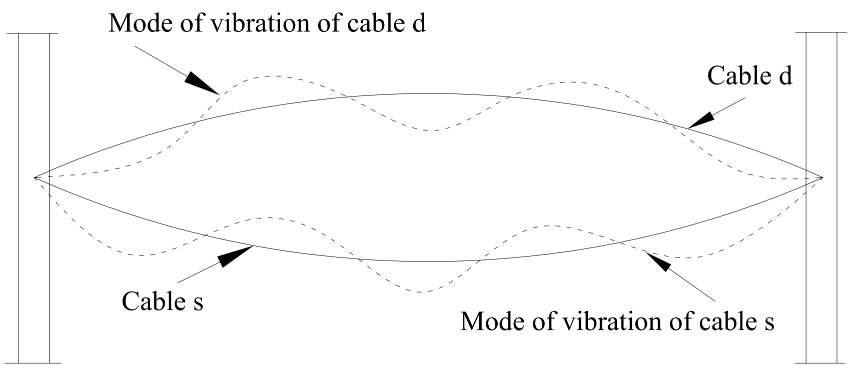

Figure 7. The geometry of vibration

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Through dead and live loading, the tension in the inferior cable increases with delta *Td* and the one in the superior cable diminishes with delta T_s. But these variations must be controlled; the excessive diminishing of the tension in the superior cable is especially dangerous. But these observations show that the pulsations of the two surfaces where the inferior one is generated by the cables underneath and the superior one by the upper cables will be different, Figure 7.

An energetic flux will be created between the two surfaces resulting in the amortization of the vibrations.

References:

1. R.Marinov, *Probleme de stabilitate in constructii*, Editura Tehnica Bucuresti, 1985 (in Romanian).
2. R.Sofronie, *Aeroleastic Stability of Wide-Span Surface Structures,* Rev. de Science Technical Mecanique, Tom22. 1977.
3. Kuneida H, *Parametric Resonance of Suspension Roofs in Wind,* International Journal Engineer Mechanic Division, Vol.34, August 1976.
4. Voinea R., Voiculescu D., Simion F.P., *Introducere in Mecanica Solidului cu Aplicatii in Inginerie*, Editura Academiei Române, Bucuresti 1989 (in Romanian).
5. Geschwindner L.F, West H.H, *Forced Vibration of Cable Networks,* Journal Structures Division, Vol.106, September 1980.
6. Chi-Mou-Ni Lee, L.M., *Forced Vibration of Cable Network*, Journal Structures Division September 1980.
7. Chisalita Adrian, *Contributii la Raspunsul neliniar, static si dinamic, al sistemelor suspendate,* Teza de doctorat, I.P.C.-N, Cluj Napoca, 1983 (in Romanian).
8. Jaroslav Kadlcak, Alois Materna, *Static's of Elastic Cable Network,* Academia Nakladatelsctvi, Prague 1978.
9. Trevor Stephen Lewis, *The form finding of structures Possessing a Constant Surface Stress,* PhD Thesis, University of Warwick, January 1997.
10. Lev Zetlin, *Steel Cable Creates Novel Structural Space System,* AISC Engineering Journal, January 1964.