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About some important changes in applied structural optimization

Jacek Boroń
Building Engineering Institute, Wroclaw University of Technology, Wroclaw, 50-370, Poland

Summary

Structural optimization is special domain of employment researching many and how different problems in the field of forming structure. In times of early computer science and computational technology, when the access to "computing time" of the machine was strongly regulated (from the point of view of considerable costs) some optimization problems were very strongly simplified, so their solution could be possible without mathematical programming methods and therefore cheaper.

In times of stormy development of informatization and almost free-for-all personal computers as well as specialized software, complication of structural optimization modeling has grown considerably.

In this paper being short recapitulation of achievements made by Division of Computational Methods in Engineering Design, it refers to these earliest problems and to these very modern both dealing with applied structural optimization, what is the domain of interest of our team from over 25 years.

KEYWORDS: structural optimization, scalar optimization problem, genetic algorithm, vector optimization problem

1. INTRODUCTION

As member of the Team for Computational Methods in Engineering and Design, I have started dealing with the applied structural optimization in the end of 70-ties in XX century. In the beginning it was research concerning steel bar structures (trusses and frames) and industrial buildings (concrete beams, silos and tanks). All of these early problems mentioned above were formulated and then solved as scalar optimization questions.

Next we started researching with vector optimization problems (steel frames and trusses) and genetic algorithms (thanks to cooperation with Carlos Coello Coello and Gregorio Toscano-Pulido).

In this paper I'm trying to bring you closer how deep were the differences between these first and last problems (exactly in this year was my personal 25th anniversary of optimization research).

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2. SCALAR OPTIMIZATION PROBLEM

2.1. Simple example of the tank welded from steel

The first example of optimization I want to present (in this case example of scalar optimization, started and conducted in 1981) is a tank (the part of steel water tower), shown on the drawing below (Figure 1).

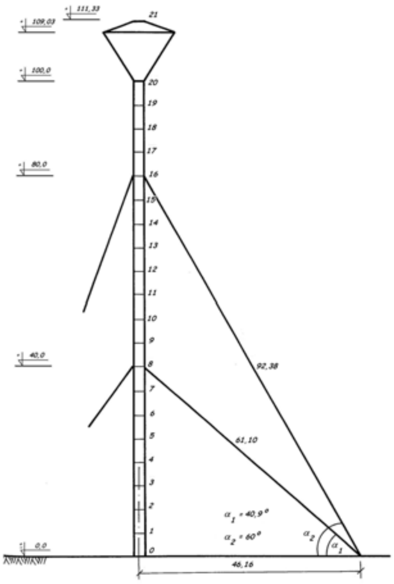

Figure 1. Steel water tower

For optimization the following lump of the tank, made from two cut off cones and one internal cylinder, has been chosen. Surface plan it in places of intersections circled wreaths stiffening. Described has resulted from capacity form highly, allocations and easy installment available methods (so called "easy" or "heavy" one).

2.2 Scalar optimization model

As criterion of optimization accept minimum of expenditure of material preliminary. Become setting up average thickness of covering above-mentioned question fetch for determination of condition of occurrence of minimum of lateral surface. Besides, it accepts following foundation and simplification:

- dimension section - they mirror middle surface,
- thickness of covering is constant (and average),

- water fulfills only maximum bottom cone,
- we use only one design variable: corner of inclination of surface for vertical α in bottom cone (see Figure 2),
- capacity of useful tank totals 600 m^3.

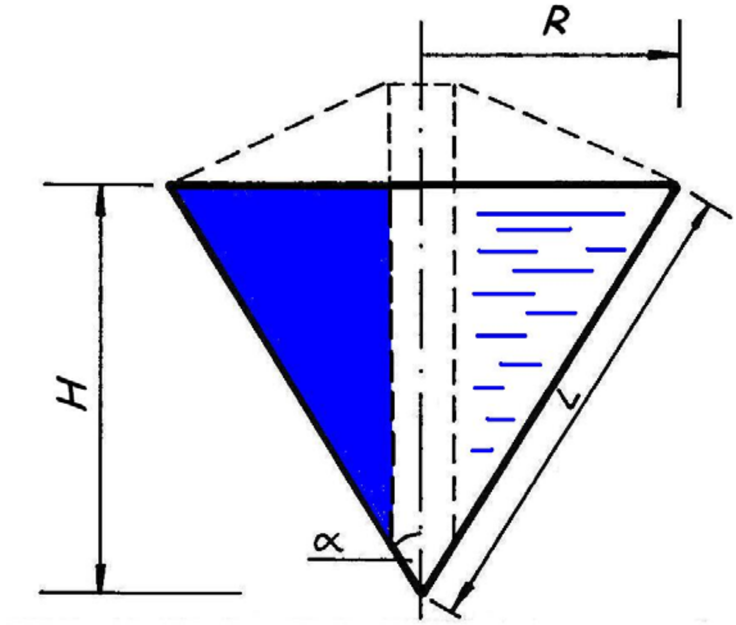

Figure 2. Steel tank – lump and design variable α

It takes into consideration, in the farthest consideration, following geometric dependences:

$$R = H \cdot tg\,(\alpha)\,, \tag{1}$$

$$L = H \,/\, cos\,(\alpha)\,, \tag{2}$$

Field of the lateral surface:

$$F = \pi \cdot R \cdot L = H \cdot tg(\alpha) \cdot H \,/\, cos\,(\alpha) = H^2 \cdot tg(\alpha) \,/\, cos\,(\alpha), \tag{3}$$

Capacity of the cone:

$$V = 600 = 1/3\ \pi \cdot R^2 \cdot H = \ldots = 1/3\ \pi \cdot H^3 \cdot tg^2(\alpha), \tag{4}$$

Basing on (4) in the function of the corner α, next H was indicated:

$$H = [1800 \,/\, \pi \cdot tg2(\alpha)]^{1/3}\ , \tag{5}$$

and it put for (1)

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$$F = A \bullet \sin^{-1}(\alpha) \bullet [tg^2(\alpha)]^{1/3},$$
$$\text{where } A = 1800 \bullet (\pi/1800)^{1/3} = \ldots = 216{,}72, \qquad (6)$$

Task of minimization solve existence of minimum of function alternate one researching F(α).

$$\min F(\alpha) \leftrightarrow F'(\alpha) = 0, \qquad (7)$$

Solution illustrate on the drawing (see Figure 3). Next it verify „candidate for minimum" (α') calculating in this point value of second derivative function F''(α):

$$F''(\alpha') = \ldots = 1{,}833 > 0 \qquad (8)$$

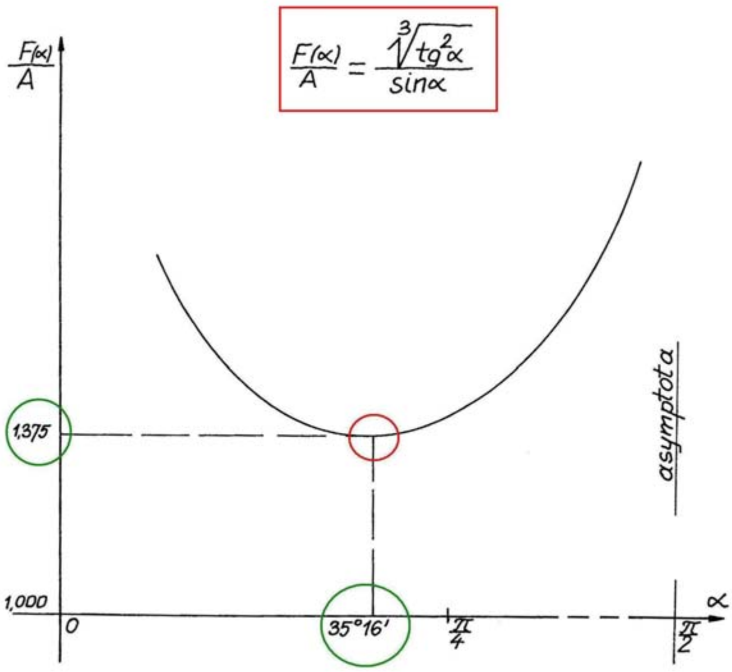

Figure 3. Solution of question of simple scalar optimization

2.3 Recapitulation and final conclusions

It exert in the first approximation, that conical tank has minimal field of lateral surface (but what behind it go, grant demanded criterion: minimum of material), when it lateral is drooping for vertical under corner creating $\alpha = 35^{o}\ 16^{''}$.

Having the corner of creating inclination α and foundations or simplifications mentioned on admission of preamble, remained dimension of the tank have been calculated from simple geometric dependences.

Fundamental geometric dimensions of the tank accepted for the farthest technical and executive design, it present on following drawing (Figure 4).

Assuring, as contact limit, the smallest surface of conical covering with aggressive environment (the water stored in this tank, so-called: industrial, with mineral small parts inclusive, about predefined temperature gone up with technological respects) we can prominently extend the constancy of maintenance of the building (water tower) in the best condition.

3. MICRO-GA AS AN EFFECTIVE SOLVER FOR MULTIOBJECTIVE OPTIMIZATION PROBLEMS

3.1. Genetic algorithms in multiobjective structural optimization

Genetic algorithms (GAs) have become very popular optimization techniques in structural optimization, but their use in multiobjective structural optimization has become less common. Additionally, only few researchers have emphasized the importance of efficiency when dealing with multiobjective optimization problems, despite the fact that its (potentially high) computational cost may become prohibitive in real-world applications.

In this paper, we present a GA with a very small population size and a reinitialization process (a micro-GA) [1] which is used for multiobjective optimization of trusses.

3.2. The micro-GA

This micro-GA approach elaborated by Toscano-Pulido [3,5] works as follows (Figure 5). It starts with a random population, it uses two memories: a replaceable (that will change during the evolutionary process) and a non-replaceable (that will not change) portion. Micro-GA uses 3 types of elitism.

The first is based on the notion that if we store the non-dominated vectors produced from each cycle of the micro-GA, we will not lose any valuable information obtained from the evolutionary process.

The second is based on the idea that if we replace the population memory by the nominal solutions (i.e., the best solutions found when nominal convergence is reached), it will gradually converge, since crossover and mutation will have a higher probability of reaching the true Pareto front of the problem over time. The third type of elitism is applied at certain intervals (defined by a parameter called “replacement cycle”). It takes a certain number of points from all the regions of the Pareto front generated so far and it uses them to fill the replaceable memory. Depending on the size of the replaceable memory, it chooses as many points from the Pareto front as necessary to guarantee a uniform distribution.

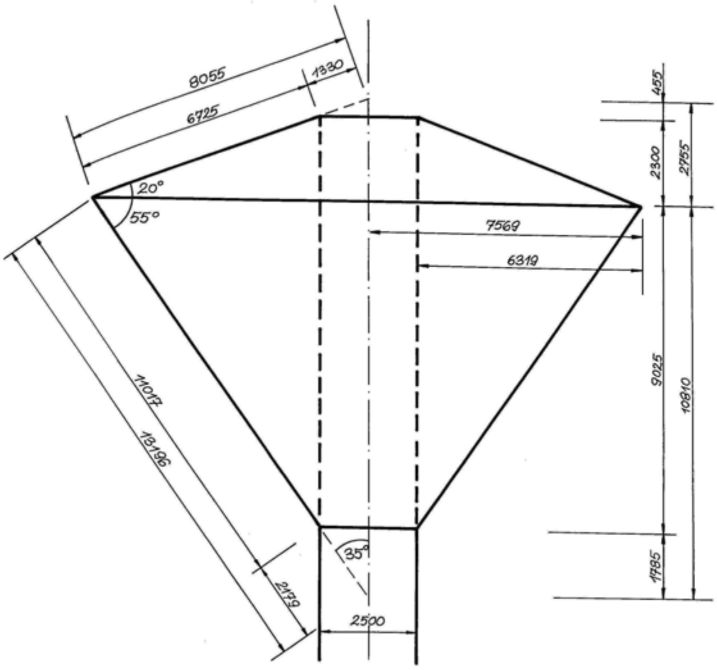

Figure 4. Conical tank accepted for the farthest technical design

This process allows us to use the best solutions generated so far as the starting point for the micro-GA, so that we can improve them (either by getting closer to the true Pareto front or by getting a better distribution along it). To keep diversity in the Pareto front, it uses an approach based on geographical location of individuals (in objective function space) similar to the adaptive grid proposed by Knowles & Corne [2]. This approach is used to decide which individuals will be stored in the external memory once it is full. Individuals in less populated regions of objective space will be preferred.

In previous work, our micro-GA has performed well (in terms of distribution along the Pareto front, and speed of convergence to the global Pareto front) with respect to other recent evolutionary multiobjective (vector) optimization approaches, while requiring a lower computational cost [3].

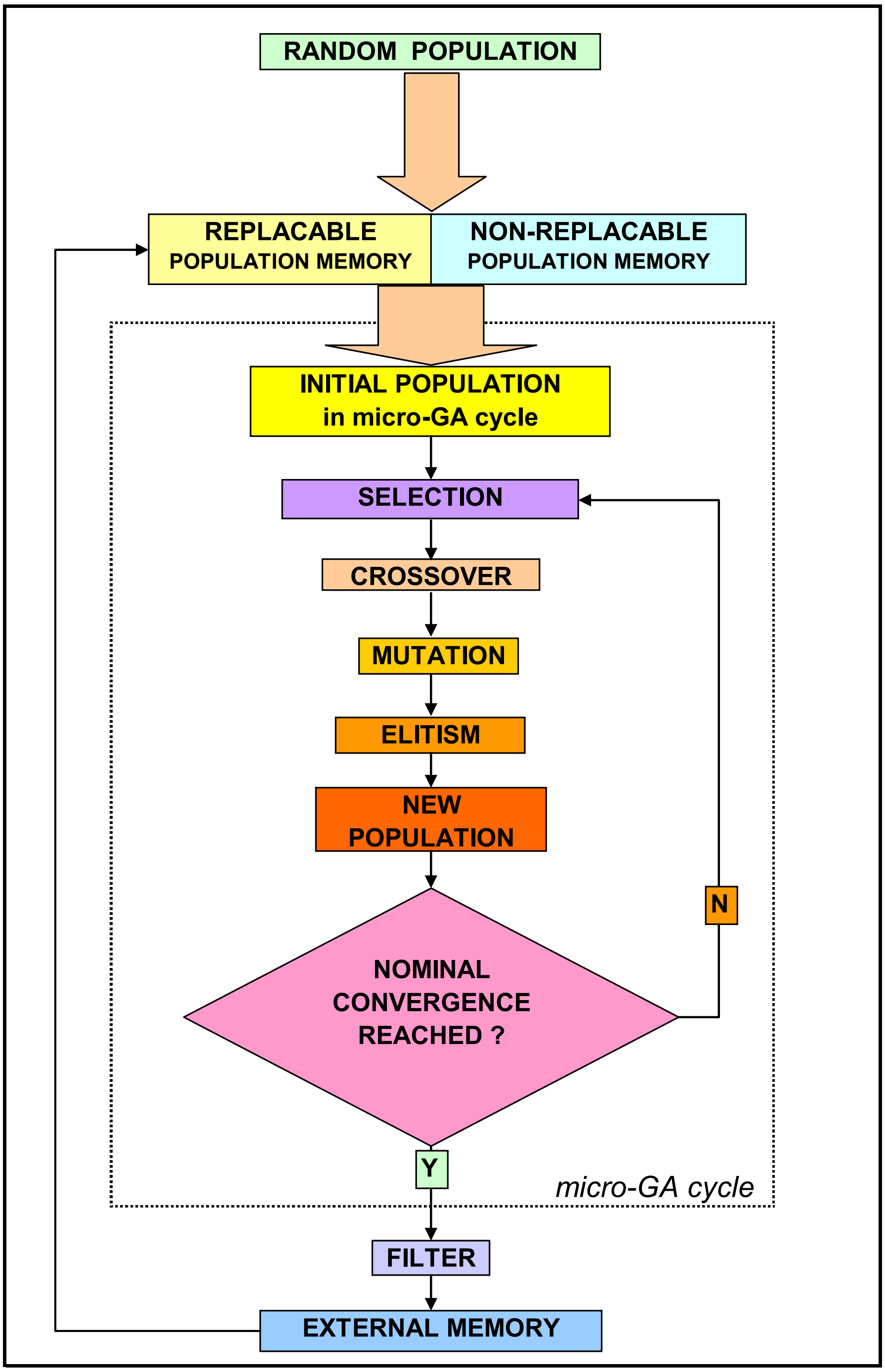

Figure 5 : Diagram of micro-GA

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3.3. Illustrative example

The 4-bar plane truss shown in below (Figure 6) is used to illustrate this approach. Two objectives were considered in this case: minimize volume and minimize its joint displacement δ. Four decision variables are considered (for details of this problem, see [4]).

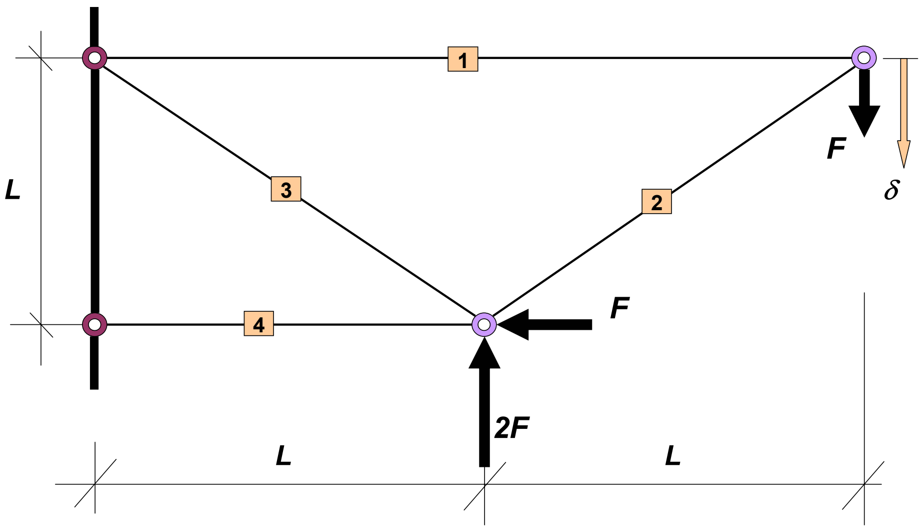

Figure 6 : Four-bar plane truss with one loading case

The Pareto front produced by micro-GA mentioned above, and its comparison against the global Pareto front (produced using an enumerative approach) is shown in the next figure (Figure 7).

3. CONCLUSIONS

The task of structural optimization is to support the constructor in searching for the best possible design alternatives of specific structures. The "best possible" or in the other words "optimal" structure means that structure which mostly corresponds to the designer's objectives meeting of operational, manufacturing and application demands simultaneously.

Compared with the "trial and error"- method mostly used in engineering practice (and based on an individual, intuitive, empirical approach) the seeking of optimal solutions by applying MOP (mathematical optimization procedures) is much more efficient and reliable. Nowadays in the time of market economy also research has

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to be "market one". In my opinion "to be market" is now the greatest challenge for applied optimization in Poland and everywhere [6].

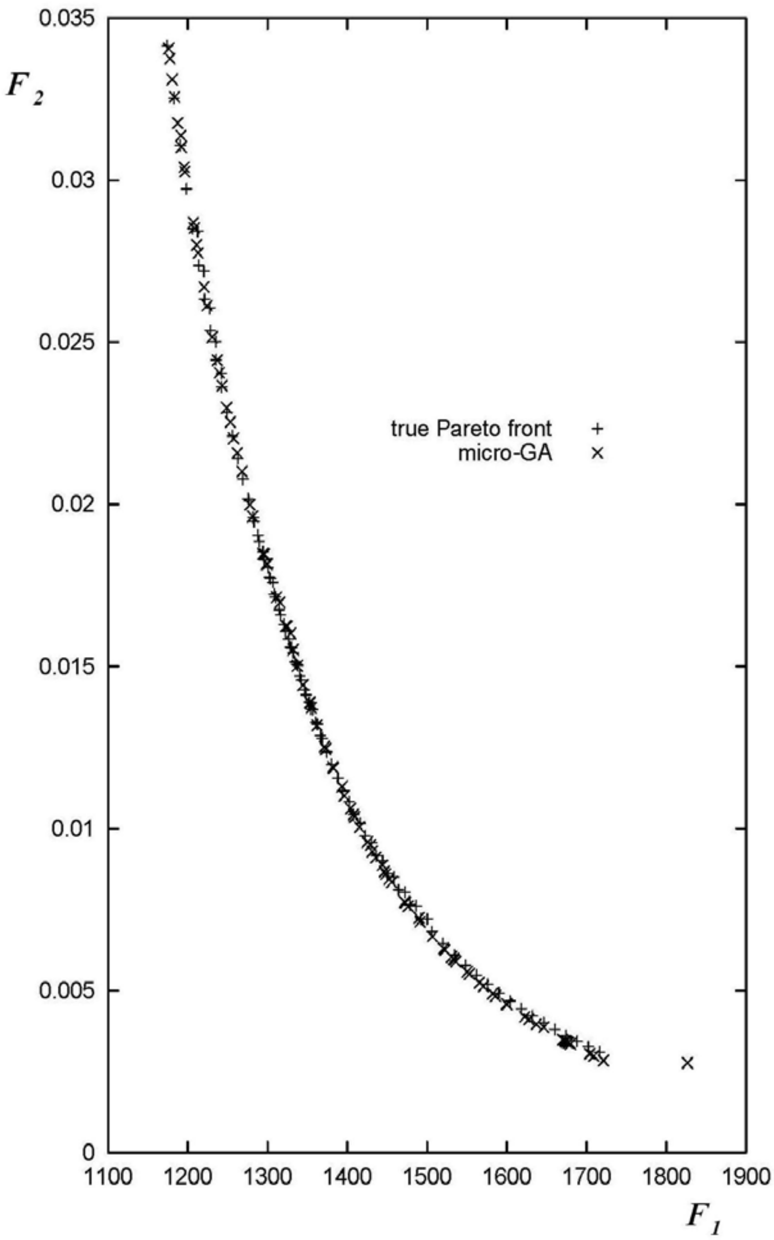

Figure 7. True Pareto front vs. front obtained by micro-GA

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Some examples of structural optimization problems modeling

Piotr Berkowski
Building Engineering Institute, Wrocław University of Technology, Wrocław, 50-370, Poland

Summary

Some examples of different optimization problems models that were under interest of the author during some past years are presented in a paper. These problems concern: discrete optimization of steel frames with accounting for the second order effects in structural analysis, shape optimization of section under torsion by using BEM, and evolutionary structural optimization method for shape and topology strength optimization.

KEYWORDS: discrete optimization, plane frames, linear analysis, P-delta method, shape optimization, torsion problem, BEM, evolutionary structural optimization

1. DISCRETE SYNTHESIS OF STEEL FRAMES ACCOUNTING FOR P-DELTA EFFECTS

1.1. Mathematical formulation of the optimization problem

In the analysis and design of multi-storey steel frames it is necessary to consider the influence of nonlinear geometrical effects which are caused by vertical loads acting on horizontal displacements of the structure and on deflections of its columns. These additional effects generally occur in the overturning and torsional moments, and are known as P-delta effects [1]. As these effects are represented by changes in the internal forces distribution over the structure, and by changes in their values, they also have an influence on the results of the minimum-weight design of high-rise frames.

The design problem is formulated as follows [2]:

Obtain the minimum-material volume design of the structure taking into consideration the influence the second-order P-delta effects.

In the proposed optimization model values representing the cross-sectional member dimensions are assumed as a design variable vector **X** and material properties as parameter vector **P**. For the I-welded sections four simple design variables are taken: web plate and flange plate width and thickness. Set of constraints is defined

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by design constraints fixed on design variables and by behaviour constraints: displacements, stresses, and local stability in form of the mathematical formulas:

Geometrical constraints: $Q_g(\mathbf{X}, \mathbf{P})$

$$x_{imin} = x_{i1} < x_{in} < x_{imax},\ x_{in} - x_{in-1} = \Delta x,\ x_{imax} - x_{imin} = 2^{q}\cdot\Delta x_i, \qquad (1)$$

$$n = 1\ (\min), \ldots, k\ (\max),\ i = i, \ldots j,\ q = 1, 2, \ldots, I.$$

Stress constraints: $Q_s(\mathbf{X}, \mathbf{P})$

$$\sigma_{k1}(\mathbf{X}, \mathbf{P})/R_{k1} - 1 <= 0,\ \ k = 1, \ldots, d. \qquad (2)$$

Local stability geometrical and stress constraints: $Q_{ls}(\mathbf{X}, \mathbf{P})$

$$\sigma_{l2}(\mathbf{X}, \mathbf{P})/R_{l2} - 1 <= 0,\ f_t(\mathbf{X}, \mathbf{P}) <= a_t,\ l = 1, \ldots, d,\ t = 1, \ldots, f. \qquad (3)$$

Displacement constraints: $Q_d(\mathbf{X}, \mathbf{P})$

$$v_{mh}(\mathbf{X}, \mathbf{P}) <= v_{mhp},\ v_{nv}(\mathbf{X}, \mathbf{P}) <= v_{nvp},\ m = 1, \ldots, g,\ t = n, \ldots, h. \qquad (4)$$

In the equations (1) to (4): x_{imin}, x_{imax}, Δx_i – minimum, maximum, and constant step between design variables; σ, R – current stresses and their permissible values, f, a – current geometrical constraint value and its permissible value, v, v_p – current displacements and their permissible values.

It is obvious that to obtain the required optimal solution – the optimal structure – the following relation must be fulfilled:

$$Q_g \cap Q_s \cap Q_{ls} \cap Q_d \neq \{0\}. \qquad (5)$$

As an objective function volume of the material used for a structure is taken. Optimization problem is then formulates in the following term:

Find the design variable vector X* in the feasible set (5) , with parameters **P**, which minimize the value of the global objective function.

1.2. Mathematical programming technique

Well-known discrete programming method called “backtrack” [3] was used to find the optimal vector **X***. This combinatorial method that can solve nonlinear constrained function minimization problem by systematical search procedure was very useful in the presented optimization problem.

1.3. Examples of synthesis process

The process of discrete synthesis of steel frames was implemented in a computer program for second-order analysis. Optimization process was conducted until the difference of values of global objective function for two successive steps was less

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than the specified tolerance (for example 0,1%). As an example the results of optimization of two-bay three-storey frame are presented in Figures 1 and 2.

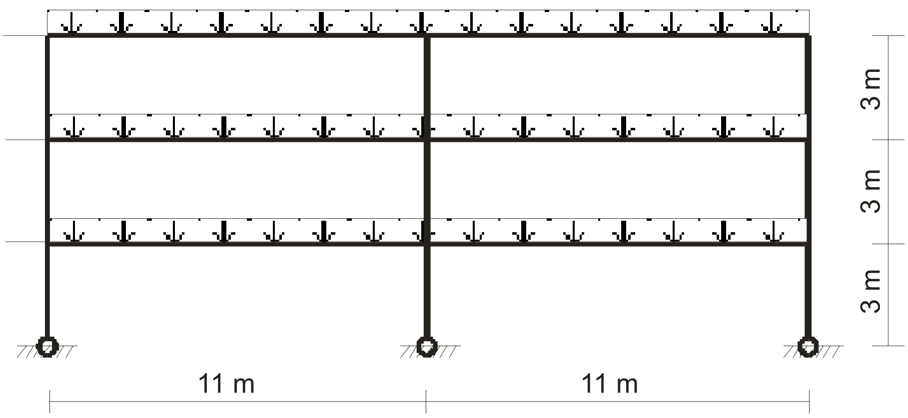

Figure 1. Optimization problem

The results of discrete synthesis of steel frames with accounting for P-delta effects in structural analysis shown that for a considered class of structures these effects led to obtain slightly “heavier” structures and, what was more important, led to material and forces redistribution in structural elements. These changes caused overstress in some elements that were optimized with linear analysis.

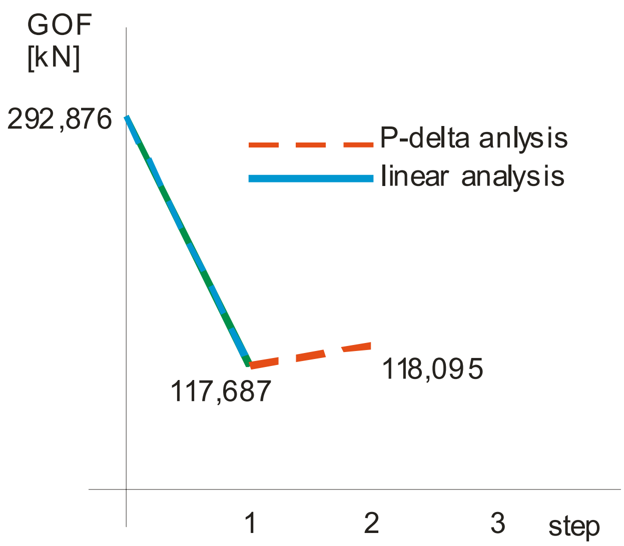

Figure 2. Synthesis results – iteration process

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2. SHAPE OPTIMIZATION OF SECTIONS UNDER SAINT - VENANT TORSION

2.1. Formulation of torsion problem for isotropic solids with BEM

The solution of the Saint-Venant torsion problem was based on that formulated by Gracia [4] and Gracia nad Doblaré in [5, 6], where a general problem of shape optimization of 2D elastic bodies based on the boundary element method was presented.

To formulate the Saint-Venant torsion problem for isotropic and homogenous solids and for multiply-connected domains $\mathbf{W}_i$ the so-called Prandtl function was used leading to a Poisson equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2 \quad \text{in } \Omega, \ \phi_i = k_i \text{ in } \Gamma_i, i = 0,1,\ldots,N, \tag{6}$$

where N is the number of boundaries $\mathbf{G}_i$,

$$\int_{\Gamma_i} \frac{\partial \phi}{\partial \mathbf{n}} d\Gamma_i = -2A_i, i = 1,\ldots,N \tag{7}$$

where:

$$A_i = \frac{1}{2} \sum_{j=1}^{N^i} (\mathbf{r}_j \bullet \mathbf{n}_j) L_j \tag{8}$$

is the area enclosed by each internal boundary, N^i is the number of elements of each internal boundary, L_j is the length of the element 'j', $\mathbf{n}_j$ is the normal to it, and $\mathbf{r}_j$ is the radius-vector between the element and the origin of coordinates.

The introduction of the second Green's identity between the Prandtl function (6) and the fundamental solution of the Laplace equation led to an alternative formulation of Equation (6) in terms of boundary integrals [10,11,12]:

$$c(Q)\phi(Q) + \int_{\Gamma} \phi \frac{\partial}{\partial \mathbf{n}} (\ln \frac{1}{r}) d\Gamma = \int_{\Gamma} \frac{\partial \phi}{\partial \mathbf{n}} \ln \frac{1}{r} - \int_{\Omega} \nabla^2 \phi \ \ln \frac{1}{r} d\Omega \tag{9}$$

where the constant *c* takes values depending on situation of point Q on the boundary and r is the radius-vector joining boundary points and the coordinate system origin.

Using (6) Equation (9) was transformed to the BEM basis:

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$$c(Q)\phi(Q) - \int_{\Gamma} \phi \frac{\mathbf{r} \bullet \mathbf{n}}{r^2} d\Gamma = -\int_{\Gamma} \frac{\partial \phi}{\partial \mathbf{n}} \ln r + \int_{\Gamma} (\frac{1}{2} - \ln r)(\mathbf{r} \bullet \mathbf{n}) d\Gamma \tag{10}$$

and, by the similar way, a formulation for torsional stiffness was obtained:

$$D = -G\left[\frac{1}{4}\int_{\Gamma} r^2 (\mathbf{r} \bullet \mathbf{n}) d\Gamma + \frac{1}{2}\int_{\Gamma} \frac{\partial \phi}{\partial \mathbf{n}} r^2 d\Gamma\right]. \tag{11}$$

The approximation functions ϕ were formulated was by using the simplest constant and linear approximations that led to the torsional stiffness expressed by:

$$D = -G\left[\frac{1}{4}\sum_{i=0}^{N}\sum_{j=1}^{N^{ei}} \int_{\Gamma_j} r^2 (\mathbf{r} \bullet \mathbf{n}) d\Gamma_j + \frac{1}{2}\sum_{i=0}^{N}\sum_{j=1}^{N^{ei}}\sum_{k=1}^{2} q_j^k \int_{\Gamma_j} \varphi_k r^2 d\Gamma_j\right]. \tag{12}$$

2.2. Formulation of optimization problem

The problem of shape optimization of sections under the Saint-Venant torsion can be stated in the considered case as it was formulated with details in [4, 5, 6, 7]:

Obtain the shape of the section with minimum area having a given torsional stiffness, and that fulfils some constraints related to the section geometry. It should be mentioned that as its dual problem the problem of finding the section with a given area and maximum torsional stiffness could be considered.

These additional constraints are as follows:

- some coordinates of the nodes can be bound;
- some boundary nodes can be fixed;
- the boundaries can not intersect;
- symmetry conditions have to be fulfilled.

The objective function f($\mathbf{x}$) is defined as:

$$f(\mathbf{x}) = \frac{1}{2}\sum_{i=0}^{N}\sum_{j=1}^{N^{ei}} (\mathbf{r}_j \bullet \mathbf{n}_j) L_j, \tag{13}$$

and the restriction corresponding to the torsional stiffness:

$$h(\mathbf{x}) = D_o + G\left[\frac{1}{4}\sum_{i=0}^{N}\sum_{j=1}^{N^{ei}} (I_j^i)_2 + \frac{1}{2}\sum_{i=0}^{N}\sum_{j=1}^{N^{ei}} q_j (I_j^i)_1\right], \tag{14}$$

where D is defined by Equation (12), and as design variables $\mathbf{x}$, in this first approximation to the optimization problem, the non-restricted boundary node coordinates were taken.

The method used in the optimization problem was based on the feasible direction method and the gradient projection one as it was described in [4, 5]. The restriction imposed on the torsional stiffness was transformed into "the constraint strip" by using the error bound ε_r and though the restriction is satisfied when:

$$(1-\varepsilon_r) \le \frac{D}{D_o} \le (1+\varepsilon_r), \tag{15}$$

where D is the torsional stiffness of the current design and D_o is the constraint stiffness. The method of the automatic constraint strip adjusting at each iteration step was applied.

2.3. Interactive graphical program for shape optimization

The overworked interactive program [8] for definition, visualization and modification of the shape optimization problem's data and results consists of two fundamental parts: graphical unit used to define and redefine the problem geometry (graphical pre- and postprocessors) and optimization unit that performs a design process, basing its analysis part on the BEM.

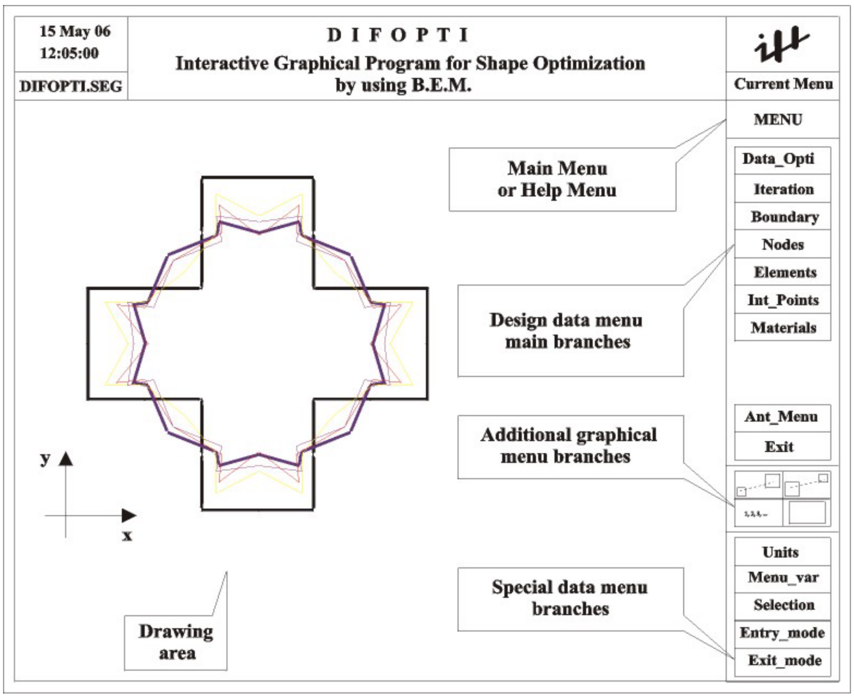

Figure 3. Optimization program graphical interface and section's shape evaluation during optimization process for the so-called "Greek cross"

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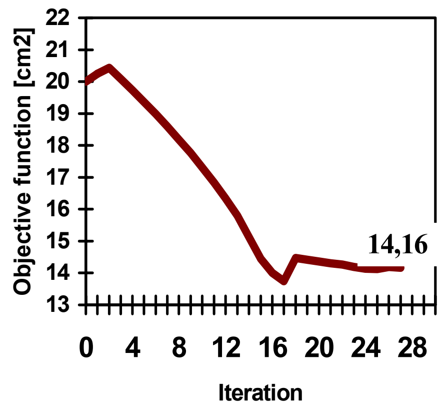
A general idea of an interactive work with the program consists in giving to a program user a possibility to define graphically the geometry of an initial shape to be optimized and then to have a chance to observe the optimization process at any iteration step.



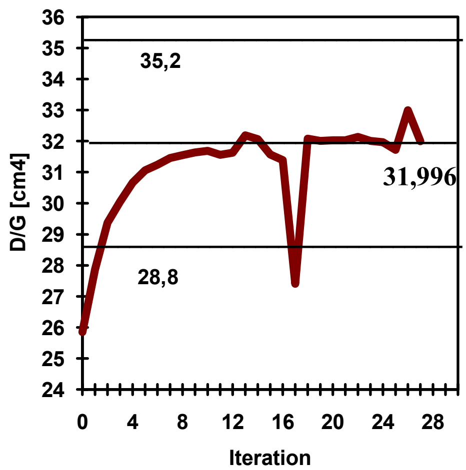


Figure 4. Evaluation of objective function and restriction during optimization

The graphical unit of the program contains a group of geometrical tools (calculating of boundary elements' length, areas closed by boundaries and angles between elements, detection of boundary intersections and mesh redefinition), completed by those of the visual presentation of optimized shapes (drawing of boundaries with different zoom levels, graphical presentation of the objective function and restriction evolution).

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An example of realized optimization process refers to the shape optimization of simply-connected domain that is so-called "Greek cross" with an obvious final result in the shape of a circle (Figure 3).

The optimization process converged rapidly with the constraint value D_0/G always inside the constraint strip (Figure 4). However, the boundary of final design is not very smooth because of lack of the mesh redefinition in case of appearance of geometrical mesh irregularities (Figure 3).

3. EVOLUTIONARY STRUCTURAL OPTIMIZATION

3.1. Formulation of the method

Evolutionary Optimization Method (ESO) is based on the simple concept that removing step-by-step inefficient material [9] leads to the optimal shape of the structure. This process is controlled by coefficients that define from which part of the structure, how many, and when the material is removed from the structure. In every case all the restrictions are fulfilled. This method is very useful in shape optimization problems.

This method was applied to solve strength shape optimization problems [9, 10]. Optimization with minimum material criteria leads to structures shaped in accordance with principal stresses trajectories in equivalent shield structure (structural domain) with identical geometrical and boundary conditions as searched structure. ESO process for that kind of problems can be described as follows:

1. Creation of finite element mesh in a initial domain.
2. Structural analysis with FEM to find stress distribution. In cases presented below [8, 9] for plane stress model Huber- von Mises stress was defined:

$$\sigma^{hvm} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \cdot \sigma_y + 3 \cdot \tau_{xy}^2} \qquad (16)$$

3. Calculation of stresses in every finite element σ_e^{hvm} and definition of material rejection criteria:

$$\sigma_e^{hvm}/\sigma_{max}^{hvm} < RR_i, \qquad (17)$$

   where σ_{max}^{hvm} is domain maximum stress value and RR_i is a current rejection ratio (for example 1%).

4. Removal of finite elements with stress that satisfy equation (17) until the process is steady (by assigning zero stiffness value to the element). It means that with the same values of R_i there are no elements that can be deleted.

5. Introducing the so-called “evolutionary rate” ER ($RR_{i+1} = RR_i + ER$) and repetition of FEM and stress analysis, and element removal until new steady stage is reached (for example E = 1%).
6. Optimization process is conducted until, for example, when there are no elements in the domain with stresses $\sigma_e^{hvm} < 25\% \cdot \sigma_{max}^{hvm}$.

3.2. ESO optimization examples

First example is a well-known structural optimization problem of the two-bar frame subjected to a single load placed in the middle of the long domain side. The optimal ratio of H/L can be obtained analytically and is H/L =2 and a structure is a pin-jointed frame.

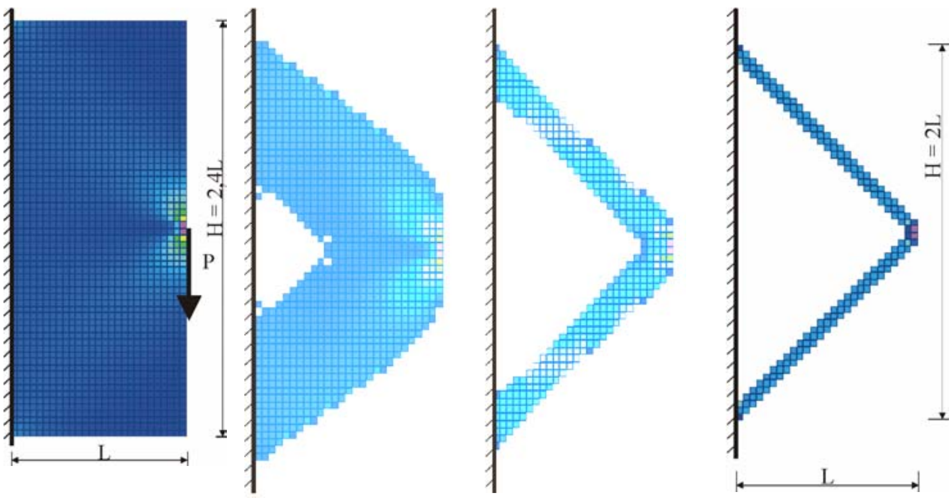

Figure 5. Design domain. ESO solution for R = 5%, 12,5%, & optimal solution for R= 30%

Second example is ESO solution for a Michell type structure with two fixed ends.

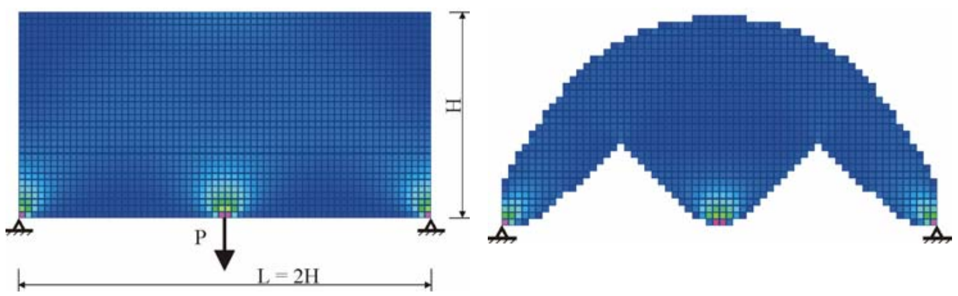

Figure 6. Design domain and ESO solution for R = 5%.

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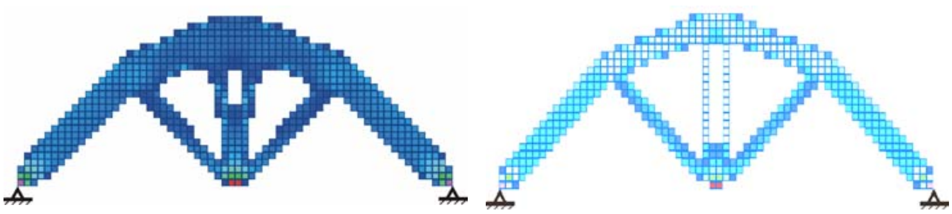

Figure 7. ESO solution for R = 10% and optimal solution for 15%.

4. FINAL REMARKS

A summary of author's interest in different problems of optimization that evaluated from simple discrete frame structural optimization with second order analysis, by shape optimization with BEM to the shape optimization with evolutionary method was presented above.

All the described optimization problems were also introduced into didactic programs in course of structural optimization and computer aided engineering allowing students to know a different way of structural designing.

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