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Numerical modeling for homogenization of masonry structures

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Summary

In the present study, equivalent elastic properties, strength envelope and different failure patterns of masonry material are homogenized by numerically simulating responses of a representative element under different stress conditions.

The representative volume element provides a valuable dividing boundary between the discrete model and the continuum model. This paper presented a computational homogenization technique of masonry.

KEYWORDS: masonry structures, homogenization, computational simulations.

1. INTRODUCTION

In recent years growing attention has been paid by researches in structural mechanics to masonry structures with the intent to provide theoretical and numerical tools for better understanding the complex mechanical behavior of such structures. The complex mechanical behavior of masonry structures depends strongly on the composite nature of masonry material.

Masonry is constituted by blocks of natural or artificial material jointed by dry or mortar joints; the latter are the weakness – areas of such a composite material and notably affect the overall response of the assembly with a number of kinematical modes at joints such as sliding, opening – closing and dilatancy.

Generally, two different methods have been developed to perform linear and non-linear analyses of masonry structures (Szołomicki 1997). The macro-modeling approach intentionally makes no distinction between units and joints but smears the effect of joints presence through the formulation of a fictitious homogeneous and continuous material equivalent to the actual one which is discrete and composite. The alternative micro-modeling approach analyses the masonry material as a discontinuous assembly of blocks, connected each other by joints at their actual position, being simulated by appropriate constitutive models of interface (Zucchini 2002).

Another direction, presented in this paper, is a method which resorts to homogenization technique. The homogenization method that would permit to

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establish constitutive relations in terms of averaged stresses and strains from the geometry and constitutive relations of the individual components would represent a major step forward in masonry modeling.

In this paper, a typical unit of masonry is selected to serve as a representative volume element. Both the bricks and mortar joints are idealized as isotropic material having their own properties such as stiffness, strength and damage characteristics. In the homogenization process, failure of the individual material in the unit is divided into three modes: tensile failure of mortar joint, shear failure of mortar joint and brick, and compressive failure of brick. A fracturing law is associated with the tensile failure of the mortar joint, while the shear failure accounts for the variations of shear strength as a function of normal stress.

2. FORMULATION OF REPRESENTATIVE VOLUME ELEMENT

The homogenized constitutive law is determined by studying the behaviour of the representative volume element (RVE) which is the cell of periodicity in the case of periodic media (Galvanetto 1997).

Such a RVE plays in the mechanics of composite material the same role as the classical elementary volume of continuum mechanics; therefore, in general, a homogenized approach is successful if the size of the cell is small compared with that of the structure.

The representative volume element of masonry should include all the participant materials, constitute the entire structure by periodic and continuous distribution, and be minimum unit satisfying the first two conditions. Under conditions of an imposed macroscopically homogeneous stress or deformation field on the representative volume element, the average stress and strain fields are respectively:

$$\overline{\sigma}_{ij} = \frac{1}{V}\int_V \sigma_{ij} dV \tag{1}$$

$$\overline{\varepsilon}_{ij} = \frac{1}{V}\int_V \varepsilon_{ij} dV \tag{2}$$

where V is the volume of the representative volume element.

Based on the constitutive relations of the brick and the mortar materials, the equivalent stress-strain relations of the RVE are homogenised by applying various compatible displacement conditions on the RVE surfaces (Luciano 1997).

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3. DAMAGE MODEL FOR MORTAR JOINT AND BRICK

3.1. Damage of mortar joint

Most of the non-linear deformation in brick masonry, until failure, occurs only in the joints. Establishing a reliable material model for the mortar joint is very important for analysing the masonry load-bearing and deformation capacity.

The degradation of tensile strength in mortar joint can be expressed by an exponential approximation as:

$$\sigma = E_n \varepsilon_n, \qquad \varepsilon_n \le \varepsilon_0 \tag{3}$$

$$\sigma = \sigma_0 e^{-\alpha_n(\varepsilon_n - \varepsilon_{n0})/\varepsilon_{n0}}, \qquad \varepsilon_n \ge \varepsilon_{n_0} \tag{4}$$

where: α_n is a material parameter, ε_{n0} – threshold strain that initiates tensile fracturing of the material, σ_0 is the elastic limit stress of the mortar joint.

The exponential decay leads to the following total fracture release energy:

$$G_f^I = \int_0^{+\infty} \sigma d\varepsilon_n . \tag{5}$$

The differential of fracture energy is obtained as:

$$dG^I = \frac{1}{2}\sigma d\varepsilon_n - \frac{1}{2} d\sigma\varepsilon_n . \tag{6}$$

Thus tensile damage can be defined as the ratio of released fracture energy to the total fracture release energy as:

$$D^I = \frac{\int_0^{\varepsilon_n} dG^I}{G_f^I} . \tag{7}$$

In the compression – shear region of mortar joints is reasonable to use the Mohr – Coulomb criterion, which can be expressed as:

$$F(\sigma,\tau) = |\tau| + \mu\sigma - c(G^{II}) = 0 , \tag{8}$$

where: μ and c are frictional coefficient and cohesion, respectively; σ is a compressive stress; G^{II} is the dissipated plastic work that results from shear failure of mortar joint.

The incremental strain vector of mortar can be divided into elastic and plastic parts:

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p} \tag{9}$$

To avoid excessive plastic dilatancy, a non-associated flow rule is proposed. It is expressed as:

$$Q(\sigma,\tau) = \boldsymbol{\eta}(\tau) + \mu\sigma \tag{10}$$

where: η is parameter that scales the dilatancy.

The direction of plastic relative displacements is governed by the flow rule as:

$$d\varepsilon^{p} = d\lambda \frac{\partial Q}{\partial \sigma} \tag{11}$$

where $d\lambda$ is the plastic multiplier.

According to the traditional plastic flow rule the complete elasto-plastic incremental stress-strain relationship is presented as:

$$d\sigma = E^{ep} d\varepsilon \tag{12}$$

where:

$$E^{ep} = E^{e} - \frac{E^{e} \frac{\partial Q}{\partial \sigma} \frac{\partial F}{\partial \sigma^{T}} E^{e}}{-A + \frac{\partial F}{\partial \sigma^{T}} E^{e} \frac{\partial Q}{\partial \sigma}} \tag{13}$$

and A is a hardening parameter.

The plastic work done by the shear stress τ depends on the lateral compression σ. When the combination of (τ, σ) on the strength surface is expressed as (8), the incremental plastic work is the following:

$$dG^{II} = (|\tau| + \mu\sigma) d\varepsilon_{t}^{p} = \{|\tau| + \mu\sigma \quad 0\}\{d\varepsilon_{t}^{p} \quad 0\}^{T}. \tag{14}$$

The damage value at compressive-shear region can be expressed as:

$$D^{II} = \frac{G^{II}}{G_{f}^{II}} \tag{15}$$

where:

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$$G^{II} = \int_0^{+\infty} (|\tau| + \mu\sigma) d\varepsilon_t^p \tag{16}$$

The total dissipated energy due to friction can be calculated by the relationship of the shear stress and strain with no lateral compression on the surfaces. The relationship can be written as:

$$\tau = E_t \varepsilon_t \text{ when } \varepsilon_t \le \varepsilon_{t_0} \tag{17}$$

$$\tau = E_t \varepsilon_{t_0} e^{-\alpha_t(\varepsilon_t - \varepsilon_{t_0})/\varepsilon_{t_0}} \text{ when } \varepsilon_t \ge \varepsilon_{t_0} \tag{18}$$

where α_t is a material parameter and ε_{t0} is threshold shear strain of mortar joint without lateral compression. The cohesion c is expressed as:

$$c = \left(1 - \frac{G^I}{G_f^I}\right)\left(1 - \frac{G^{II}}{G_f^I}\right) c_0 = \left(1 - D^I\right)\left(1 - D^{II}\right) c_0 . \tag{19}$$

3.2. Damage of bricks

Damage of bricks is composed of compressive crushing and tensile splitting due to high compression. According to the isotropic damage theory, the secant constitutive tensor can be written as:

$$\Lambda_{ijkl}(D) = \Lambda_{0ijkl}(1 - D) \tag{20}$$

where: Λ_{0ijkl} is initial stiffness of material and D is a damage.

The damage scalar consists of two parts, namely D_t due to tensile damage and D_c due to compressive damage. It is evaluated by the combination of:

$$D = A_t D_t + A_c D_c, \ \dot{D} > 0 \tag{21}$$

where: A_t and A_c are the balancing coefficient characterizing tension and compression, respectively. Damage scalars D_t and D_c corresponding respectively to damage measured in tension and compression can be expressed again in exponential approximation as:

$$D_t = 1 - e^{-\beta_t(\tilde{\varepsilon}^+ - \varepsilon_0^+)/\varepsilon_0^+} \tag{22}$$

$$D_c = 1 - e^{-\beta_c(\tilde{\varepsilon}^- - \varepsilon_0^-)/\varepsilon_0^-} \tag{23}$$

where $\tilde{\varepsilon}^+$ and $\tilde{\varepsilon}^-$ are the equivalent tensile and compressive strains.

4. FINITE ELEMENT IMPLEMENTATION

The material model proposed above is used into finite – difference program. The finite element analysis is based on this decoupling of the macroscopic and microscopic displacement fields. In the present analysis, the material is considered in elastic state before the directional strain of the mortar joint reaches the threshold strain.

In post-failure state, the stress will decrease with increase of uniaxial strain. The material parameters of brick can also be determined by uniaxial tensile and compressive test data. The computational models of RVEs in the present numerical analysis are shown in Figure 1. The brick and mortar are discretized individually. The brick size is 250 x 65 x 120 mm and mortar thickness is 10 mm.

In numerical simulation, vertical and horizontal displacements are applied to the RVE surfaces. The advantage of using displacement boundaries can not only avoid incompatible deformation between RVEs, but also makes it possible to obtain a complete monotonic stress-strain curve through the homogenization process.

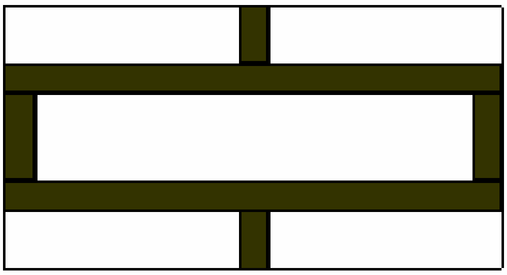

Figure 1. Representative volume element of masonry

5. NUMERICAL APPLICATION

The theory presented above has been used in the following example. The masonry panel schematically reported in Figure 2, subjected to pure shear loading is analyzed. This is characterized by the following geometrical parameters: H = 3000 mm, B = 2000 mm and material characteristics (Tables 1 and 2).

In Figure 3, the damage and the minimum principal stress maps for the prescribed displacement u = 5 mm is plotted for analyzed panel. It can be noted that the failure mechanism is characterized by the formation, growth and propagation of inclined damage bands, as it typically occurs in structures subjected to horizontal forces. Damaging process in analysed example is concentrated in a single band.

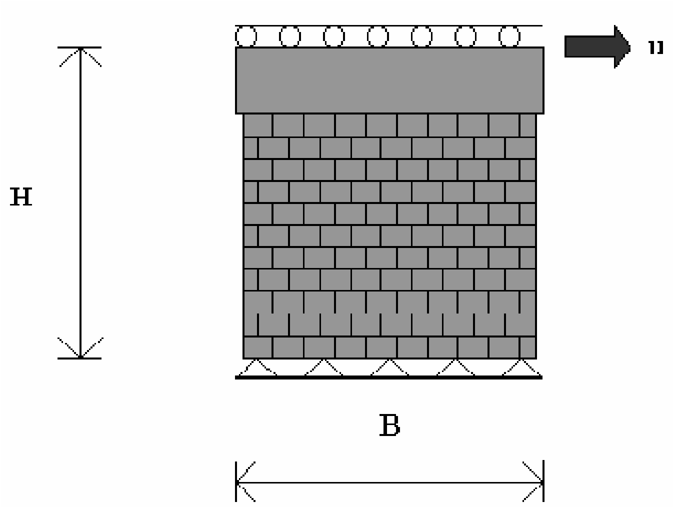

Figure 2. Analyzed masonry panel

Table 1. Material parameters for brick

Material	E MPa	G MPa	ν -	σ_t MPa	σ_c MPa	ε_0^+ -	ε_0^- -
Brick	12000	4700	0,15	2	50	0,0002	0,0004

Table 2. Material parameters for mortar

Material	E_n MPa	E_t MPa	σ_0 MPa	c_0 MPa	μ -	ε_{n0} -	ε_{t0} -
Mortar	2,0	1,0	0,9	1,1	0,8	0,0004	0,001

It can be noted that the mechanical response of wall subjected to shear loading is characterized by:

- an initial elastic response,
- a first step softening branch due to the damage propagation concentrated in place where the maximum tensile strains occur,
- a hardening phase during which the plastic evolution process becomes more significant than the damage one.

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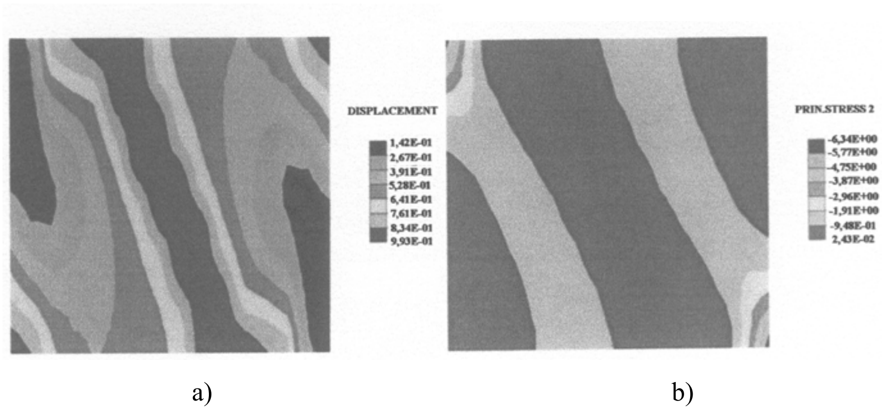

Figure 3. Numerical analysis of masonry panel: a) damage distribution and b) minimum principal stress distribution for u = 5 mm

6. CONCLUSION

In this paper the authors presented a numerical homogenization technique of masonry. The strength behavior and damage behavior have been obtained by numerical analysis of the representative volume element responding to various boundary conditions. They have been implemented into a continuum plastic damage model. Failure of masonry can be described into three types: tensile failure due to mortar tensile damage, shear failure of brick and mortar and compressive failure of brick. It should be noticed that the RVE employed in a general homogenization process should be smaller than the whole masonry structure. Otherwise the edge effect will affect the macro-material properties significantly, and in that case the discrete element method is preferred.

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