Structural Mechanics

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The numerical model for reinforced concrete structures and analysis using finite elements method

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Summary

The purpose of this paper is to review model for finite element techniques for non-linear crack analysis of reinforced concrete beams and slabs. The non-linear behavior of concrete and steel were described. Some calculations of "self-stress" for concrete and reinforced concrete beam was made. Current computational aspects are discussed. Several remarks for future studies are also given.

The numerical model of the concrete and reinforced concrete was described. The paper shows the results of calculations on a reinforced concrete plane stress panel with cracks. The non-linear, numerical model of calculations of reinforced concrete was assumed. Using finite elements method some calculations were made. The results of calculations like displacements, stresses and cracking are shown on diagrams. They were compared with experimental results and other finding. Some conclusions about the described model and results of calculation are shown.

1. NON-LINEAR BEHAVIOR OF CONCRETE AND STEEL

Reinforced concrete structures exhibit very complicated behavior different from steel structures. The structural system is composed of different materials, such as cement, steel bars, aggregate etc. Moreover each material shows various physical phenomena.

The non-linear behavior of entire structures can be considered to be accumulated from cracking of concrete, non-linear material properties of concrete under compression, time-dependent deformations due to creep and shrinkage, bond behavior, yielding and strain hardening of steel essentially. Progressive cracking of concrete is surely the most important component of the non-linear response of reinforced concrete structures in normal service state.

The experiments [3,4] show that under cyclic loads concrete and reinforced concrete structures response like linear-elastic materials. The linear-elastic behavior of concrete elements under cyclic loads is the base to assume that total deformations is the sum of elastic, residual, plastic and creep deformations as shown in Figure 1.

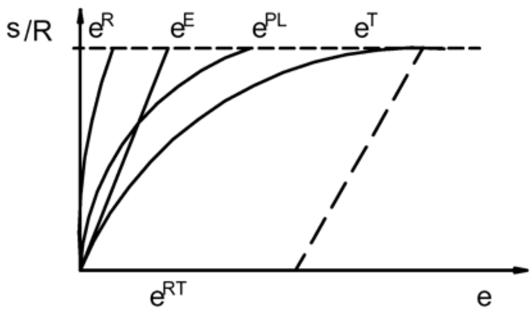

Figure 1. Stress-strain relationship for concrete

The total residual ε^{RT} deformation is the sum:

$$\varepsilon^{RT} = \varepsilon^{R} + \varepsilon^{PL} + \varepsilon^{R\varphi} \tag{1}$$

where:

ε^{RT} - residual deformation,

ε^{PL} - plastic deformation,

$\varepsilon^{R\varphi}$ - creep deformation.

The creep deformation described using Rush [2] theory. It was assumed that residual creep deformation is 80% of total. The non-linear residual ε^{R} deformation, because of plane cross section, gives "self-stress" σ^{R} inside the element:

$$\int \sigma^{R}\, dF = 0, \qquad \int \sigma^{R} z\, dF = 0 \;. \tag{2}$$

Using the equilibrium of forces and moments in the cross section (Eq. 2), the constants A, B and "self-stress" was calculated:

$$\sigma^{R} = E_b \varepsilon^{ED} = E_b (A + Bz - \varepsilon^{R}) \tag{3}$$

We can write the total stress as a sum of self-stress σ^{R} and easy to obtain linear elastic stress σ^{E}:

$$\sigma^{RT} = \sigma^{R} + \sigma^{E} \tag{4}$$

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The steel reinforcement is stressed only in one direction. The material is represented by a bilinear model which may either be elastic - perfectly plastic or strain-hardening as shown in Figure 2.

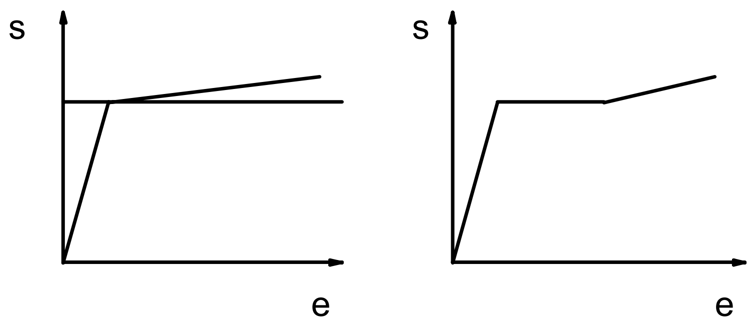

Figure 2. Stress strain model for steel

2. SELF-STRESS IN CONCRETE ELEMENT

The self-stress was calculated for bending concrete beam Figure 3.

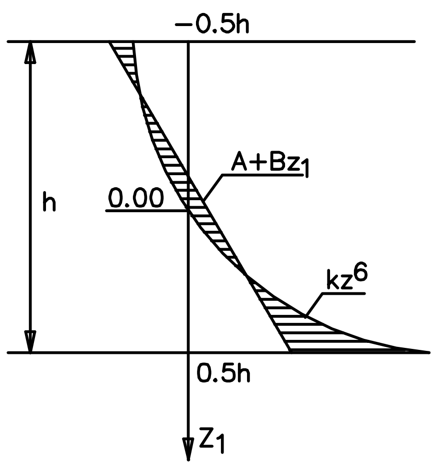

Figure 3. Self-stress in concrete beam

The course of residual strain was assumed as known and described:

$$\varepsilon^R = \begin{cases} -kz^6 & \text{for } z<0 \\ kz^6 & \text{for } z>0 \end{cases}, \qquad (5)$$

where: $z = \sigma / R_{bk}$ for $\sigma < 0$,

$z = \sigma / R_{bzk}$ for $\sigma > 0$.

Using the equilibrium of forces and moments (Eq. 2) the constants A, B was calculated:

$$\int \sigma^R \, dF = \int \left[A + Bz_1 - \varepsilon^R(z_1) \right] b \, dz_1 = 0 \quad ,$$

$$\int \sigma^R z_1 \, dF = \int \left[A + Bz_1 - \varepsilon^R(z_1) \right] bz_1 \, dz_1 = 0 \quad , \tag{6}$$

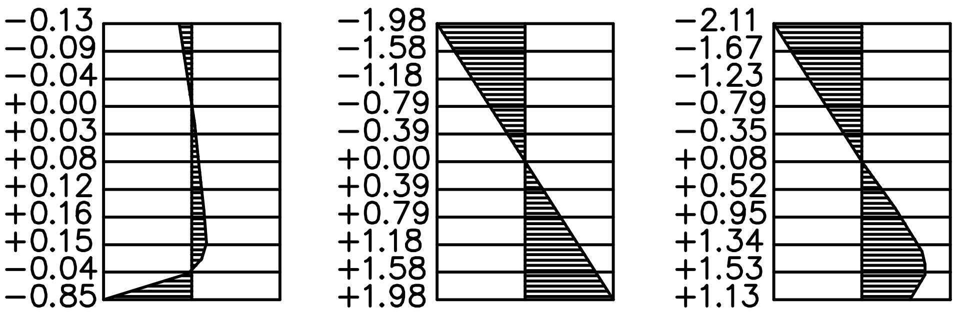

Figure 4. Results of calculation the self-stress for concrete B-15 (f_c=15.0 Mpa, f_{ct}=1.40 Mpa, E_c=23.1*10^3MPa)

$$A = 1 / F_b \int \varepsilon^R(z_1) b \, dz_1 \; ; F_b = bh \, ,$$

$$B = 1 / I_b \int \varepsilon^R(z_1) z_1 b \, dz_1 \; ; I_b = bh^3/12 \, . \tag{7}$$

Some calculations for different concrete was done The results of calculations of self-stress, elastic stress and total stress are shown on Figure 4.

3. SELF-STRESS IN REINFORCEMENT CONCRETE ELEMENT WITH CRACK

The self-stress in reinforcement concrete bending beam was calculate in the same way as in concrete element Figure 5.

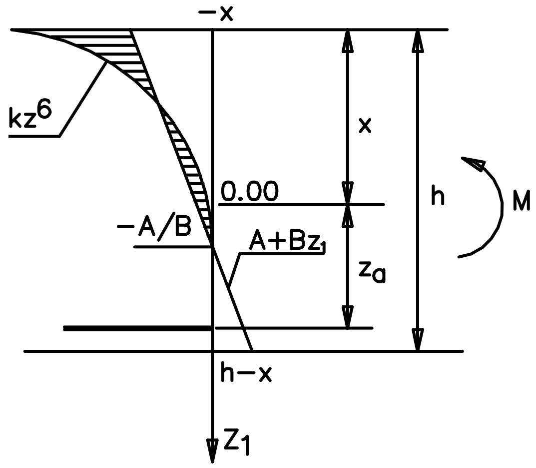

Figure 5. Self-stress in reinforced concrete beam with crack

The residual strain-stress relationship described the same equals (Eq. 4) like for concrete element. Using condition (2) the constants A, B was calculated:

$$E_b\int(A+Bz_1-\varepsilon^R(z_1))\,dF\ +\ E_aF_a(A+Bz_a-\varepsilon_a^R)=0\ ,$$

$$E_b\int(A+Bz_1-\varepsilon^R(z_1))\,z_1b\,dz_1\ +\ E_aF_a(A+Bz_a-\varepsilon_a^R)z_a=0\ ,\qquad(8)$$

where: ε_a^R - residual strain for steel.

Some numerical calculations for different causes were done. The results of these calculations are shown in Figure 6.

4. NON-LINEAR ANALYSIS USING F.E.M.

The Finite Elements Method was used to calculate the non-linear effects in reinforced concrete elements. The rectangular elements were used with stiffness matrix calculated by Rockey [5]. The first steep is taken as the linear solution using known relationship of F.E.M. [6]:

$$[K]\{\delta\}\text{-}\{R\}=0\ ,\qquad(9)$$

where: [K] - stiffness matrix,

{δ} - displacement,

{R} - external forces.

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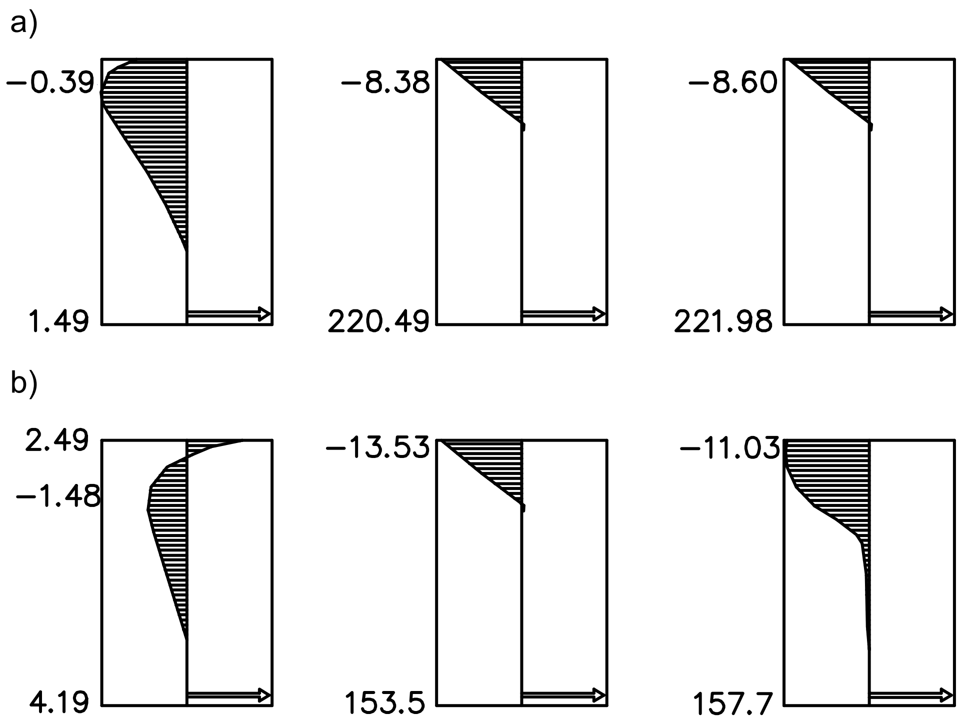

Figure 6. Stresses in cracked concrete element, concrete B-15 (see Figure4), steel f_y = 220 MPa. a) μ = 0.5%, M = 29 kNm, b) μ = 2%, M = 75 kNm

In Eq. 9 the linear behaviour of materials was assumed:

$$\{\sigma\}=[D](\{\varepsilon\})-\{\varepsilon o\})+\{\sigma o\} \quad , \tag{10}$$

$$F(\{\sigma\},\{\varepsilon\})=0 \quad . \tag{11}$$

The non-linear effects were calculated using iteration and changing of the external forces {R}. The external forces were calculated on the basis of initial strain and initial stress which described the cracks or non-linear material behavior. This method needs no necessity of changing of the stiffness matrix.

The reinforcement was described as linear elements which are added to the stiffness matrix. Before the cracks appear the concrete strain and the steel strain are equal. After cracking the steel elements take over the stresses from the concrete. This stresses are added to the external forces as initial stresses.

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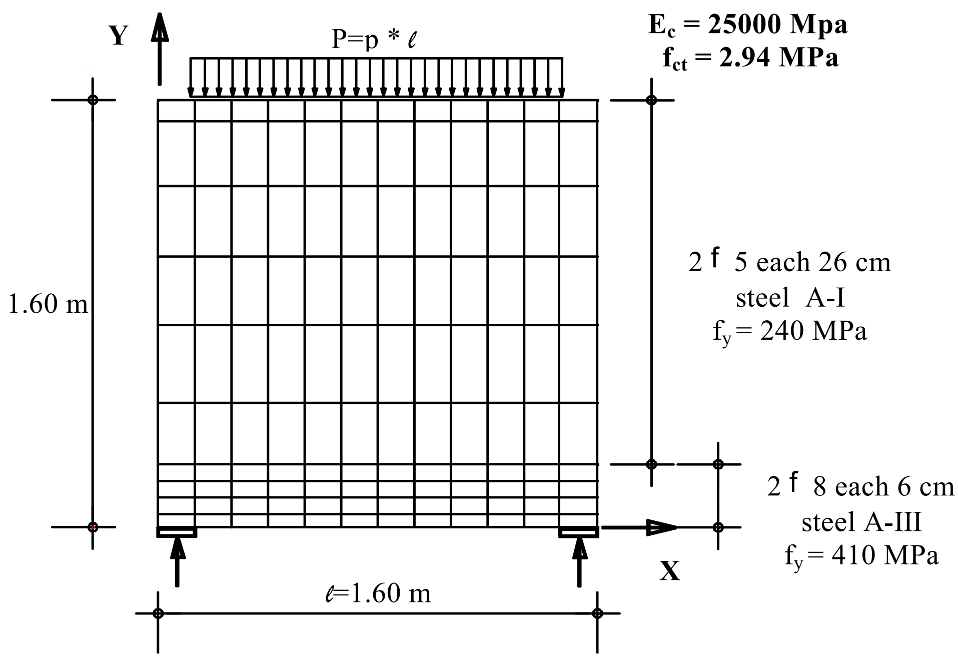

Figure 7. The schema of calculated RC panel

5. ANALYSIS OF RC DISK WHICH WAS TESTED BY LEONHARDT AND WALTHER [7]

The purpose method was examined on calculations of the RC panel (Figure7) tested by Leonhardt and Walther [7]. The same panel was calculated by Floegl [8], Buyukozturk [9] and Lewiñski [10]. So, there is the material to compare the results of analysis.

The results of numerical calculations are shown on Figures 8-11. Figure 8 shows the propagation of cracks under different loading. The first cracks was observed for P=400 kN. In each level of the loading the number of cracks and the width of the cracks vary as shows the figure.

In Figure 9, the relationship between loading and displacement of the panel as compared with other finding is shown. The good compatibility was notice.

Figure 10 and 11 show the comparison of stresses in steel bars calculated by authors with experimental findings and other calculations.

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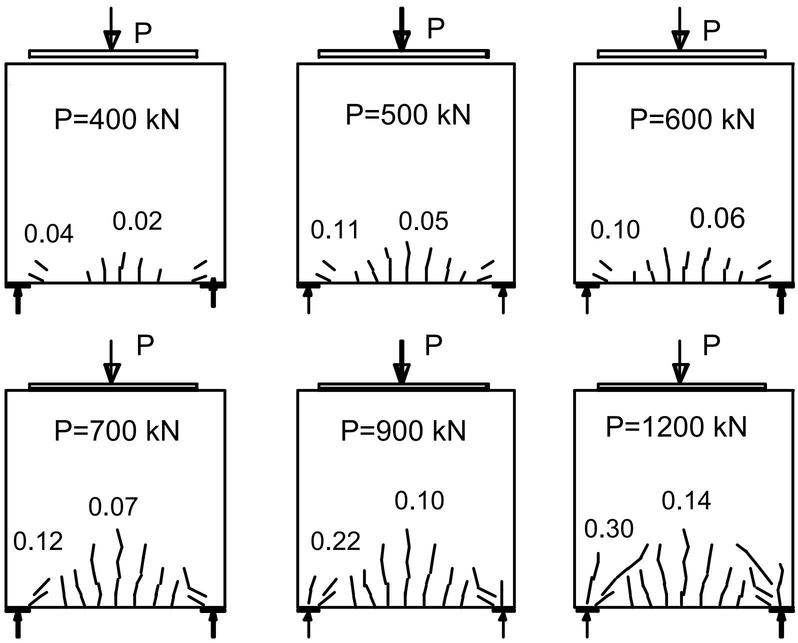

Figure 8. The propagation of the cracks under different level of loading

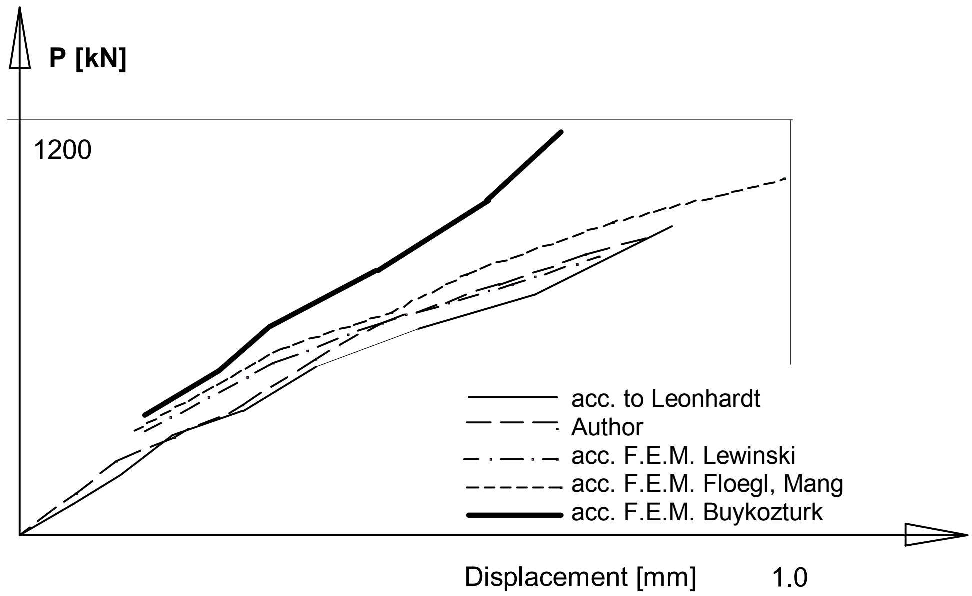

Figure 9. The relationship between loading and displacement

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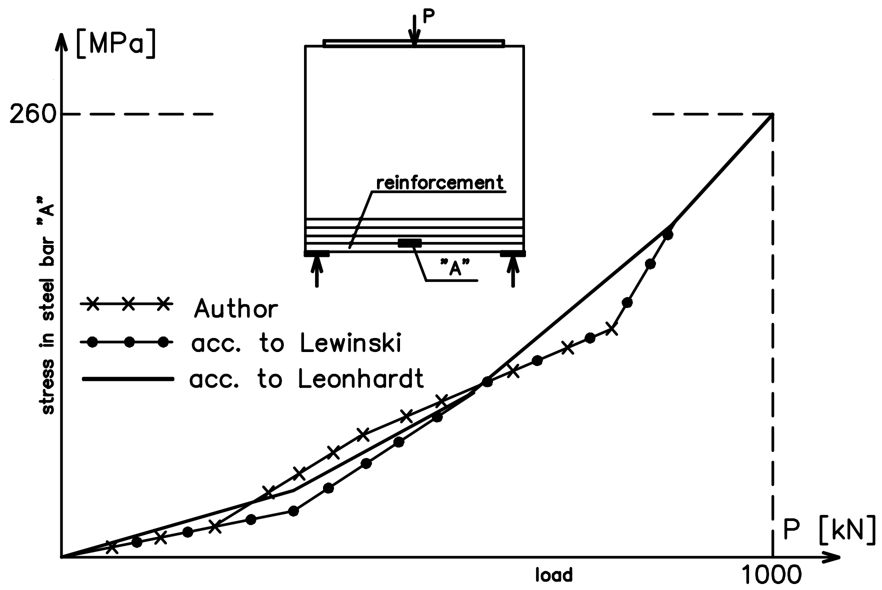

Figure 10. Comparison of the stresses in reinforcement of the panel with other finding

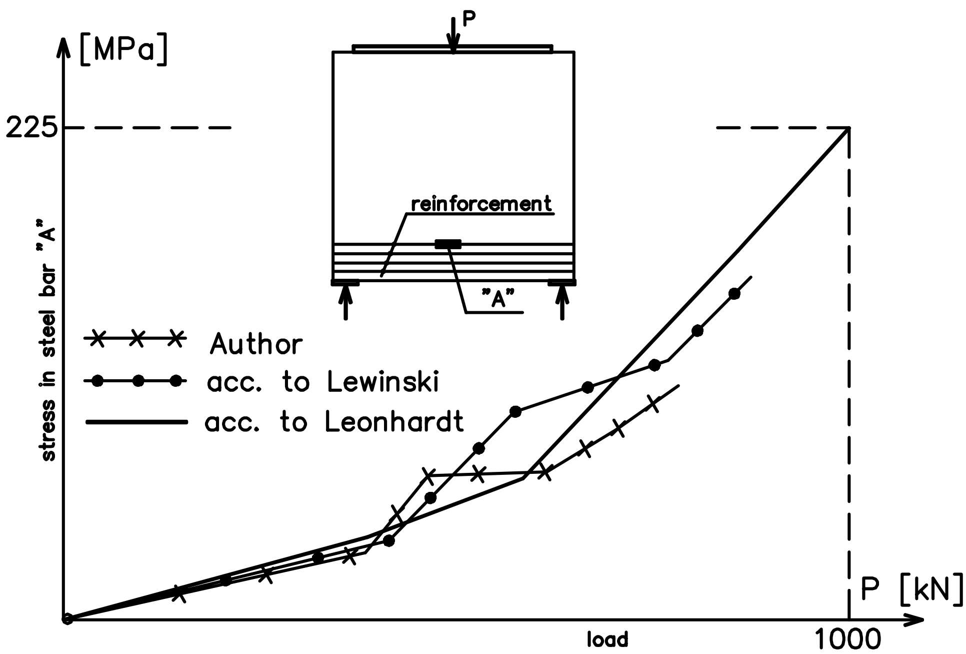

Figure 11. Comparison of the stresses in reinforcement of the panel with other finding

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4. CONCLUSIONS

The mathematical idealization of cracked reinforced concrete structures is very difficult. The presented model of non-linear behavior of reinforced concrete can be used to numerical analysis with the finite element method. It may give relatively quick solution because of no necessity to change the stiffness matrix and solve equations several times.

Descriptions of all non-linear behavior of materials, cracks, self-stresses treated as initial strains gives only different right sides of standard equals. Therefore the iteration of cracks and loads do not change the stiffness matrix. All that may preference finite element method to numerical analysis.

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