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Numerical analysis of historical constructions

Cristina Emanuela Lanivschi[1], Graça Vasconcelos[2], Paulo Lourenço[2]

[1] *"Gh. Asachi" Technical University, Iasi, 700050, Romania*
[2] *Universidade do Minho, Campus of Azurém, Guimarães, 4800, Portugal*

Summary

All European countries are rich of monumental buildings and there is a considerable amount of existing residential masonry buildings in rural areas. It is frequent that old historical masonry structures present significant level of damage and must be repaired. In fact, conservation, rehabilitation and strengthening of the built heritage and protection of human lives are clear demands of modern societies [1]. This requires the identification of deficiencies and of damage of existing structures and appropriate intervention techniques. Numerical modelling of masonry structures is an important tool for evaluation of damage extent and a basis for a decision about the design of appropriate remedial measures.

The paper addresses the scope of numerical analysis of ancient masonry structures. The numerical modelling intends to assess the stability conditions of the salient elements of the façade of the Cathedral of Porto dated from the middle of 12th century. Although no important visible cracking is present, the deep decay of the granite stone and corrosion of steel bars that connect the stone pieces can induce stability problems of the local salient elements of the façade. The numerical modelling is performed with the help of a commercial finite element software program DIANA.

KEYWORDS: historical constructions, finite element analysis, smeared cracking model, load-displacement diagram, safety factor

1. INTRODUCTION

The Cathedral of Porto, as it is found today, is hybrid and frozen in a perpetual past, where re-combinations or architectural redesign seem hardly acceptable in the near future. In fact, the cathedral is a result of the previous restoration that represents today the real self of the cathedral, transformed into a monument. This is forged identity that cannot be considered a minor representation of the strength of a specific cultural period [1].

The restoration works carried out on the Cathedral during 20th century were mostly concentrated in the towers and in the roofs and façades of west and south wings.

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The central façade is composed of several salient elements, namely a central balcony and two lateral pinnacles, see Figure 1. The advanced degradation state of the granite of the columns below the referred structures led to the positioning of an auxiliary structure in order to avoid injury of people. This problem represented the main reason for the need of assessment the stability conditions of the main salient structural elements of the façade: the balcony and the pinnacles.

Figure 1. View of the two salient elements from the central part of the main façade of the Cathedral

2. DESCRIPTION AND VISUAL INSPECTION OF THE SALIENT ELEMENTS

The balcony substructure consists of an almost semi-circular slab, made of granite ashlars that seem to be bounded together with iron dowels. The slab is stiffened by three ribs of the same granite, a central one and two marginal. The loads that are considered to be applied in the balcony include the own weight and the massive granite balustrade at the top of seven granite columns that also seem to integrate steel dowels, see Figure 2.

Pinnacles are massive granite elements, but it is difficult to see whether the ashlars are connected with steel dowels or not. The main structural element of these assemblies is the shell, which takes over the loads from the two pinnacles and transmit them to the columns, see Figure 3.

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Figure 2. Details of the connection of the stones in the balcony's balustrade and between the ashlars of the slab

Even if no important cracking is observed on these elements, granite presents signs of weathering, which may result from combined action of cyclic variations of the moisture content, salts (considering also that the cathedral is situated close to the Douro river) and pollution.

As is shown in Figure 3, besides the grey spots on the surface of granite, superficial breaking off is also visible, which can be associated to some physical and chemical actions, leading to high porosity and consequently to the lowering of the mechanical parameters.

It is probable that the pinnacles' stone ashlars were connected to the battlement by mortar joints and iron dowels. However, a detachment of the pinnacles form the battlement is observed and can be attributed to deterioration of the mortar joints and corrosion of the steel bars. This can affect the deformation of the structure occurs and, thus, must necessarily be considered on the geometric model of the structure in terms of proper boundary conditions, see Figure 4.

Where structures have been changed, differences on the behaviour of the different periods of masonry may produce signs of distress. For investigating the degradations that occur in the salient elements of the façade and their connections, a GPR inspection was performed and is described in the next section.

3. GPR INSPECTION OF THE SALIENT ELEMENTS OF THE FAÇADE

The main objective of the geotechnical radar inspection (GPR) was the detection of the steel connectors that connect the various stone elements that constitute the church façade. The inspection was performed in the connection between the central balcony and the lateral battlements.

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Figure 3. Deterioration of the granite – details in the slab, pinnacles and columns

Figure 4. Detail of the detaching of pinnacles' structure from the battlement and view of the iron dowels on the back of the elements connecting the granite ashlars

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Generally, relative short radar profiles were obtained due to the small dimensions of the testing places. The balcony is composed of overlapped stone ashlars, fixed to the wall and supported, in the inferior part by stone ribs that seem to be connected with metallic dowels. The metallic connectors that may be seen in the surface of the balcony remained uncovered due to the corrosion leading to the disintegration of the stone covering layer. The objective was also to detect the steel connectors (visible or not) that existed in the elements. The lateral battlements were also investigated; see Figure 5 and Figure 6.

(a) (b)

Figure 5. Central balcony: (a) inferior ribs of the slab, (b) steel connectors in the balcony's slab

Figure 6. Left battlement of the facade

On the top surface of the balcony various radar longitudinal (parallel to the façade) and transversal profiles (perpendicular to the façade) were performed. It was observed that the ashlars are not connected to the wall by steel connectors. The unique steel connectors detected were the ones visible on the balcony's surface.

The most relevant results of the inspection may be seen in the Figure 7, where the radargram of the longitudinal profile of the slab is shown. In this profile, the ribs that support the base of the balcony are visible as reference hyperboles. The average propagation velocity was about 11.5 cm/ns, which is the usual value for the granite stone. No signs of cracking or any other deterioration were detected.

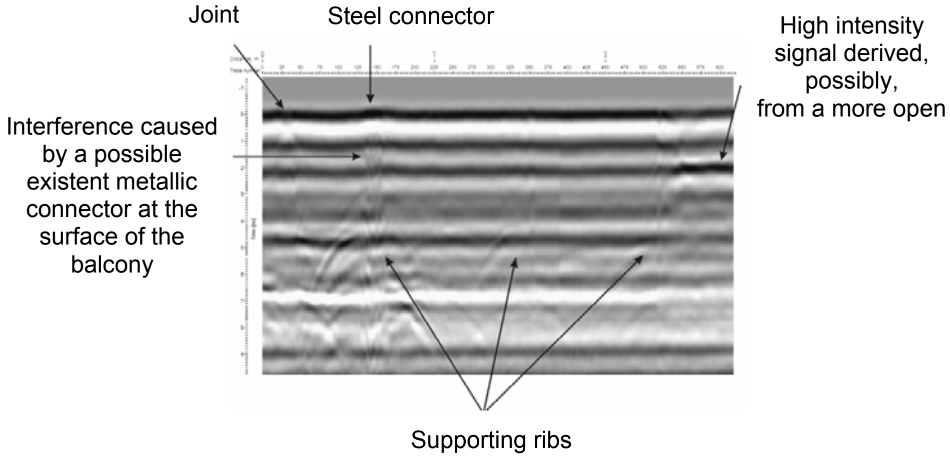

Figure 7. Longitudinal radar profile at 15 cm from the facade's wall

On the bottom surface of the balcony some transversal profiles over and between the ribs were performed. The results obtained in the profiles over the ribs are shown in Figures 8, 9. From both profiles, it is possible to observe the thickness of the ashlars and the position of the steel connectors on the opposite surface (that corresponds to the top surface of the balcony). It is also possible to verify the presence of steel connectors in a depth of around 3 to 6 cm from the top.

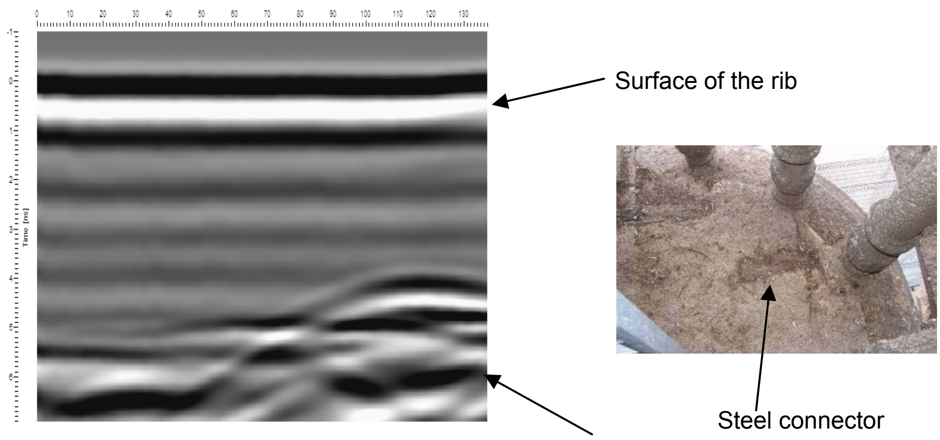

Figure 8. Radar profile on the left supporting rib

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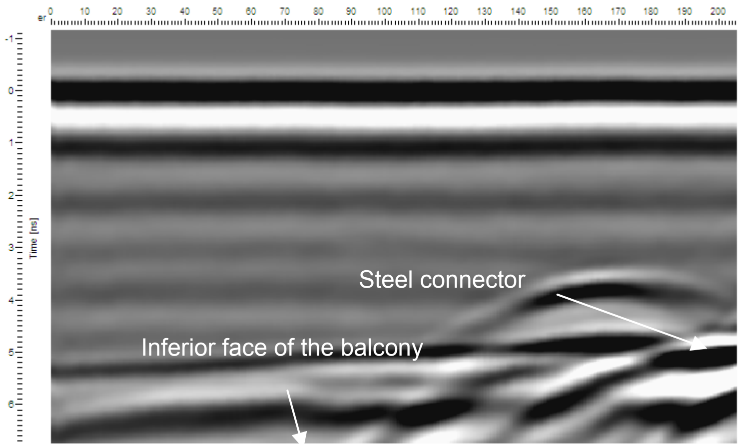

Figure 9. Radar profile of the middle supporting rib

In the left side battlement four tests (two horizontal and other two vertical) were performed. The existence of two steel salient elements that connect two granite independent ashlars made impossible the radar inspection in that respective influence area. Even so, neither deficiency nor steel connectors were detected in this zone, see Figures 10, 11.

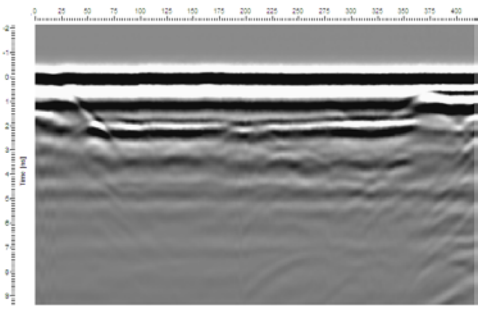

Figure 10. Horizontal radar profile

4. STRUCTURAL ANALYSIS OF THE TWO SALIENT ELEMENTS OF THE MAIN FAÇADE OF THE CATHEDRAL OF PORTO

In general, historic heritage buildings have complex geometrical shape, which is also the case of the two elements of the façade under analysis: the balcony and the pinnacles. As both structures are clearly three-dimensional and for better simulating their mechanical behaviour, solid elements were adopted for the

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calculations. Thus, finite quadrilateral (type CHx60, 20 nodes) and triangular (CTP45, 15 node) volume elements were used to create the finite element mesh, see Figure 12.

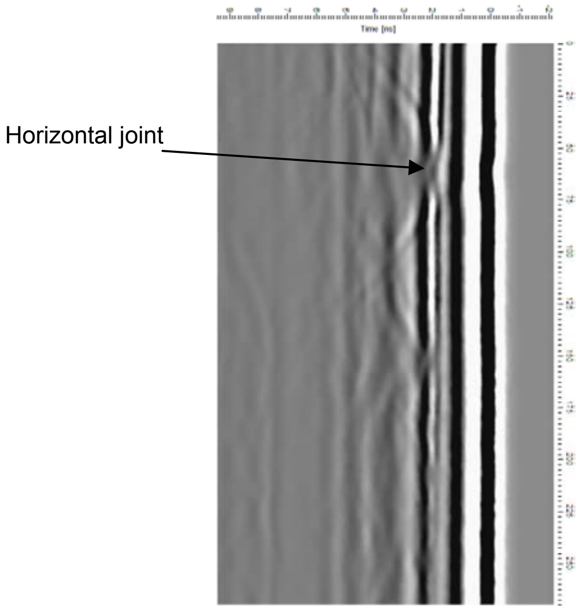
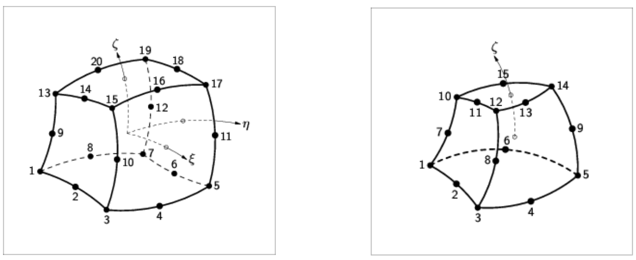


Figure 11. Vertical radar profile

The three dimensional geometrical models of the balcony and the pinnacles were developed in AUTOCAD. Coarse meshes composed of quadrilateral and prismatic shape solid bricks were defined. This procedure was followed so that refinement of the mesh in DIANA could be easier. The final mesh of the structures obtained in DIANA are indicated in Figure 13.



(a) *(b)*

Figure 12. CHX60 finite element – brick, 20 nodes and (b) CTP45 - wedge, 15 nodes

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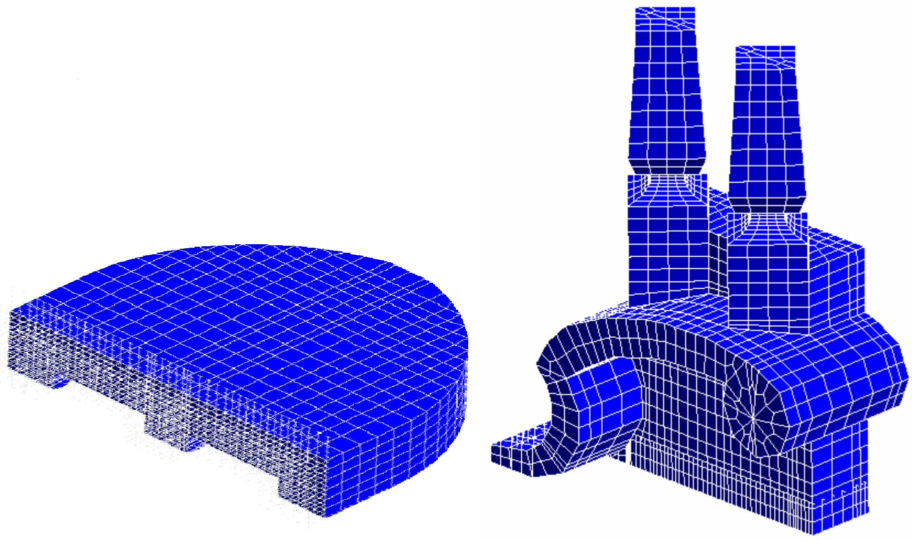

(a) (b)

Figure 13. Final mesh and the boundary conditions for (a) the balcony and (b) the pinnacles

The boundary conditions considered for the salient structural elements under analysis aim at simulating the real connection conditions of these elements to the façade. Thus, for the balcony, the constraints were defined in the surface corresponding to the edge connected to the wall. The constraint degrees of freedom were X, Y, Z translations, to describe the actual fixed connection, see Figure 13(a).

For the pinnacles, the constraints were defined at the basis of the column and also of the shell, considering the X, Y, Z translations restrained, see Figure 13 (b).

The material used in the construction of the cathedral and namely of the façade is a two mica medium grained granite. It is believed that it has a high weathering degree, taking into account its yellow colour and its visible superficial detachments.

Based on the simplified petrographical description of the granite used for construction, the mechanical properties adopted were similar to the properties found in Vasconcelos [2]. This author carried out an extended experimental characterization on distinct types of granite with different petrographical characteristics. The mechanical properties considered for weathered two mica medium grained granite are summarized in Table 1.

Table 1: Mechanical properties considered for the granite

f_c (N/mm^2)	E (N/mm^2)	f_t (N/mm^2)	G_f (N/mm)
60.40	15008	1.56	0.234

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The compressive strength, f_c, and Young modulus, E, were obtained from uniaxial compressive tests, whereas, the tensile strength, f_t, and fracture energy, G_f, resulted from direct tensile tests. The stress-displacement diagram exhibiting the tensile behaviour of the granite for different specimens is presented in Figure 14.

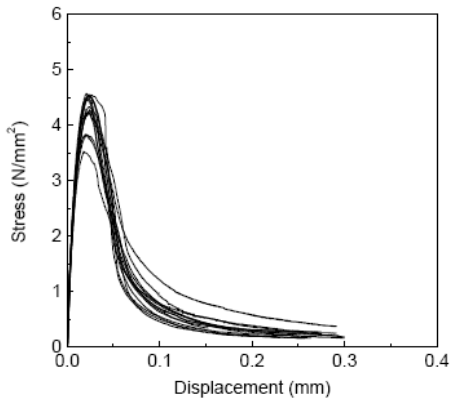

Figure 14. Stress-displacement diagrams of granite obtained in direct tensile tests, Vasconcelos [2]

Both elements, the balcony and the pinnacles, were subjected basically to their own weight.

For the Balcony, the weight was defined as an external load pressure acting on all upper horizontal surface. The loads were calculated by multiplying the volume of the slab and ribs by the dry density of the granite of about 2600 kg/m^3. The weight of the massive balustrade was also applied to the balcony, as element pressure loads in correspondence with the columns.

Due to its complex shape, the own weight of the pinnacles was calculated automatically by the program being given the information about the dry density of granite (2600 kg/m^3) and the gravity acceleration of 9.81 m/s^2.

4.1. –Linear analysis - isotropic material approach

In this section, an overview of the results obtained in the linear elastic calculations is given. This is important so that the initial condition of the structures and the understanding of the stress and strain field distributions and, thus, the detection of the possible critical points where maximum principal stresses occur (feasible cracked zones) can be identified. For the linear analysis only information about Young modulus and Poisson‘s ratio was required.

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The stress distribution on the Y global direction, the maximum principal strains, the minimum principal stresses on deformed mesh and the stress diagram for the linear elastic approach are displayed from Figure 15 to Figure 19.

From the normal stress distribution, it is observed that the maximum values appear at the support's section, where maximum bending moments occur. It is noted that in fact the structure behaves as a cantilever.

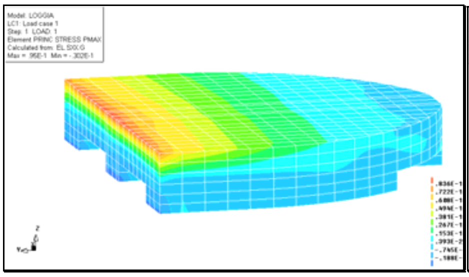

Figure 15. Syy stress distribution

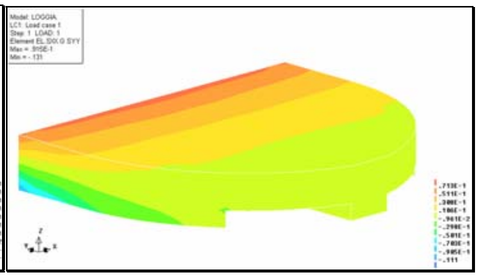

Figure 16. Maximum principal stresses

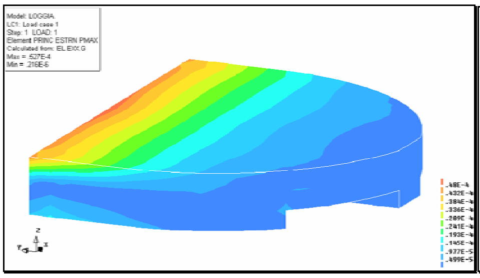

Figure 17. Maximum principal strains

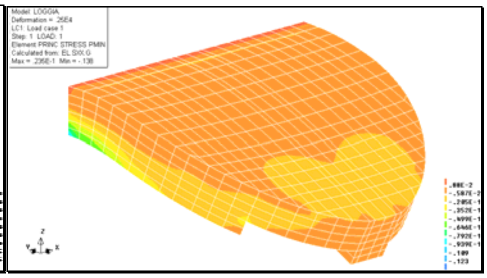

Figure 18. Minimum principal stresses

As expected, the normal stress diagram along a cross section defined at the support is linear, see Figure 19. Tensile stresses occur at the upper part of the section, whereas the maximum normal compressive stresses occur at the lower fibre.

From the distribution of the maximum and minimum principal stresses it can be seen that low values of both tensile and compressive stresses are present, when compared with the mechanical strengths.

Similarly to the Balcony, also for the pinnacles, the linear analysis is performed to emphasize the actual state of the salient elements of the cathedral's façade and also as a preliminary step for running the nonlinear analysis. The results, in terms of displacements, stresses and strains, can be seen from Figure 20 to Figure 24.

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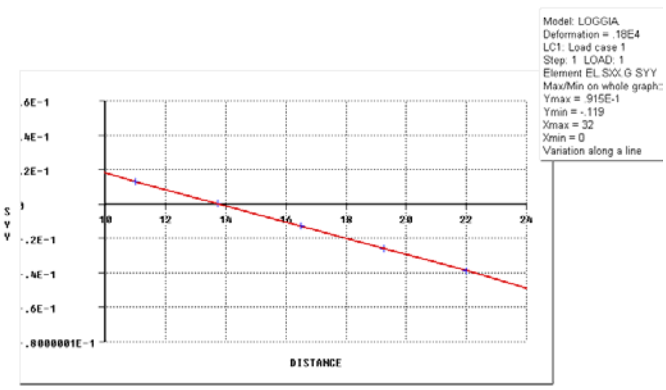


Figure 19. Stress diagram in the critical section

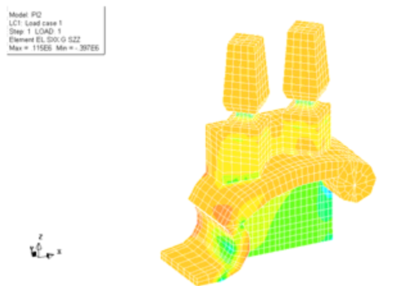


Figure 20. S_{zz} stress distribution

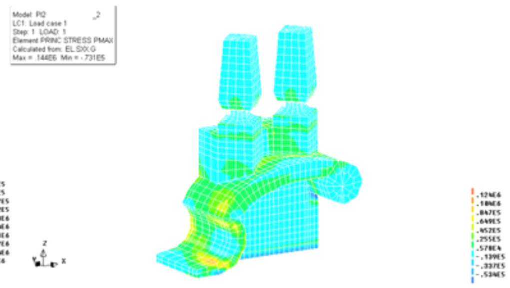


Figure 21. Maximum principal stresses

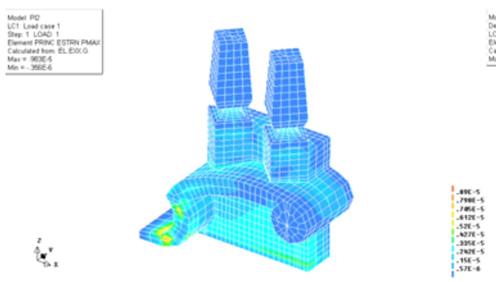


Figure 22. Maximum principal strains

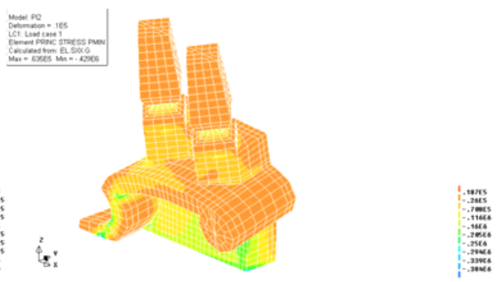


Figure 23. Minimum principal stresses

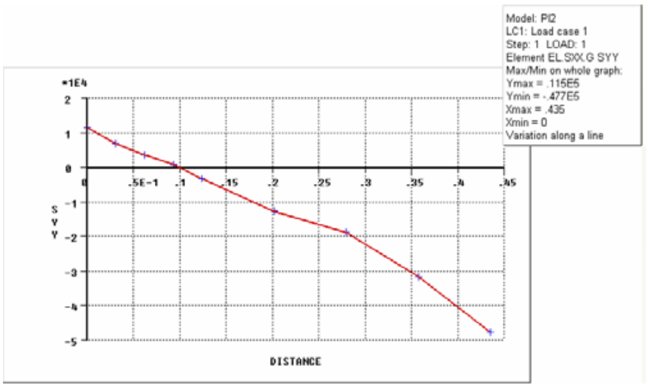
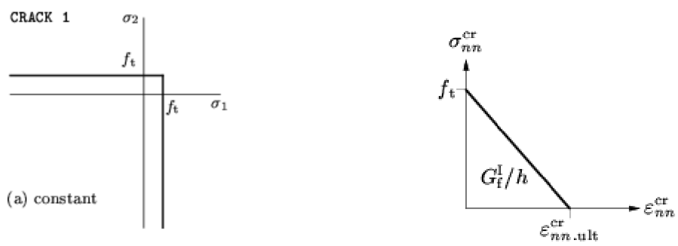


Figure 24. Stress diagram in a critical section

4.2. Non linear analysis

From the results of the linear analysis and based on the mechanical properties of granite, it is possible to observe that material nonlinearity should occur due to tensile cracking. In fact, similarly to concrete, the granite exhibits a much lower tensile strength than the compressive strength.

Therefore, the failure is expected to occur due to tensile stresses that might occur in the upper fibres of the balcony and in the shell belonging to the pinnacles substructure. Thus, for modelling the behaviour of the granite, a smeared cracking model was considered [3]. Cracking is specified as a combination of tension cut-off, tension softening and shear retention, [4]. The fundamental feature of the decomposed crack model is the decomposition of the total strain ε into an elastic strain ε_e and a crack strain ε_{cr}: $\varepsilon = \varepsilon_e + \varepsilon_{cr}$ [5]. Granite is modeled as a combination of constant tension cut-off criterion, linear tension softening and constant shear retention, see Figure 25. Mode I fracture energy is released in the element if the tensile stress is violated and the deformations localize in the element, depending the results on the mesh refinement [5].



Constant tension cut-off | Linear tension softening

Figure 25. Failure criteria granite used for nonlinear analysis of the structures

Balcony

The relation between the load factor and the maximum vertical displacement obtained at the border of the balcony is shown in Figure 26. The onset of the nonlinear behaviour occurs for a load factor of about 1.8, being the cracking localized in the upper edge of the support's section, see Figure 27 and Figure 28, where the principal stress and strain distributions associated to cracking are displayed. For the tensile strength of granite, the load factor is 4.8. This factor represents the scalar that multiplies the gravity load, indicating the safety level of the structure. Note also that the vertical displacement (2.5mm) is rather reduced. This state corresponds to tensile and compressive maximum stresses of respectively 2.79 and 14.6 MPa. It is important to stress that the compressive stress obtained in the nonlinear analysis is clearly below the compressive strength adopted for the granite, which means that the linear model under compression is valid, see Figure 29. Given the weathered state of the granite, an additional nonlinear analysis was performed considering a tensile strength of granite of about 0.5MPa and a fracture energy of about 0.078N/mm, (corresponding approximately to safety partial coefficient for the material γ_M of 3.0). As expected, the initial behaviour is similar but a considerably lower load factor of 1.8 is obtained, see Figure 26. This means that considering 1/3 of the tensile strength, a load factor 80% higher than the own weight is still achieved. For this situation, the maximum values of the tensile and compressive stress are respectively 0.85MPa and 5.33 MPa.

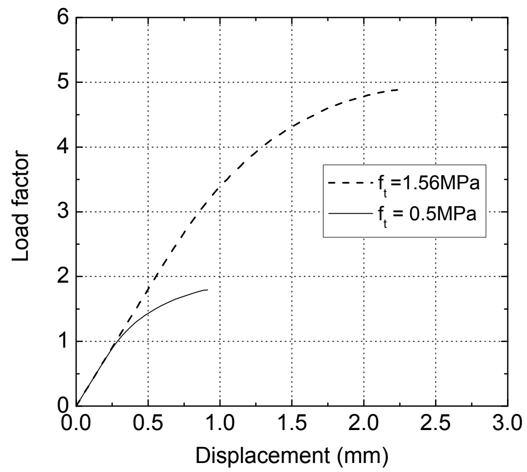


Figure 26. Relation between the load factor and the displacement at the free edge of the balcony

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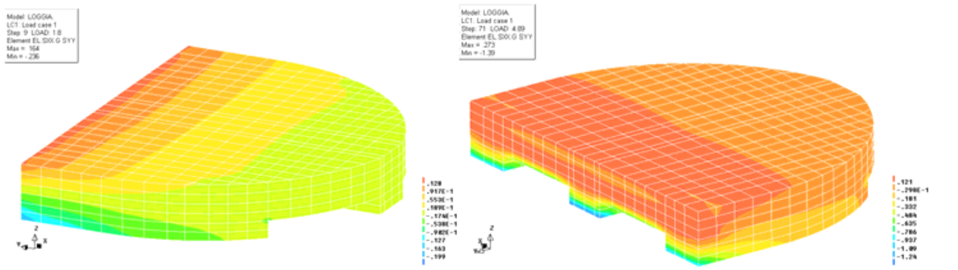

Figure 27. S_{yy} stress distribution for the first crack stage and for the failure stage of the balcony

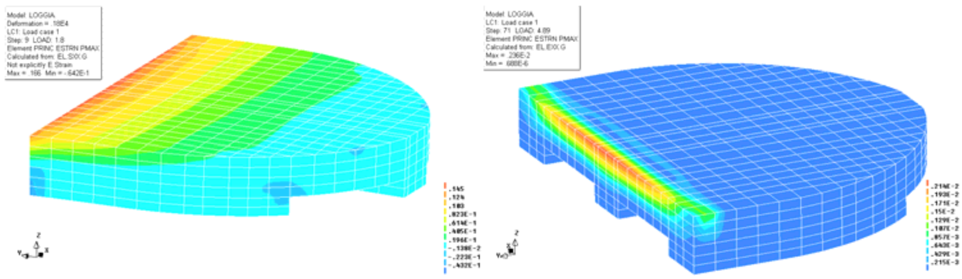

Figure 28. Maximum principal strains for the first crack stage and for the failure stage of the balcony

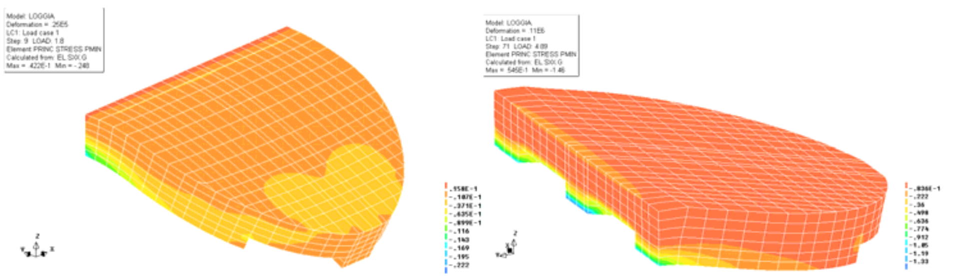

Figure 29. Minimum principal stresses and the deformed shape of the balcony for the first crack stage and for the failure stage

The normal stress distribution at the support's section that crosses the central rib corresponding to the collapse state obtained with f_t=1.56MPa is shown in Figure 30. It is observed that there is possibility of tensile stress transfer in the upper fibre.

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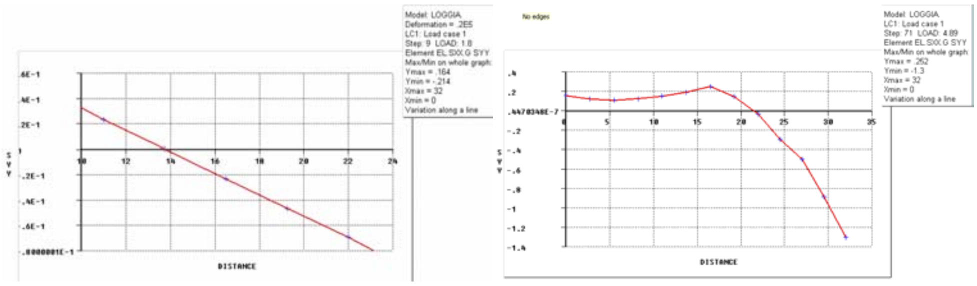
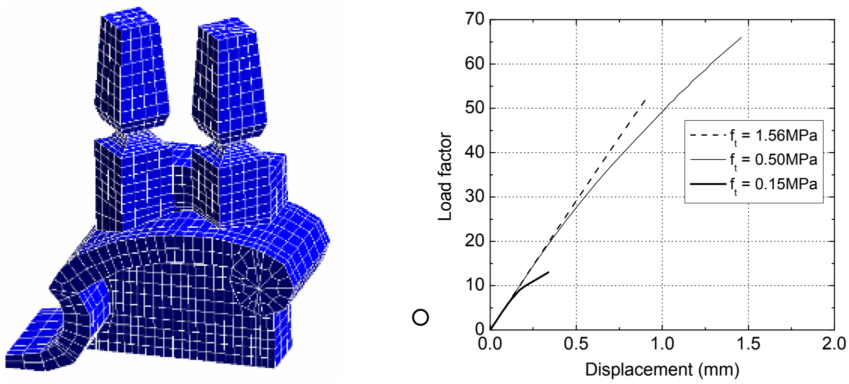


Figure 30: Stress diagram in the critical section for the first crack stage and for the failure stage for the balcony

Pinnacles

The relation obtained between the load factor and the control displacement based on the tensile strength of granite of 1.56MPa is displayed in Figure 31. An additional nonlinear analysis based on a tensile strength of 0.5MPa was also carried for the pinnacles. It can be seen that the response of the pinnacles exhibits reduced nonlinearity when the tensile strength of 1.56MPa is considered, see Figure 31. The high load factor found for both conditions results from the lower level of stresses associated to the own weight.



(a) (b)

Figure 31. – Results of the nonlinear analysis of the pinnacles; (a) vertical control displacement; (b) relation between the load factor and the displacement control

The nonlinear behaviour of the structure for a tensile strength of 0.5MPa is more evident, which is attributed to some cracking presented by the structure, see Figure 32a. Aiming at evaluating the collapse mode, a nonlinear analysis was carried out taking into account a tensile strength of 0.15MPa. It can be seen that the collapse of the structure occurs in the zone of separation between the support and the

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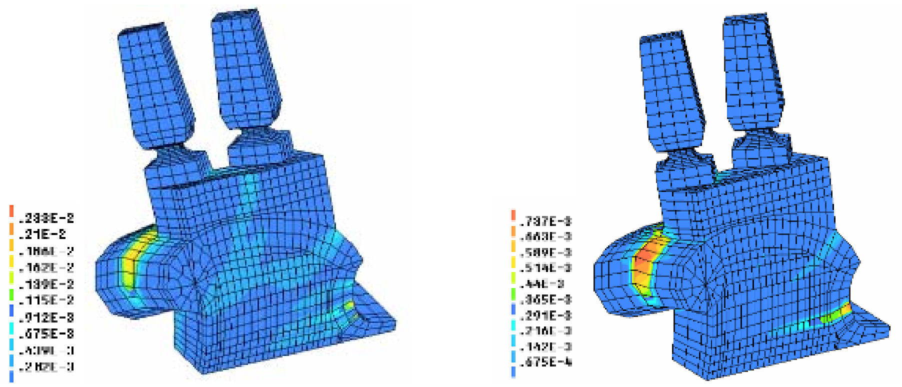
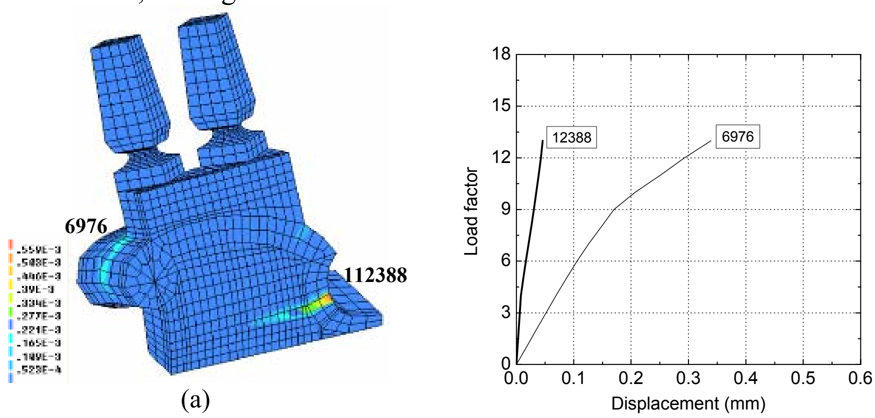
decoration of the pinnacles after the crack opening at the right lower corner of the support, see Figure 32b.



(a) (b)

Figure 32 – Plastic strains for the last load increment: (a) f_t=0.5MPa; (b) f_t=0.15MPa

In the collapse mode the cracking initiates in the right lower corner. As the load factor increases, the crack at the lower corner stabilizes and there is crack propagation between the support decoration of the pinnacles and the lower support. This collapse mechanism can also be observed through the comparison of the force-vertical displacement diagrams from different points located in the crack zones. The displacement of node 12388 exhibits a much more slow increase than the node 6976, see Figure 33.



(a) (b)

Figure 33 – Analysis of results of pinnacles for a tensile strength of 0.15MPa: (a) plastic strain distribution and control nodes (load factor 10) (b) comparison of force-displacement diagrams for the different cracked zones of the structure

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It is important to stress that even for a much low tensile strength, the load factor is considerably high (higher than 13.0). This allows concluding about the safety of the structure even if the granite presents high levels of deterioration.

5. CONCLUSIONS

The assessment of the stability conditions of the salient elements of the facade of the cathedral of Porto was based in two steps: (a) investigation on the presence of steel connector between stone ashlars and decorative elements and (b) numerical simulation of the balcony and pinnacles.

From the visual inspections it was seen the existence of steel connector between several elements of the façade, having some of them signs of corrosion. From radar inspection, it could be observed that is possible the existence of steel connector into the decorative elements.

The numerical simulation allowed verifying that the maximum tensile and compressive stresses are very low. The load factors resulting from the multiplication of the own weight of the structure are considerably high and in particular for the pinnacles, which guarantees their stability. It should be noticed that the adopted models are homogeneous and the boundary conditions are adequate.

Even so, considering the deep degradation of the granite, it is recommended a surface treatment of the stone, consisting in cleaning the superficial layer and water-proofing the granite surface, so that the polluting factors not to have access to the structure of the stone.

References

1. Lourenço, Paulo B., Ângela Melo, Mafalda Carneiro - "Remedial measures for the Cathedral of Porto: a post-modern conservation approach", Department of Civil Engineering, Universidade do Minho, Guimarães, Portugal, 2000
2. Vasconcelos, Graça – "Experimental Investigations on the mechanics of stone masonry: Characterization of granites and behaviour of ancient masonry shear walls", PhD. Thesis, Universidade do Minho, Guimarães, Portugal, 2005
3. Litton, R. W. - "A Contribution to the Analysis of Concrete Structures Under Cyclic Loading", PhD thesis, University of California, Berkeley, 1974
4. Rots, J. G. - "Computational Modelling of Concrete Fracture", PhD thesis, Delft University of Technology, 1988
5. DE Borst,R. - "Smeared cracking, plasticity, creep and thermal loading - a unified approach", Comp. Meth. Appl. Mech. Eng. 62 (1987), 89-110.

ISSN 1582-3024